Hedging the World:

Assessing the Performance of Dynamically Hedged Long-Short Equity Investments

by

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Abstract

The extraordinary conditions in the financial world of late 2008 caused severe market dislocations and consequently many asset managers experienced significant portfolio losses, partly due to ineffective hedging techniques. In order to examine the effect of the credit crisis on investment strategies, we create a diverse set of long-short equity portfolios with domestic equity sectors and an array of MSCI indices by extending Engle (2008). Each domestic sector is hedged against the others and the S&P 500, while the MSCI indices are hedged against the MSCI World index and S&P 500 over 1/1/02 – 2/27/09. Hedge ratios determined via GARCH-DCC and TGARCH-DCC are used for daily portfolio reallocation. We find that hedging in this method generally results in a significant volatility reduction and thus a benefit to the investor. Further inspection reveals that DCC may underestimate correlation during times of high volatility, a condition that is exacerbated for the international indices due to a non-synchronicity in information.

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Introduction

As 2006 came to a close, financial markets were in their prime: equity valuations had peaked, oil prices were poised to rise, commodities had experienced several years of growth and economies worldwide were at their apexes. Superficially, both qualitative and quantitative measures of analysis coincided and indicated a well-positioned global marketplace. However, unbeknownst to most, the early 2000s sowed the seeds for categorically extraordinary events that in summation caused in the globe's worst economic crisis since the Great Depression. As underwriters continued packaging and selling securities that consisted of mortgages on the brink of default, investment banks incessantly purchased said products resulting in bloated balance sheets chock-full of nothing. The financial implications of those investments, though, would not be realized until several years later.

On February 27th, 2007, the Shanghai Stock Exchange Composite Index fell 9% and sent shocks through markets as following an announcement from the government that the trading tax would be tripled. Termed the Chinese Correction, it represented the government's attempt to cool a market that more than doubled in the last year. Theoretically a tax hike should not significantly affect volatility since its paramount driver—the news—is unaffected. The devaluation was representative of and foreshadowed the market behavior currently seen throughout the world. The equity drop rippled through many economies, including the United States, where the S&P 500 lost 3.5%. When the dust settled in China and investors moved on, Bear Stearns announced in June it was bailing out two of its hedge funds that invested primarily

in subprime loans and CDOs.[2] In July, the firm informed investors the funds had filed for Chapter 15 bankruptcy protection due to losses exceeding 90%. This bankruptcy was effectively the first major casualty of the credit crisis. One month later, previously inconceivable 25-sigma equity market movements caused massive losses at several long-short market-neutral quantitative hedge funds. Khandani and Lo (2007) argued that funds at Goldman Sachs, Renaissance Technologies and Highbridge Capital saw substantial portfolio fluctuations during the early part of August, likely due to the failure of their quantitative models.[3] Even though 2007 contained several events of economic importance, their implications paled in comparison of those to come.

Shortly into 2008, Bear Stearns, the 123 year old investment bank and at the time 7th largest securities firm, announced it was being sold at a fire-sale price of $2 per share to J.P. Morgan due to a severe devaluation in its real estate positions. Although the firm was eventually sold for $10 per share, the sheer thought of a sale at such a price incited panic. As the housing crisis worsened, many feared a collapse was imminent. The ebb and flow of the equity market netted a gradual decline over the quarter, but triggered little of significance until that summer. Starting in July and continuing through September, the federal government placed several GSEs[4] such IndyMac, Fannie Mae, and Freddie Mac into conservatorships due to poor liquidity conditions that were exacerbated. Then, on the weekend of September 13th, two announcements rocked Wall Street to its core: Merrill Lynch was to be sold to Bank of America and Lehman Brothers had filed for bankruptcy protection. Both companies had essentially suffered from a rapid devaluation of their subprime investments due to the ongoing mortgage conditions. After

[2] A collateralized debt obligation (CDO) is an asset-backed security comprised of an underlying set of fixed income assets tranched by risk profile

[3] In their paper, "What Happened to the Quants in August 2007?" Khandani and Lo describe the financial world during the week of August 6, 2007 and discuss at length its effect on a number of quantitative hedge funds. They argue that a massive unwind at a large asset manager caused undue stress on a number of quantitative models, which resulted in equity market disruption.

[4] Government-sponsored enterprises are organizations Congress creates in order to take on the role of a financial institutions on behalf of the government

weeks of speculation and both companies' stocks approaching $0, two of the world's most storied investment banks virtually became extinct. The falls of these institutions were products of not only inefficient and cursory regulation, but also irrational investing.

As the housing sector continued its snowball contraction through the 4th quarter of 2008, contagion took effect as bets on every asset class faltered at staggering velocities. Equities, fixed income, commodities, and derivatives for both domestic and international markets suffered steep declines. The ensuing unprecedented volatility and gross uncertainty created a market that traded on unfounded rumors and baseless decisions. What resulted was a liquidity crisis, unrealistic $20 one-day movements in oil prices, and the CBOE VIX[5] trading at levels that implied 5% daily movements in the S&P 500. While some of that volatility has subsided, a significant portion remains. Immediately prior to the Bear Stearns debacle, annualized volatility for the S&P 500 was 10% to 20%. At its peak, between October and December, the same figure was upwards of 75%. Currently, it hovers between 30% and 40%, but is far from levels seen in late 2007.[6] Consequently, asset managers saw their returns vary dramatically.

This paper will explore how estimates for volatility and correlation affected those investment decisions by extending the hedging experiment of Engle (2008).[7] It will also determine whether a dynamically hedged portfolio could have averted those oscillations by analyzing a diverse set of long-short portfolios. Principally, we attempt to evaluate the performance of a set of dynamically hedged positions in comparison to an expanding window method, both of which are explained in the following section. In theory, if we estimate

[5] Chicago Board Options Exchange Volatility Index, which tracks implied volatility on the S&P 500 using weighted blend of prices for a range of options on the S&P 500.
[6] From Rob Engle's Vlab: http://www.vlab.stern.nyu.edu
[7] In "High Dimension Dynamic Correlations," Engle develops a new method for forecasting correlations and performs a hedging experiment to compare the results of various correlation estimation methods. Arrays of long-short portfolios are hedged dynamically with various forecasting models and the volatilities for each are averaged to determine which method performs best.

tomorrow's volatility and correlation and re-hedge an investment accordingly, returns for that investment should be less volatile. The paper aims to delve into that concept and examine the volatility implications of dynamic hedging on a variety of investments.

Method

Composition of a Long-Short Portfolio

In order to examine the implications of heightened volatility, we will analyze several long-short portfolios, or hedged investments, created by investing in a security and shorting a certain share of another such as the S&P 500. The portfolios are constructed from three inputs: log returns,[8] standard deviation, and correlation.

Standard deviation or σ is a measure of a dataset's variability and can be found like so:

$$(1) \qquad \sigma = \sqrt{\frac{\sum_{i=1}^{t}\left(r_{A,t} - \overline{r_A}\right)}{n-1}}$$

where

$r_{A,t}$ = log return of asset A at time t

$\overline{r_A}$ = average log return of asset A

Similarly, volatility or σ_T is standard deviation over a period:

$$(2) \qquad \sigma_T = \sigma\sqrt{T}$$

Both σ and σ_T attempt to quantify the variability in a data set, but the difference between the two is best illustrated with an example. Assume a return series with three-month duration. The standard deviation is found using (1), but what if instead one wants to find the annualized standard deviation? Over the course of a year, based on the three-month data, how much should one expect that return to vary? This is where volatility comes into play. Since there are 4 three-month periods in a year, (2) is used to compute an annualized figure.

[8] Log returns: $r_A = \log(A_t / A_{t-1})$, where A represents the price of asset A

Correlation, in contrast, measures how one security moves in relation to another, but does not take into account magnitude. It ranges from -1 to +1 with +1 representing perfect movement in the same direction and -1 being the opposite (this is in contrast to β which is theoretically any real number). Mathematically, it is as follows:

$$(3) \qquad \rho_{AB} = \frac{\operatorname{cov}(r_A, r_B)}{\sigma_{r_A}\sigma_{r_B}} = \frac{\sum_{i=1}^{t}\left[\left(r_{A,t} - \overline{r_A}\right)\left(r_{B,t} - \overline{r_B}\right)\right]}{\sigma_{r_A}\sigma_{r_B}}$$

Combining (2) and (3) yields β, which in formal terms represents a stock's movement in terms of both direction and magnitude against a security. Technically, any security can be used, but the S&P 500 is most common because it is widely considered the best representation of market performance. Quantitatively, it is a combination of the two securities' standard deviations and correlation. The formula that follows is for an investment in Stock A hedged with Stock B:

$$(4) \qquad \beta_{AB} = \frac{\sigma_{r_A}}{\sigma_{r_B}}\rho_{AB}$$

For the purposes of this paper, β is actually a hedge ratio, but the two terms will be used interchangeably and are assumed synonymous. Using (4) we construct a long-short portfolio:

$$(5) \qquad r_P = r_A - \beta_{AB} r_B = r_A - \left(\frac{\sigma_{r_A}}{\sigma_{r_B}}\rho_{AB}\right) r_B$$

where

r_P = Portfolio return
r_A = Log return on asset A
r_B = Log return on asset B
β_{AB} = Hedge ratio of asset A for B

Thus, we receive r_A less a proportion of r_B. If B is the S&P 500, our portfolio would yield Stock A less market fluctuations based on the hedge ratio. Using the S&P 500 as a hedge is not only a

long-short strategy, but also a market-neutral one since it hedges broader market movements. Therefore r_P is a function of a manager's stock picks rather than market gyrations. It is abundantly clear that accurately calculating the hedge ratio is vital to the success of a long-short strategy as it is the principal driver for r_P. Various econometric models exist to predict σ and ρ, but this paper focuses on GARCH-DCC and TGARCH-DCC.

GARCH-DCC and TGARCH-DCC

Two common econometric models are introduced to forecast volatility. GARCH, or generalized autoregressive conditional heteroscedasticity, models were proposed by Bollerslev (1986)[9] as an extension of Engle (1982)[10] as an alternate way to model volatility in the financial markets. Engle originally developed ARCH models for this purpose, whose basic principles stem from mean-reverting equations that predict the next period's variance. A formal ARCH(q) process:

(6) $$h_t = \omega + \sum_{j=1}^{q} \alpha_j r_{t-j}^2$$

where

ω = Long-run volatility
r_{t-1} = Return of previous period

and

(7) $$\alpha_j \geq 0 \forall\ j = 0,1,2,...,q$$

Thus, according to an ARCH model, variance is a function of long-run volatility and returns of the previous period(s). Two points of interest in the ARCH model are the methods by which to

[9] Bollerslev's 1986 paper generalized Engle's ARCH models by allowing for past conditional variances to affect the calculation for the current conditional variance
[10] Engle's seminal work that sparked a momentous interest in the study of econometric volatility modeling

determine lag and the constants. In his paper, Engle proposed the Lagrange multiplier test to specify the lag, which regresses the squared errors on a constant for q lagged values. For the constant, one maximizes the log likelihood function, which determines α_i in such a way that makes the observed sample most likely occur under a specified distribution. It follows that an ARCH(1) model is simply

(8) $$h_t = \omega + \alpha r_{t-1}^2$$

Bollerslev later added a term to the ARCH(q) process to include past conditional variances in the calculation for the next term's variance. His generalized form:

(9) $$h_t = \omega + \sum_{j=1}^{q} \alpha_j r_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$$

The q and p indicate the number of past observations (lags) used to predict h_t. A deeper analysis reveals that in actuality, it is simply another form of ARCH. This is best explained using a GARCH(1,1) model:

(10) $$h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1}$$

If we use this model to estimate h_{t-1}, the GARCH(1,1) model becomes

(11) $$h_{t-1} = \omega + \alpha r_{t-2}^2 + \beta h_{t-2}$$

Then, by combining (10) and (11), the following is obtained:

(12) $$h_t = \omega + \alpha r_{t-1}^2 + \beta\left(\omega + \alpha r_{t-2}^2 + \beta h_{t-2}\right) = \omega\left(1+\beta\right) + \alpha r_{t-1}^2 + \alpha\beta r_{t-2}^2 + \beta^2 h_{t-2}$$

Continuing this process for an infinite number of lags, we observe the last term is approximately 0 since β^t eventually reaches 0. The remaining terms, ω and r_{t-j}^2, are gathered like so:

(13) $$h_t = \frac{\omega}{1-\beta} + \alpha \sum_{j=1}^{t} \beta^{j-1} r_{t-j}^2$$

Thus GARCH is simply a special situation of ARCH since the estimated variance from the previous term vanishes; they do, however, differ in their α-selection methods for r_{t-j}^2. GARCH weighs them in a decreasing geometric fashion, thereby giving recent observations relatively substantial influence over the current period's estimate. Both ARCH and GARCH models attempt to predict volatility, but the latter is more common due to its inclusion of the variance-dependent term (or weighing schema). Threshold GARCH, or TGARCH, of Zakoin (1994) is one of the many extensions of GARCH that incorporates the asymmetric effect of stock returns. It has been well documented that negative news affects stock prices (and thus volatility) with a greater magnitude than an equally positive result. This had been discussed at length by Christie (1982), Nelson (1991), and Schwert (1989) when Zakoian proposed adding yet another term to the GARCH model that allowed for asymmetric effects:

(14) $$h_t = \omega + \alpha r_{t-1}^2 + \gamma d_{t-1} r_{t-1}^2 + \beta h_{t-1}, d_{t-1} = I_{r_{t-1}<0}$$

We see that when the previous day's return is negative, d_{t-1} is unity (otherwise zero), thereby allowing for asymmetric effects. Oftentimes, α will be close to zero, implying that positive returns have little effect on volatility, whereas γ will be much greater, implying the converse.

Correlation is predicted using a DCC model, as developed by Engle (2002). DCC is in some ways an adaptation of the GARCH model, with a few key subtle differences. There are 3 steps to the DCC model: 1) estimate volatility-adjusted returns;[11] 2) dynamically estimate quasi-correlations; 3) rescale the quasi-correlations.

The volatility-adjusted returns are formed by estimating volatilities for each asset using a GARCH or TGARCH model described in (9) or (14), respectively. This is often termed "DE-GARCHING" the data because the residual series should be ~N(0,1). The empirical distribution

[11] Also known as standardized residuals, calculated via $\varepsilon_t = y_t / \sqrt{h_t}$

will not be exactly Gaussian, and depending on the series may be either leptokurtic or platykurtic. We can then estimate quasi-correlations using the standardized series. Several estimation methods exist, but only the mean-reverting model is discussed and its generalized specification follows:

(15) $$Q_t = \Omega + \alpha \varepsilon_{t-1} \varepsilon'_{t-1} + \beta Q_{t-1}$$

There are clear analogs to the GARCH model, and it functions similarly as well. The intercept of the formula, or the long run correlation Ω, can be estimated by correlation targeting, which effectively amounts to using an estimator for Ω

(16) $$\hat{\Omega} = (1 - \alpha - \beta)\overline{R}$$

where

(17) $$\overline{R} \equiv \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \, \varepsilon'_t$$

Through substitution of (16) into (15) we arrive at the mean-revering model, which has only 2 parameters instead of the original 3, thus making it more parsimonious:

(18) $$Q_t = \overline{R} + \alpha(\varepsilon_{t-1} \varepsilon'_{t-1} - \overline{R}) + \beta(Q_{t-1} - \overline{R})$$

Finally, we rescale the quasi-correlation matrix, Q_t. This is necessary because the diagonal elements of a true correlation matrix should be unity because an asset is perfectly correlated with itself; however, in the quasi-correlation matrix, they are not. On average, the diagonal elements will be 1, even though individual observations may not. To remedy this and obtain the true estimator of our correlation matrix R_t, the following process is employed:

(19) $$R_t = diag\{Q_t\}^{-1/2} Q_t \, diag\{Q_t\}^{-1/2}$$

And thus we have estimates for each asset's correlation for use in portfolio analysis. GARCH and TGARCH are coupled with DCC to form the two dynamic methods for predicting volatility

and correlation: GARCH-DCC and TGARCH-DCC. In summation, after estimating volatility and correlation daily for each asset pair, we can construct a series of portfolios to examine the effects of hedging across different markets.

Breadth of Data Set

The data in this paper encompasses worldwide equity returns from 1/1/02 to 2/28/09, obtained via Rob Engle's Vlab.[12] Domestic Equities incorporate the S&P 500 and the following SPDR ETFs: Consumer Staples (henceforth Staples), Consumer Discretionary (henceforth Discretionary) Energy, Financials, Healthcare, Industrials, Materials, Technology and Utilities. International Equities are comprised of the S&P 500 and the following MSCI indices: Australia, Belgium, BRIC, Canada, EAFE, Emerging Markets, Germany, Hong Kong, Italy, Japan, Mexico, Netherlands, Singapore, Sweden, Switzerland, Spain, UK, and World.

Explanation of Analysis Outputs

The model reports standard deviation, beta, and kurtosis for each investment. Standard deviation is calculated for both the dynamic and expanding window hedge and compared for a given interval to determine which method better reduced volatility. Average beta can provide insight as to how the models compute their hedge ratios. Kurtosis quantifies the degree of "peakedness" in the data, where a Gaussian distribution has a kurtosis of 3. A low kurtosis signifies data are clustered toward the mean, whereas a high kurtosis implies more outliers, also known as fat tails. Mathematically, kurtosis is the fourth standardized moment about the mean

$$(20) \qquad \alpha_4 = \frac{\mu_4}{\sigma^4}$$

[12] http://vlab.stern.nyu.edu

A moment is defined as per the following, where X is a random variable:

$$(21) \qquad \mu_k = E\left[\left(X - E\left[X\right]\right)^k\right] = \int_{-\infty}^{+\infty} \left(x-\mu\right)^k \, f(x)dx$$

Thus the general form for kurtosis in (19) translates into

$$(22) \qquad \alpha_4 = \frac{\mu_4}{\sigma^4} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \mu}{\sigma}\right)^4$$

Most financial distributions are leptokurtic (kurtosis > 3) because large shocks tend to occur more frequently than in the normal (platykurtic distributions, in contrast, have kurtosis < 3). Those shocks are exactly what econometric models attempt to capture in their predictions.

Construct of Analysis Model

The model allows for five inputs: investment security, security to hedge against, start date, end date, and comparison date. There are two separate analyses for each security combination denoted as the dynamically hedged portfolio, whose parameters are calculated conditionally, and the expanding window portfolio whose parameters are calculated unconditionally. The major difference between the two is the method by which the parameters—standard deviation and correlation—are computed. When we say the parameters are determined conditionally (or dynamically), we mean an econometric model such as GARCH-DCC or TGARCH-DCC is used to estimate values. In contrast, unconditional metrics weigh each historical observation equally, thereby producing a different result than predictive models. Since we re-hedge daily, the expanding window utilizes more and more data points with each passing day. Effectively it merely summarizes instead of predicts. While this hedging method is not particularly advanced, it is still a relatively robust technique compared to a hedge updated weekly or monthly. In summary, the dynamic hedge (under GARCH-DCC and TGARCH-DCC)

attempt to predict volatility and correlation, while the expanding window is an amalgam of the entire history. Using those quantities, the model cycles through all portfolio combinations for the data sets and provides results for each. For example, we can analyze the volatility effect of hedging MSCI Japan with either the S&P 500 or the MSCI World Index to see how both hedges would have performed. In this manner, we can observe the impact of dynamic hedging on many portfolios and quantify the benefit for several assets.

Data Analysis

Overview

This analysis provides performance metrics for three estimation methods for an array of equities. In essence, we quantify the absolute and relative (each dynamic model as compared to expanding window) reduction in average standard deviation for a certain hedging combination. This reduction is compared across several periods to assess how GARCH-DCC and TGARCH-DCC perform during times of high and low volatility. In order to test the statistical significance of this benefit, a Diebold-Mariano test is employed whereby the difference in squared returns is regressed on a constant:

(22) $$\pi_t^2 - \varphi_t^2 = c + \varepsilon$$

with

H_0: $c = 0$
H_A: $c < 0$

where

π_t = return on dynamically hedged portfolio at t
φ_t = return on expanding window hedged portfolio at t

In this test, we determine whether the difference in squared returns between the two methods is significantly less than 0 using heteroskedasticity- and autocorrelation-consistent (HAC) standard errors. Ideally the regression coefficient, or c, is negative for all portfolios, which implies the dynamically hedged portfolio had less volatility than the expanding window. The remaining regression outputs (standard error, t-statistic, and p-value) also aid in evaluating significance. HAC standard errors function as robust estimators when data are dependent and not iid. Since predictions of σ and ρ are based on historical information, the r_P series is not iid. A HAC

standard error accounts for this and in a sense reduces the magnitude of the regression's t-statistic. In theory, if data are dependent and iid, they should vary less. Thus, the p-value of a HAC standard error is in a sense more stringent and better assesses the significance of the volatility reduction.

Domestic Equities

Domestic sectors and the S&P 500 were hedged against one another from 1/1/02 – 2/28/09. Standard deviation, β, and kurtosis of each investment are computed for both the dynamic and expanding window, with the data split at 9/1/08 signified by Pre and Post. The split date was chosen to show the effect of late 2008's turmoil after the financial system's large-scale collapse. According to many estimates, this collapse began during the later part of 2008, close to September.

Table 1: Select GARCH-DCC Hedges

Summary Analysis for Equity Sectors Hedged With S&P 500, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Cons. Discretionary	0.59%	1.49%	0.59%	1.46%	0.61	0.96	0.59	0.98	6.66	5.44	6.97	5.89
Consumer Staples	0.54%	1.12%	0.55%	1.09%	0.34	0.45	0.31	0.51	6.62	3.55	6.37	2.98
Energy	1.23%	2.34%	1.29%	2.76%	0.59	0.95	0.33	0.82	4.06	4.08	3.86	4.30
Financials	0.67%	2.91%	0.75%	3.22%	0.68	1.75	0.65	1.23	15.01	5.76	26.77	4.48
Healthcare	0.61%	1.32%	0.61%	1.51%	0.49	0.52	0.50	0.74	6.55	4.35	5.99	4.08
Industrials	0.50%	1.23%	0.50%	1.25%	0.59	0.87	0.56	0.91	5.24	3.99	5.35	5.17
Materials	0.83%	1.87%	0.88%	1.80%	0.68	0.86	0.45	0.86	5.46	3.29	5.14	3.45
Technology	0.71%	1.14%	0.80%	1.67%	0.72	0.90	0.92	1.28	7.45	4.88	6.29	5.28
Utilities	0.90%	1.57%	0.93%	1.60%	0.41	0.55	0.33	0.62	10.75	4.49	12.01	4.44
Average	**0.71%**	**1.68%**	**0.75%**	**1.84%**	**0.59**	**0.91**	**0.54**	**0.92**	**7.13**	**4.42**	**8.34**	**4.46**

Summary Analysis for Equity Sectors Hedged With Consumer Staples, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.71%	1.98%	0.77%	2.44%	0.53	1.31	0.34	0.74	6.41	3.14	6.08	3.64
Cons. Discretionary	0.91%	2.20%	0.97%	2.53%	0.55	1.35	0.37	0.78	6.12	4.44	5.66	3.92
Energy	1.42%	3.76%	1.45%	4.28%	0.45	1.04	0.25	0.62	4.14	3.71	3.99	4.12
Financials	1.14%	4.60%	1.21%	5.14%	0.61	2.26	0.41	0.96	11.35	3.74	12.67	3.52
Healthcare	0.75%	1.67%	0.80%	1.83%	0.45	0.77	0.27	0.55	6.70	6.75	6.87	9.94
Industrials	0.86%	2.06%	0.91%	2.30%	0.51	1.15	0.32	0.68	6.36	3.52	6.00	3.63
Materials	1.12%	2.81%	1.17%	3.03%	0.58	1.02	0.32	0.70	6.03	3.79	5.57	4.25
Technology	1.23%	2.14%	1.32%	2.59%	0.50	1.18	0.23	0.60	7.86	3.33	7.17	4.19
Utilities	1.01%	1.89%	1.05%	2.24%	0.40	0.82	0.27	0.58	13.63	5.63	12.39	5.95
Average	**1.06%**	**2.75%**	**1.12%**	**3.10%**	**0.52**	**1.25**	**0.31**	**0.70**	**6.94**	**4.18**	**6.85**	**4.79**

Table 2: Select TGARCH-DCC Hedges

Summary Analysis for Equity Sectors Hedged With S&P 500, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Cons. Discretionary	0.60%	1.38%	0.59%	1.46%	0.61	0.98	0.59	0.98	6.41	4.85	6.97	5.89
Consumer Staples	0.55%	1.13%	0.55%	1.09%	0.35	0.45	0.31	0.51	6.72	3.61	6.37	2.98
Energy	1.23%	2.49%	1.29%	2.76%	0.57	0.94	0.33	0.82	4.01	4.51	3.86	4.30
Financials	0.67%	2.78%	0.75%	3.22%	0.68	1.77	0.65	1.23	16.17	5.43	26.77	4.48
Healthcare	0.59%	1.38%	0.61%	1.51%	0.48	0.50	0.50	0.74	6.10	5.28	5.99	4.08
Industrials	0.51%	1.23%	0.50%	1.25%	0.60	0.87	0.56	0.91	5.24	4.26	5.35	5.17
Materials	0.83%	1.87%	0.88%	1.80%	0.66	0.83	0.45	0.86	5.44	4.13	5.14	3.45
Technology	0.70%	1.18%	0.80%	1.67%	0.74	0.88	0.92	1.28	7.47	5.34	6.29	5.28
Utilities	0.90%	1.64%	0.93%	1.60%	0.41	0.52	0.33	0.62	10.74	4.87	12.01	4.44
Average	**0.71%**	**1.68%**	**0.75%**	**1.84%**	**0.58**	**0.90**	**0.54**	**0.92**	**7.19**	**4.68**	**8.34**	**4.46**

Summary Analysis for Equity Sectors Hedged With Technology, 1/1/02 - 2/28/09, Split at 9/1/08

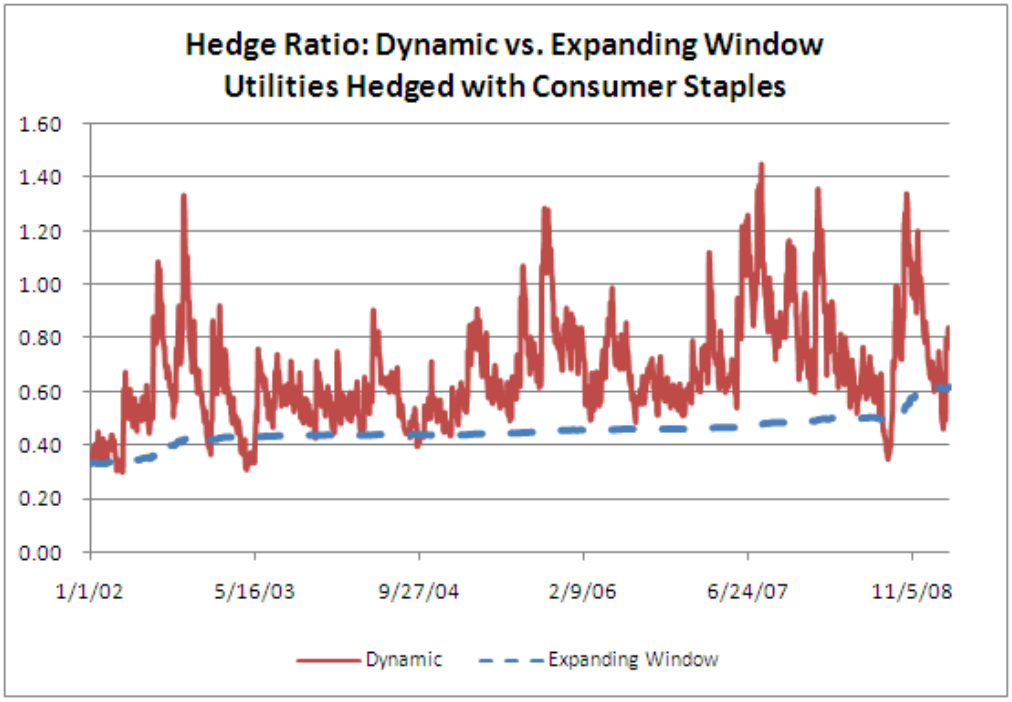
	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.51%	1.16%	0.57%	1.83%	0.39	0.92	0.29	0.55	7.86	5.15	6.07	4.09
Cons. Discretionary	0.80%	1.87%	0.89%	2.30%	0.38	0.88	0.24	0.49	5.48	4.13	5.50	3.58
Consumer Staples	0.66%	1.23%	0.73%	1.53%	0.18	0.40	0.07	0.18	6.79	4.26	6.29	4.40
Energy	1.38%	3.12%	1.47%	4.26%	0.28	0.78	0.09	0.31	3.92	5.55	3.97	4.67
Financials	1.04%	3.60%	1.17%	4.84%	0.40	1.51	0.25	0.56	14.79	4.83	14.65	3.55
Healthcare	0.77%	1.63%	0.77%	1.72%	0.28	0.45	0.22	0.39	9.62	5.72	7.52	8.12
Industrials	0.73%	1.56%	0.79%	1.91%	0.37	0.79	0.25	0.48	5.24	3.38	5.20	3.23
Materials	1.02%	2.17%	1.15%	2.82%	0.38	0.73	0.16	0.39	5.47	4.57	5.07	4.35
Utilities	1.01%	1.79%	1.08%	2.34%	0.23	0.46	0.11	0.27	10.67	4.94	11.35	5.89
Average	**0.92%**	**2.17%**	**1.00%**	**2.77%**	**0.32**	**0.79**	**0.18**	**0.40**	**7.33**	**4.63**	**6.89**	**4.56**

Comparing the dynamic and expanding window standard deviation for each investment reveals reductions in almost every scenario, with some exceeding 25%. Unsurprisingly, volatility was dramatically higher in the crisis (post) period, by 100% to 200%. Generally, even though conditions severely worsened, dynamic hedging demonstrated a clear volatility reduction through most situations. While is it not yet evident, it is worth nothing that the statistical

significance of the volatility reductions under TGARCH-DCC are stronger than that of the GARCH-DCC (Appendices A and B shows the outcomes of hedging with the remaining SPDRs for GARCH-DCC and TGARCH-DCC, respectively).

The hedge ratio provides an average measure of how each model computes r_P. In almost all situations, the dynamic hedge had a higher hedge ratio than the expanding window; however, a time series plot reveals a deeper relationship:

Chart 1: GARCH-DCC Hedge



(A plot of the same hedge with TGARCH-DCC would not have been meaningfully different) Interestingly, the dynamic hedge ratio's volatility is vastly greater than expanding window's smoothness. This is likely due to large one-day returns having a greater impact on forecasts, compared to a miniscule effect for the expanding window. The chart illustrates movements of more than 20% fairly quickly; however, the dynamic is still almost always greater than the expanding window, which is fairly common in most other combinations (Appendix C).

Counter-intuitively, kurtosis actually dropped during the crisis irrespective of hedge method. The fact that we are hedging does not necessarily influence this outcome as the kurtosis for the log returns of the S&P 500 (un-hedged) dropped from 14.3 pre- to 4.0 post- the 9/1/2008 split. Although the number and frequency of 4- and 5-sigma events certainly increased during the crisis, the kurtosis suggests that on a standardized basis, these events were not rare. While this is a retrospective commentary, it serves to illustrate that high-sigma market movements during the credit crunch were actually the norm and thus—in some sense—should have been priced into financial models.

The statistical significance of these results, determined via the Diebold-Mariano test, generally proved the advantages of dynamic hedging:

Table 3: GARCH-DCC Domestic Equities Hedges

Number of Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	81	69	63	50	90
% total	90%	77%	70%	56%	
1/1/02 - 8/31/08	79	70	70	57	90
% total	88%	78%	78%	63%	
9/1/08 - 2/27/09	67	50	41	31	90
% total	74%	56%	46%	34%	
Total	227	189	174	138	270
% total	84%	70%	64%	51%	

Table 4: TGARCH-DCC Domestic Equities Hedges

Number of Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	85	73	70	58	90
% total	94%	81%	78%	64%	
1/1/02 - 8/31/08	83	76	74	61	90
% total	92%	84%	82%	68%	
9/1/08 - 2/27/09	74	53	47	34	90
% total	82%	59%	52%	38%	
Total	242	202	191	153	270
% total	90%	75%	71%	57%	

We see from the Diebold-Mariano results (Appendix E) that Consumer Staples, Healthcare, and Technology were generally best to hedge with, while investments in the S&P

500, Energy, and Financials were most easily "hedge-able." It seems that industries with low volatilities provided the best hedge, whereas indices with high volatilities saw the most significant reductions in volatility, potentially because they had the most "room" for improvement. There is also a drastic decline in performance for the crisis period, which may be a function of both relatively few data points and the unprecedented market movements during that time, but most likely the latter.

In order to determine why some hedges performed well while others did not, we compare the percent difference in average hedge ratio for the dynamic and expanding window for each investment (for example, what is the percent difference between the dynamic beta and expanding window beta for each industry hedged with the S&P 500?). Juxtaposed with the Diebold-Mariano summaries, we find the percent difference between the two hedge ratios is far greater for hedges with statistically significant test coefficients. In other words, using the S&P 500 hedge under TGARCH-DCC for the pre-crisis interval as an example, observe that 6 hedges showed a significantly lower volatility through dynamic hedging, and 3 did not. The dynamic hedge ratios are 21.7% higher than the expanding window for the former group, but only 7.6% higher for the latter. This is logical because the more similar the two hedge methods are, the less benefit we expect to find with dynamic hedging. Potentially, it may imply that DCC is under-hedging its investments relative to the expanding window. In order to test this hypothesis, we artificially increase correlations for a select group of investments and repeat the Diebold-Mariano test. Increasing correlations by 1% – 3% improves coefficients slightly in some cases, but not to a statistically significant degree (Appendix F). When we exclude (1/1/02 – 8/31/08) or isolate (9/1/08 – 2/27/2009) the crisis we find the correlation threshold either increases, or does not exist, meaning test coefficients were improving even at 5% increases. While this provides

only slight evidence for the under-hedging hypothesis, there are no statistically significant results and thus we cannot form a conclusion. It would be interesting to determine whether the poor performance of the models can be attributed to times of either high volatility or changing volatility. One could examine this by introducing an exogenous variable into the DCC model that accounts for the difference in the last two periods' volatility estimates. Perhaps positive differences ($h_{t-1} > h_{t-2}$) have a more significant impact than negative differences on the estimate of Q_t. It could be that during times of increasing volatility, correlation should have been higher than DCC estimated, and the converse.

International Equities

The international equities data consisted of the MSCI Indices hedged against both the S&P 500 and MSCI World from 1/1/02 – 2/227/09:

Table 5: GARCH-DCC Hedges

Summary Analysis for International Equities Hedged With S&P 500, 1/1/02 - 2/27/09; Split 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Fixed Window		Dynamic		Fixed Window		Dynamic		Fixed Window	
MSCI Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Australia	1.10%	2.73%	1.14%	2.94%	0.53	1.04	0.32	0.88	4.59	4.28	4.53	4.12
Belgium	1.17%	2.06%	1.19%	2.11%	0.49	0.85	0.37	0.84	7.76	4.13	8.15	4.09
BRIC	1.22%	3.47%	1.21%	3.53%	0.43	0.62	0.20	0.51	6.41	4.47	6.56	4.04
Canada	1.02%	2.19%	1.01%	2.23%	0.50	0.79	0.34	0.74	4.72	3.63	4.68	3.46
EAFE	0.64%	1.28%	0.65%	1.41%	0.58	0.98	0.49	0.94	4.98	3.30	4.72	3.68
Emerging M	1.01%	2.97%	1.01%	2.97%	0.25	0.38	0.17	0.38	6.37	4.33	6.38	4.19
Germany	0.94%	1.85%	0.96%	1.76%	0.70	0.99	0.68	1.09	5.59	5.06	5.18	3.77
Hong Kong	1.11%	2.14%	1.14%	2.09%	0.70	1.08	0.54	1.09	5.21	3.51	5.43	3.71
Italy	0.96%	1.91%	0.96%	1.92%	0.52	0.88	0.43	0.84	5.92	3.34	5.64	3.22
Japan	1.04%	1.63%	1.07%	1.66%	0.62	0.75	0.45	0.84	4.73	5.79	4.68	4.47
Mexico	1.18%	2.28%	1.20%	2.05%	0.59	0.97	0.53	1.08	4.21	6.86	4.30	4.75
Netherlands	1.03%	1.78%	1.03%	1.74%	0.60	0.95	0.55	0.98	6.37	4.05	6.35	3.75
Singapore	1.25%	2.32%	1.29%	2.32%	0.68	0.98	0.46	0.97	4.62	3.41	4.63	3.63
Spain	1.02%	1.91%	1.03%	1.94%	0.57	0.96	0.43	0.89	7.18	3.29	7.21	3.35
Sweden	1.33%	2.43%	1.33%	2.40%	0.70	1.19	0.62	1.13	5.60	2.68	5.53	2.64
Switzerland	1.02%	1.56%	1.02%	1.47%	0.48	0.69	0.42	0.76	5.03	2.98	4.98	2.85
UK	0.85%	1.82%	0.86%	1.91%	0.55	0.99	0.46	0.93	5.98	2.94	5.47	3.21
World	0.42%	1.35%	0.41%	1.31%	0.44	0.70	0.45	0.72	6.98	4.38	6.78	4.37
Average	**1.02%**	**2.09%**	**1.03%**	**2.10%**	**0.55**	**0.88**	**0.44**	**0.87**	**5.68**	**4.02**	**5.62**	**3.74**

Summary Analysis for International Equities Hedged With MSCI World, 1/1/02 - 2/27/09; Split 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Fixed Window		Dynamic		Fixed Window		Dynamic		Fixed Window	
MSCI Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Australia	1.01%	2.94%	1.08%	3.04%	0.71	1.36	0.42	1.14	4.84	3.79	4.58	3.54
Belgium	1.08%	2.30%	1.09%	2.27%	0.67	1.10	0.57	1.11	7.34	4.67	8.00	4.90
BRIC	1.04%	2.76%	1.07%	2.63%	0.45	0.75	0.37	0.97	5.42	4.40	5.60	4.21
Canada	0.94%	2.10%	0.94%	2.07%	0.67	1.04	0.47	0.98	4.47	3.61	4.55	3.24
EAFE	0.51%	1.86%	0.51%	1.82%	0.75	1.23	0.66	1.19	6.53	4.88	6.01	5.10
Emerging M	0.83%	2.18%	0.86%	2.18%	0.53	0.81	0.33	0.80	5.11	4.60	5.82	4.44
Germany	0.84%	2.26%	0.84%	2.17%	0.90	1.25	0.90	1.38	6.34	7.55	6.31	6.20
Hong Kong	1.18%	3.15%	1.20%	3.02%	0.78	1.18	0.62	1.20	4.88	3.30	5.06	3.41
Italy	0.90%	1.90%	0.89%	1.83%	0.69	1.14	0.63	1.11	5.54	4.13	5.30	3.97
Japan	0.96%	2.08%	1.00%	1.98%	0.81	0.94	0.61	1.08	6.40	5.59	5.37	4.44
Mexico	1.21%	2.55%	1.21%	2.21%	0.70	1.15	0.64	1.29	4.32	6.67	4.26	3.70
Netherlands	0.90%	2.06%	0.90%	1.99%	0.79	1.20	0.81	1.29	5.29	4.81	5.36	4.98
Singapore	1.26%	3.02%	1.27%	2.93%	0.83	1.14	0.60	1.14	4.80	4.61	4.85	4.47
Spain	0.94%	2.21%	0.94%	2.14%	0.75	1.21	0.64	1.15	7.90	4.78	8.02	5.25
Sweden	1.25%	2.75%	1.25%	2.74%	0.92	1.53	0.86	1.45	6.64	4.45	6.52	4.73
Switzerland	0.93%	1.83%	0.92%	1.73%	0.65	0.88	0.61	1.00	4.91	4.21	4.88	4.74
UK	0.78%	2.17%	0.77%	2.13%	0.71	1.26	0.62	1.18	6.32	4.20	6.16	4.28
Average	**0.97%**	**2.36%**	**0.99%**	**2.29%**	**0.72**	**1.13**	**0.61**	**1.14**	**5.71**	**4.72**	**5.68**	**4.45**

Table 6: TGARCH-DCC Hedges

Summary Analysis for International Equities Hedged With S&P 500, 1/1/02 - 2/27/09; Split 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Fixed Window		Dynamic		Fixed Window		Dynamic		Fixed Window	
MSCI Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Australia	1.11%	2.84%	1.14%	2.94%	0.53	0.97	0.32	0.88	4.61	4.09	4.53	4.12
Belgium	1.17%	2.06%	1.19%	2.11%	0.49	0.84	0.37	0.84	7.74	4.48	8.15	4.09
BRIC	1.23%	3.46%	1.21%	3.53%	0.44	0.57	0.20	0.51	6.43	4.43	6.56	4.04
Canada	1.01%	2.22%	1.01%	2.23%	0.48	0.75	0.34	0.74	4.76	3.63	4.68	3.46
EAFE	0.64%	1.37%	0.65%	1.41%	0.58	0.93	0.49	0.94	4.81	3.73	4.72	3.68
Emerging M	1.01%	2.97%	1.01%	2.97%	0.25	0.36	0.17	0.38	6.35	4.31	6.38	4.19
Germany	0.94%	1.90%	0.96%	1.76%	0.71	0.93	0.68	1.09	5.67	4.85	5.18	3.77
Hong Kong	1.11%	2.17%	1.14%	2.09%	0.71	1.02	0.54	1.09	5.02	3.61	5.43	3.71
Italy	0.96%	1.96%	0.96%	1.92%	0.52	0.84	0.43	0.84	5.91	3.50	5.64	3.22
Japan	1.03%	1.80%	1.07%	1.66%	0.61	0.71	0.45	0.84	4.84	7.21	4.68	4.47
Mexico	1.12%	2.10%	1.20%	2.05%	0.75	0.93	0.53	1.08	4.08	4.68	4.30	4.75
Netherlands	1.03%	1.81%	1.03%	1.74%	0.60	0.95	0.55	0.98	6.50	4.21	6.35	3.75
Singapore	1.25%	2.38%	1.29%	2.32%	0.65	0.92	0.46	0.97	4.56	3.37	4.63	3.63
Spain	1.02%	1.96%	1.03%	1.94%	0.57	0.94	0.43	0.89	7.52	3.35	7.21	3.35
Sweden	1.34%	2.46%	1.33%	2.40%	0.70	1.15	0.62	1.13	5.74	2.74	5.53	2.64
Switzerland	1.02%	1.61%	1.02%	1.47%	0.48	0.68	0.42	0.76	5.07	3.16	4.98	2.85
UK	0.85%	1.88%	0.86%	1.91%	0.56	0.96	0.46	0.93	5.86	3.28	5.47	3.21
World	0.42%	1.35%	0.41%	1.31%	0.44	0.70	0.45	0.72	6.83	4.35	6.78	4.37
Average	**1.01%**	**2.13%**	**1.03%**	**2.10%**	**0.56**	**0.84**	**0.44**	**0.87**	**5.68**	**4.06**	**5.62**	**3.74**

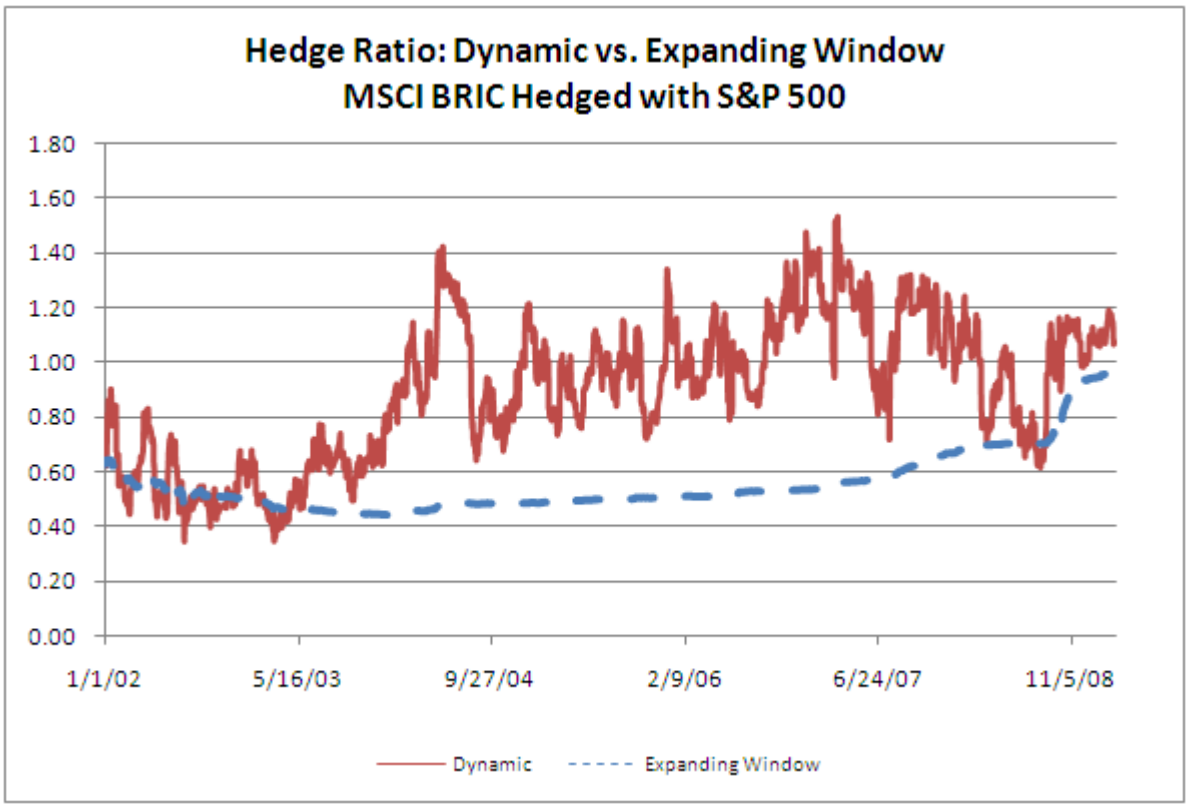
Summary Analysis for International Equities Hedged With World, 1/1/02 - 2/27/09; Split 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Fixed Window		Dynamic		Fixed Window		Dynamic		Fixed Window	
MSCI Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Australia	1.02%	2.97%	1.08%	3.04%	0.71	1.28	0.42	1.14	4.83	3.75	4.58	3.54
Belgium	1.09%	2.27%	1.09%	2.27%	0.67	1.09	0.57	1.11	7.42	4.55	8.00	4.90
BRIC	1.04%	2.78%	1.07%	2.63%	0.44	0.70	0.37	0.97	5.47	4.31	5.60	4.21
Canada	0.94%	2.13%	0.94%	2.07%	0.64	1.01	0.47	0.98	4.50	3.36	4.55	3.24
EAFE	0.51%	1.86%	0.51%	1.82%	0.75	1.19	0.66	1.19	6.30	4.94	6.01	5.10
Emerging M	0.83%	2.18%	0.86%	2.18%	0.53	0.77	0.33	0.80	5.16	4.58	5.82	4.44
Germany	0.84%	2.27%	0.84%	2.17%	0.91	1.19	0.90	1.38	6.41	7.51	6.31	6.20
Hong Kong	1.18%	3.12%	1.20%	3.02%	0.79	1.13	0.62	1.20	4.71	3.47	5.06	3.41
Italy	0.90%	1.93%	0.89%	1.83%	0.69	1.10	0.63	1.11	5.52	4.31	5.30	3.97
Japan	0.95%	2.14%	1.00%	1.98%	0.79	0.90	0.61	1.08	6.03	6.02	5.37	4.44
Mexico	1.16%	2.37%	1.21%	2.21%	0.88	1.11	0.64	1.29	4.15	4.32	4.26	3.70
Netherlands	0.90%	2.04%	0.90%	1.99%	0.79	1.20	0.81	1.29	5.33	4.83	5.36	4.98
Singapore	1.25%	2.97%	1.27%	2.93%	0.78	1.09	0.60	1.14	4.85	4.26	4.85	4.47
Spain	0.94%	2.22%	0.94%	2.14%	0.75	1.19	0.64	1.15	8.09	4.96	8.02	5.25
Sweden	1.26%	2.76%	1.25%	2.74%	0.92	1.49	0.86	1.45	6.69	4.47	6.52	4.73
Switzerland	0.93%	1.86%	0.92%	1.73%	0.65	0.89	0.61	1.00	4.95	4.08	4.88	4.74
UK	0.78%	2.18%	0.77%	2.13%	0.72	1.23	0.62	1.18	6.27	4.30	6.16	4.28
Average	**0.97%**	**2.36%**	**0.99%**	**2.29%**	**0.73**	**1.09**	**0.61**	**1.14**	**5.69**	**4.59**	**5.68**	**4.45**

The S&P reduced return volatility on average, and for most individual hedges, while the World index actually amplified volatility during the crisis period. Interestingly the results for this group are considerably weaker than those for the domestic equities. They are, however, more promising when the crisis period is excluded, which is to be expected since hedging with predictive models is more difficult during increasingly turbulent environments. Some hedges (MSCI Australia) performed well irrespective of time period and hedging model, while others (MSCI Switzerland) were consistently poor. The overall view is that hedging with dynamic models is not nearly as effective for this data set, and even though there are some satisfactory results, none are as strong as we saw with the Domestic Equities. Similar issues regarding the lack of volatility-dependent correlation estimates contribute to this, but considering these results are much worse than previously seen, it is clearly a multifaceted problem. Potential explanations will be provided in the final paragraph of this section.

As was the case with domestic equities, dynamic hedge ratios were higher on average. A time series of the two also reveals similar a relationship:

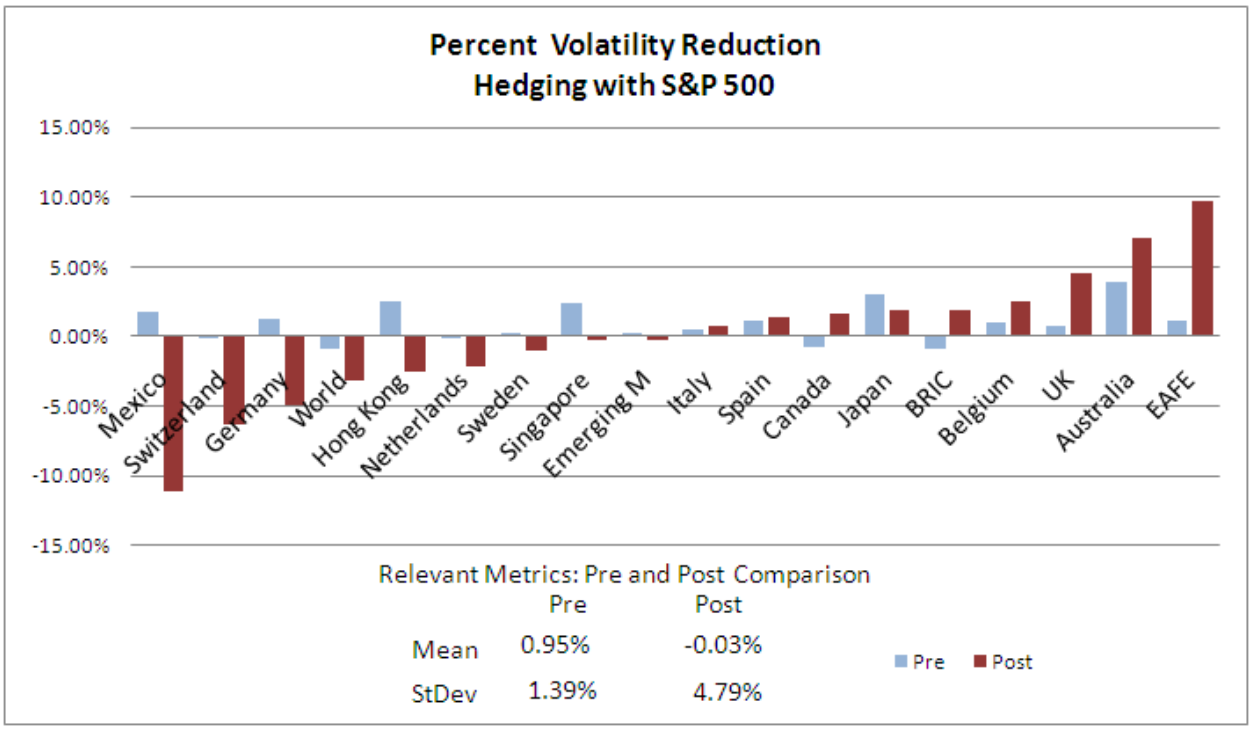
Chart 2: GARCH-DCC Hedge

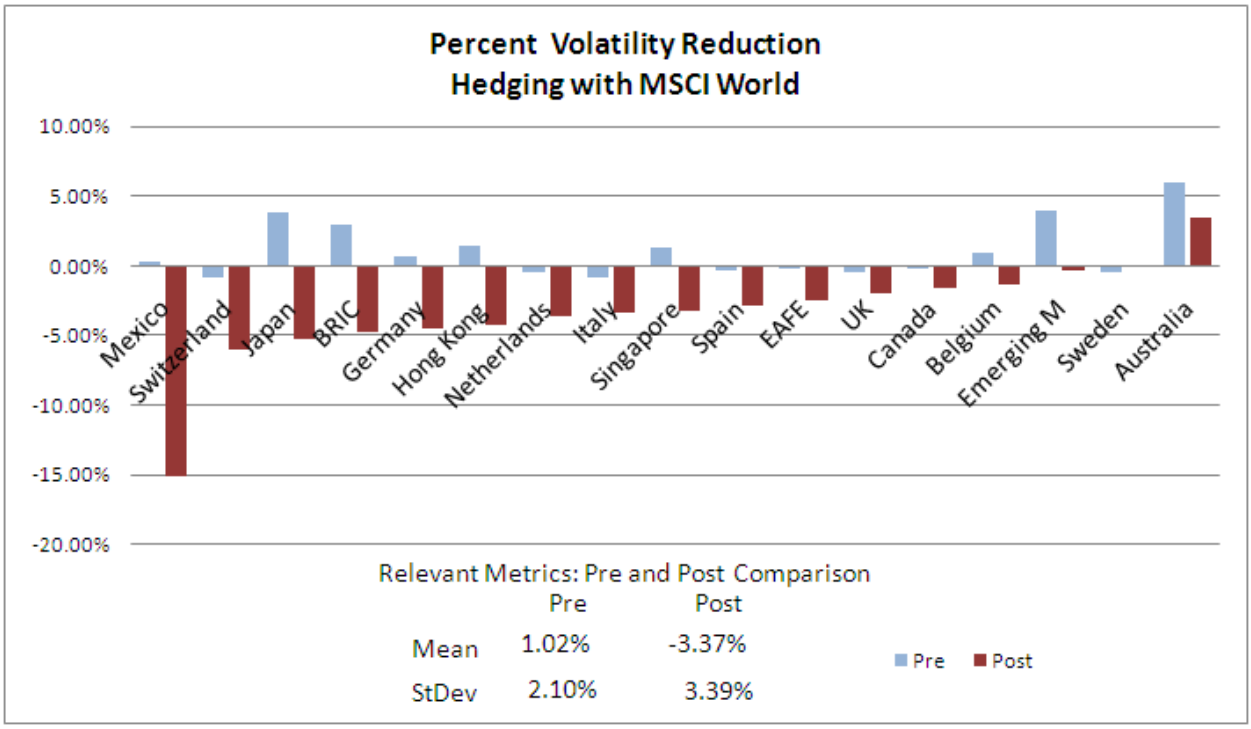


Here, the expanding window is more active than the domestic, partly due to the greater movements in the dynamic hedge, which implies relatively more volatility in the underlying securities. In general the dynamic hedge ratio fluctuates frequently, while the expanding window is smoother. This is evidence for the significant changes in portfolio allocation that dynamic hedging necessitates. Considering how slowly the expanding window moves, one would probably see a similar overall result if a portfolio hedged in that manner were updated weekly, monthly and at times, quarterly (more examples of these charts can be found in Appendix G). Finally, kurtosis is generally lower during the crisis period, which again points to the abundance of multi-sigma events. Even though there were many more, they were effectively the norm.

A bar chart of the volatility reductions further illustrates the difference in performance for the two sets of hedges:

Chart 3: GARCH-DCC Hedges





Both the S&P500 and MSCI World graphs are shown because of the dramatic differences in outcomes. The distribution in the S&P chart is noticeably more uniform than the MSCI World,

which shows high overall volatility increase for the latter months. One can infer, based on the above information, that the MSCI World Index is a poor hedging choice for the international markets. It is somewhat ironic that an index that more closely resembles its corresponding investments underperforms one that is largely based on the United States.

Accordingly the S&P considerably surpasses the MSCI World in the significance tests:

Table 7: GARCH-DCC International Equities Hedges

Number of S&P 500 Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	15	10	8	3	18
% total	83%	56%	44%	17%	
1/1/02 - 8/31/08	13	8	7	4	18
% total	72%	44%	39%	22%	
9/1/08 - 2/27/09	10	4	2	1	18
% total	56%	22%	11%	6%	
Total	38	22	17	8	54
% total	70%	41%	31%	15%	

Number of MSCI World Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	6	4	3	2	17
% total	35%	24%	18%	12%	
1/1/02 - 8/31/08	9	7	6	5	17
% total	53%	41%	35%	29%	
9/1/08 - 2/27/09	2	1	1	1	17
% total	12%	6%	6%	6%	
Total	17	12	10	8	51
% total	33%	24%	20%	16%	

Table 8: TGARCH-DCC International Equities Hedges

Number of S&P 500 Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	12	7	4	3	18
% total	67%	39%	22%	17%	
1/1/02 - 8/31/08	14	8	7	4	18
% total	78%	44%	39%	22%	
9/1/08 - 2/27/09	7	1	1	0	18
% total	39%	6%	6%	0%	
Total	33	16	12	7	54
% total	61%	30%	22%	13%	

Number of MSCI World Hedges at Various Significance Levels

	<0 Coeff.	0.10	0.05	0.01	Total Hedges
1/1/02 - 2/27/09	7	4	2	2	17
% total	41%	24%	12%	12%	
1/1/02 - 8/31/08	10	7	7	6	17
% total	59%	41%	41%	35%	
9/1/08 - 2/27/09	3	1	1	0	17
% total	18%	6%	6%	0%	
Total	20	12	10	8	51
% total	39%	24%	20%	16%	

While the results are not as impressive as the domestic set, these hedges still indicate a general volatility diminution (Appendix I). Explaining the results for this data is similar to our explanation for domestic equities: Artificial increases in correlation slightly improve the outcome of our results (Appendix J), but again not statistically significantly. This test consistently reveals that 1% – 5% increases in correlation make test coefficients decidedly worse for the ex-crisis period. However, since the overall hedging results are much poorer here than for the Domestic Equities, there are clearly other issues in question.

Equity market timing may explain why we see such drastic differences. An MSCI index tracks a group of stocks in a given country, and an ETF is a tradable stock on an American

exchange that tracks the MSCI index. Since it trades on the American market, it does not follow the trading hours of its home country. For example, because the MSCI Belgium (and implicitly its constituents) trades in America while the Belgian market is closed, there exists a non-synchronicity in pricing information. If information surfaces at 14:00 EST it priced into the US market, but not the Belgian one since the latter closes at 10:00 EST. Thus, the closing prices of a security that trades on both markets can be different. That same information is not accounted for in Brussels until the BEL20 opens the following day. Since the MSCI World is composed of several country indices, the information that is (not) priced into each one is unclear. Thus, properly hedging a portfolio is challenging because we are unsure of when to re-hedge and reassess performance. Is it more appropriate to re-hedge after the Brussels market closes, or after US trading completes? One would assume that markets are efficient enough to price in these differences, but the extent to which they do is currently unknown. Furthermore, since the S&P 500 has no such timing issues, this explanation helps elucidate not only the absolute poor performance of the MSCI World, but also the relatively poor performance against the S&P 500. This information asymmetry effectively reduced correlations and thus hedge ratios which accords with the Diebold-Mariano test results.

Conclusion

It is fairly obvious that the global economic meltdown had an overwhelming and pervasive effect on securities across the financial world. Investors experienced unprecedented losses and oscillations in their portfolios, primarily beginning in the fourth quarter of 2008. We have, however, determined that dynamically hedging those investments could have reduced—to a large extent in some cases—the volatility of many assets' returns. This benefit was most pronounced in domestic equities, but also present in others. The domestic equities saw the best performance from hedging, but the international set was less promising. What we found from the latter, however, is our hedge ratios for the MSCI World were systematically lower than they should have been due to deflated correlations, thereby causing relatively poor performance.

Armed with this knowledge, portfolio managers could have averted some of their losses had they implemented these dynamic hedging measures. It would be interesting to see how a quantitative hedge fund, similar to those Lo and Khandani (2007) discuss, would have performed under a similar hedging mechanism. Alas, since their holdings are private, this result will probably never be known. Nevertheless, we can be relatively certain that said funds were not hedging investments via a simple index and hedge ratio. Many were using not only leverage, but also derivates to reduce volatility. An alternate analysis may hedge with options as opposed to indices, choose another set of securities to analyze, or even hedge using alternative forecasting models, like the Factor Double ARCH of Engle (2008) or a GARCH model with an exogenous variable dependent on changes in volatility estimates. In the former model, correlation estimates are affected by changing volatility, which is exactly what DCC lacks. Perhaps if one had access to equity sector data for each country, and created an analysis similar to our domestic equities

one, we would observe a more pronounced reduction in volatility. It would also be interesting to see the effect of hedging a single investment with multiple assets; presumably this would result in an even better hedge.

Any investor with significant funds in the market is effectively required to manage and carefully allocate risk. Risk for any group of assets is quantifiable fairly simply through volatility or VaR measures and thus can be implemented across many investments. Better asset allocation and prediction techniques could have helped avert the massive losses many fund managers suffered. It is likely that analogous forms of the volatility reduction techniques used in the paper can be applied to other investments and strategies. Consequently, one must question the quality and quantity of risk measures used during the heart of the financial crisis. Although the portfolios created in the paper hedged idiosyncratic risk, the remaining systematic and unsystematic risk still caused several to perform poorly. Even so, daily portfolio re-allocation and assessment have proven to be effective tools in reducing the variance. Methods as simple as modeling parameter estimates and updating a portfolio accordingly have improved investment outcomes, as we have shown.

Appendix A: Selected GARCH-DCC Domestic Equity Hedges

Summary Analysis for Equity Sectors Hedged With Cons. Discretionary, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.50%	1.50%	0.53%	1.64%	0.45	0.80	0.37	0.68	6.12	7.19	6.27	8.32
Consumer Staples	0.60%	1.23%	0.61%	1.22%	0.26	0.38	0.20	0.37	6.28	3.34	6.27	3.56
Energy	1.39%	3.52%	1.44%	3.90%	0.35	0.58	0.18	0.47	3.98	5.23	4.03	5.04
Financials	0.87%	3.46%	0.94%	4.00%	0.51	1.50	0.43	0.90	8.94	4.13	14.05	3.45
Healthcare	0.73%	1.72%	0.72%	1.69%	0.37	0.44	0.33	0.53	7.26	7.81	6.91	10.57
Industrials	0.67%	1.51%	0.68%	1.51%	0.46	0.71	0.39	0.68	6.14	8.48	6.11	8.41
Materials	0.98%	2.32%	1.02%	2.31%	0.52	0.64	0.35	0.65	6.02	5.83	5.69	6.52
Technology	0.97%	1.85%	1.00%	1.78%	0.51	0.72	0.48	0.79	9.08	6.55	9.02	6.66
Utilities	1.00%	2.17%	1.04%	2.25%	0.28	0.41	0.19	0.39	10.59	6.35	11.46	6.95
Average	**0.89%**	**2.23%**	**0.92%**	**2.34%**	**0.43**	**0.71**	**0.34**	**0.63**	**6.82**	**5.91**	**7.44**	**6.32**

Summary Analysis for Equity Sectors Hedged With Energy, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.80%	1.73%	0.85%	1.83%	0.22	0.55	0.20	0.41	4.62	3.55	4.90	3.23
Cons. Discretionary	1.09%	2.45%	1.12%	2.41%	0.19	0.43	0.18	0.34	5.53	4.13	5.35	2.87
Consumer Staples	0.73%	1.49%	0.74%	1.37%	0.11	0.21	0.14	0.21	4.66	3.71	4.54	3.20
Financials	1.34%	4.51%	1.37%	4.89%	0.22	0.77	0.20	0.44	14.51	3.01	12.84	3.27
Healthcare	0.85%	1.78%	0.89%	1.73%	0.16	0.24	0.16	0.26	5.30	6.34	5.62	6.55
Industrials	0.95%	1.92%	0.97%	1.84%	0.21	0.48	0.21	0.38	4.88	4.29	4.86	2.94
Materials	1.02%	1.97%	1.05%	1.97%	0.31	0.63	0.26	0.52	4.01	3.86	4.32	3.43
Technology	1.28%	2.16%	1.35%	2.12%	0.21	0.46	0.18	0.35	6.07	5.22	6.23	4.23
Utilities	1.00%	1.77%	1.03%	1.76%	0.20	0.38	0.20	0.37	11.16	4.78	11.91	4.31
Average	**1.04%**	**2.33%**	**1.07%**	**2.33%**	**0.20**	**0.46**	**0.19**	**0.36**	**6.42**	**4.37**	**6.25**	**3.78**

Summary Analysis for Equity Sectors Hedged With Financials, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.44%	1.63%	0.52%	1.72%	0.43	0.46	0.37	0.56	5.38	7.09	10.28	5.56
Cons. Discretionary	0.71%	1.94%	0.74%	2.07%	0.45	0.47	0.40	0.60	6.26	5.36	6.13	3.68
Consumer Staples	0.60%	1.38%	0.62%	1.47%	0.25	0.20	0.21	0.30	6.63	3.49	6.43	3.24
Energy	1.39%	3.75%	1.43%	3.71%	0.34	0.34	0.19	0.41	4.15	4.45	4.30	4.57
Healthcare	0.70%	1.81%	0.74%	2.06%	0.35	0.23	0.31	0.40	6.96	10.31	7.85	6.50
Industrials	0.68%	1.90%	0.72%	2.06%	0.43	0.38	0.36	0.52	4.81	5.68	5.86	4.48
Materials	1.00%	2.64%	1.06%	2.59%	0.48	0.35	0.32	0.49	5.88	5.31	5.68	4.53
Technology	0.98%	2.00%	1.08%	2.17%	0.47	0.39	0.45	0.60	6.74	5.92	7.67	4.57
Utilities	0.95%	2.31%	1.02%	2.31%	0.29	0.22	0.23	0.35	9.92	6.49	11.11	5.63
Average	**0.87%**	**2.20%**	**0.91%**	**2.30%**	**0.40**	**0.34**	**0.32**	**0.47**	**5.92**	**5.79**	**6.27**	**4.51**

Appendix A (continued): Selected GARCH-DCC Domestic Equity Hedges

Summary Analysis for Equity Sectors Hedged With Healthcare, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.64%	2.01%	0.66%	2.19%	0.47	1.27	0.42	0.79	5.66	4.21	5.54	3.85
Cons. Discretionary	0.90%	2.34%	0.91%	2.48%	0.48	1.30	0.45	0.81	5.80	3.98	5.77	2.84
Consumer Staples	0.62%	1.26%	0.64%	1.38%	0.28	0.64	0.21	0.41	5.95	6.52	5.51	5.18
Energy	1.40%	3.73%	1.44%	4.16%	0.37	1.03	0.22	0.56	4.05	3.83	3.95	3.78
Financials	1.10%	4.61%	1.15%	5.09%	0.53	2.13	0.45	0.92	15.82	4.00	15.91	3.62
Industrials	0.82%	1.94%	0.82%	1.98%	0.46	1.16	0.41	0.75	5.34	4.38	5.36	3.28
Materials	1.10%	2.62%	1.13%	2.76%	0.49	1.05	0.33	0.68	5.27	3.61	5.22	3.03
Technology	1.14%	2.32%	1.15%	2.24%	0.51	1.14	0.60	0.97	7.64	4.56	8.60	3.78
Utilities	1.00%	1.94%	1.04%	2.18%	0.31	0.77	0.23	0.50	11.38	7.34	11.58	6.10
Average	**1.01%**	**2.69%**	**1.03%**	**2.87%**	**0.45**	**1.21**	**0.38**	**0.73**	**7.12**	**4.41**	**7.19**	**3.64**

Summary Analysis for Equity Sectors Hedged With Industrials, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.45%	1.39%	0.47%	1.51%	0.48	0.94	0.45	0.81	5.15	5.49	4.96	5.69
Cons. Discretionary	0.72%	1.72%	0.72%	1.69%	0.51	0.93	0.50	0.86	7.29	4.54	7.22	3.80
Consumer Staples	0.60%	1.28%	0.61%	1.26%	0.27	0.42	0.23	0.42	5.86	3.08	5.93	3.31
Energy	1.31%	2.92%	1.35%	3.51%	0.45	0.90	0.28	0.68	3.85	5.12	3.85	4.83
Financials	0.92%	4.01%	0.96%	4.23%	0.54	1.58	0.50	1.00	13.69	5.15	16.22	3.96
Healthcare	0.70%	1.36%	0.70%	1.46%	0.39	0.51	0.39	0.63	5.55	4.64	5.63	4.84
Materials	0.84%	1.82%	0.86%	1.87%	0.62	0.90	0.47	0.87	4.99	3.63	4.70	3.41
Technology	0.95%	1.71%	0.96%	1.64%	0.55	0.84	0.62	0.98	7.30	4.38	6.82	4.27
Utilities	0.96%	1.89%	1.00%	2.05%	0.31	0.51	0.25	0.50	8.70	4.72	10.87	5.02
Average	**0.86%**	**2.12%**	**0.88%**	**2.24%**	**0.48**	**0.87**	**0.43**	**0.78**	**6.93**	**4.36**	**7.20**	**4.06**

Summary Analysis for Equity Sectors Hedged With Materials, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.64%	1.74%	0.68%	1.92%	0.32	0.73	0.28	0.54	5.63	3.34	6.08	3.47
Cons. Discretionary	0.90%	2.14%	0.91%	2.09%	0.34	0.67	0.35	0.59	6.59	4.55	6.29	3.07
Consumer Staples	0.67%	1.46%	0.67%	1.40%	0.18	0.30	0.18	0.30	5.27	4.30	5.44	4.07
Energy	1.18%	2.59%	1.24%	3.15%	0.40	0.96	0.26	0.66	3.75	6.40	3.86	5.30
Financials	1.16%	4.23%	1.17%	4.68%	0.36	1.15	0.34	0.68	15.86	2.92	15.86	3.25
Healthcare	0.79%	1.63%	0.80%	1.64%	0.25	0.36	0.24	0.40	5.71	5.38	6.31	5.84
Industrials	0.72%	1.58%	0.72%	1.50%	0.36	0.71	0.36	0.61	6.56	2.86	6.49	2.38
Technology	1.12%	1.99%	1.18%	1.94%	0.35	0.64	0.30	0.56	8.70	4.53	8.20	3.98
Utilities	0.99%	2.08%	1.03%	2.16%	0.23	0.44	0.18	0.38	13.88	5.30	12.72	5.59
Average	**0.93%**	**2.23%**	**0.96%**	**2.34%**	**0.32**	**0.68**	**0.29**	**0.54**	**7.49**	**4.42**	**7.49**	**3.99**

Appendix A (continued): Selected GARCH-DCC Domestic Equity Hedges

Summary Analysis for Equity Sectors Hedged With Technology, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.51%	1.14%	0.57%	1.83%	0.39	0.90	0.29	0.55	7.51	5.29	6.07	4.09
Cons. Discretionary	0.80%	1.92%	0.89%	2.30%	0.38	0.86	0.24	0.49	5.43	4.29	5.50	3.58
Consumer Staples	0.66%	1.22%	0.73%	1.53%	0.19	0.40	0.07	0.18	6.88	4.20	6.29	4.40
Energy	1.38%	3.06%	1.47%	4.26%	0.29	0.77	0.09	0.31	3.98	5.39	3.97	4.67
Financials	1.05%	3.67%	1.17%	4.84%	0.41	1.47	0.25	0.56	14.85	4.78	14.65	3.55
Healthcare	0.78%	1.63%	0.77%	1.72%	0.29	0.46	0.22	0.39	11.64	5.70	7.52	8.12
Industrials	0.73%	1.56%	0.79%	1.91%	0.37	0.78	0.25	0.48	5.20	3.28	5.20	3.23
Materials	1.02%	2.19%	1.15%	2.82%	0.40	0.74	0.16	0.39	5.74	4.06	5.07	4.35
Utilities	1.01%	1.76%	1.08%	2.34%	0.23	0.48	0.11	0.27	10.96	4.89	11.35	5.89
Average	**0.92%**	**2.18%**	**1.00%**	**2.77%**	**0.33**	**0.78**	**0.18**	**0.40**	**7.68**	**4.53**	**6.89**	**4.56**

Summary Analysis for Equity Sectors Hedged With Utilities, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.81%	1.94%	0.81%	2.21%	0.34	0.99	0.28	0.58	6.02	2.98	5.76	3.14
Cons. Discretionary	1.07%	2.56%	1.07%	2.67%	0.32	0.89	0.26	0.52	5.67	3.70	5.61	3.04
Consumer Staples	0.71%	1.27%	0.70%	1.27%	0.22	0.51	0.21	0.37	5.29	3.90	5.53	3.52
Energy	1.34%	2.96%	1.36%	3.55%	0.46	1.19	0.27	0.69	3.83	3.17	3.81	3.57
Financials	1.30%	4.86%	1.28%	5.19%	0.39	1.49	0.33	0.70	19.55	3.43	15.89	3.55
Healthcare	0.84%	1.56%	0.84%	1.66%	0.27	0.56	0.23	0.44	5.64	4.26	5.64	6.59
Industrials	0.96%	2.11%	0.96%	2.17%	0.32	0.85	0.27	0.52	5.61	3.15	5.50	2.93
Materials	1.15%	2.65%	1.16%	2.81%	0.40	0.92	0.25	0.57	5.11	2.99	5.12	3.12
Utilities	1.00%	1.94%	1.04%	2.18%	0.31	0.77	0.23	0.50	11.38	7.34	11.58	6.10
Average	**1.08%**	**2.52%**	**1.08%**	**2.69%**	**0.34**	**0.91**	**0.26**	**0.55**	**7.17**	**3.52**	**6.74**	**3.72**

Appendix B: Selected TGARCH-DCC Domestic Equity Hedges

Summary Analysis for Equity Sectors Hedged With Cons. Discretionary, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.50%	1.44%	0.53%	1.64%	0.44	0.79	0.37	0.68	5.93	7.25	6.27	8.32
Consumer Staples	0.59%	1.23%	0.61%	1.22%	0.27	0.37	0.20	0.37	6.30	3.39	6.27	3.56
Energy	1.39%	3.66%	1.44%	3.90%	0.33	0.55	0.18	0.47	3.98	5.26	4.03	5.04
Financials	0.86%	3.32%	0.94%	4.00%	0.51	1.50	0.43	0.90	8.97	3.79	14.05	3.45
Healthcare	0.71%	1.67%	0.72%	1.69%	0.35	0.41	0.33	0.53	6.92	9.75	6.91	10.57
Industrials	0.67%	1.52%	0.68%	1.51%	0.46	0.70	0.39	0.68	6.03	7.97	6.11	8.41
Materials	0.99%	2.36%	1.02%	2.31%	0.50	0.60	0.35	0.65	6.04	6.35	5.69	6.52
Technology	0.97%	1.85%	1.00%	1.78%	0.51	0.69	0.48	0.79	8.94	7.03	9.02	6.66
Utilities	1.00%	2.19%	1.04%	2.25%	0.28	0.38	0.19	0.39	10.55	6.70	11.46	6.95
Average	**0.88%**	**2.23%**	**0.92%**	**2.34%**	**0.42**	**0.69**	**0.34**	**0.63**	**6.74**	**6.22**	**7.44**	**6.32**

Summary Analysis for Equity Sectors Hedged With Consumer Staples, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.70%	1.97%	0.77%	2.44%	0.52	1.31	0.34	0.74	6.33	3.04	6.08	3.64
Cons. Discretionary	0.91%	2.17%	0.97%	2.53%	0.55	1.37	0.37	0.78	6.09	4.26	5.66	3.92
Energy	1.42%	3.82%	1.45%	4.28%	0.43	1.03	0.25	0.62	4.10	3.96	3.99	4.12
Financials	1.13%	4.54%	1.21%	5.14%	0.60	2.29	0.41	0.96	11.12	3.70	12.67	3.52
Healthcare	0.75%	1.70%	0.80%	1.83%	0.43	0.73	0.27	0.55	6.37	7.72	6.87	9.94
Industrials	0.86%	2.07%	0.91%	2.30%	0.51	1.14	0.32	0.68	6.26	3.41	6.00	3.63
Materials	1.12%	2.83%	1.17%	3.03%	0.56	0.99	0.32	0.70	5.98	4.31	5.57	4.25
Technology	1.23%	2.15%	1.32%	2.59%	0.50	1.14	0.23	0.60	7.66	3.45	7.17	4.19
Utilities	1.01%	1.92%	1.05%	2.24%	0.40	0.78	0.27	0.58	13.54	5.67	12.39	5.95
Average	**1.06%**	**2.75%**	**1.12%**	**3.10%**	**0.51**	**1.24**	**0.31**	**0.70**	**6.80**	**4.40**	**6.85**	**4.79**

Summary Analysis for Equity Sectors Hedged With Energy, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.78%	1.57%	0.85%	1.83%	0.22	0.53	0.20	0.41	4.54	3.01	4.90	3.23
Cons. Discretionary	1.08%	2.37%	1.12%	2.41%	0.19	0.41	0.18	0.34	5.52	3.56	5.35	2.87
Consumer Staples	0.73%	1.45%	0.74%	1.37%	0.12	0.20	0.14	0.21	4.61	3.60	4.54	3.20
Financials	1.33%	4.40%	1.37%	4.89%	0.22	0.75	0.20	0.44	15.14	3.15	12.84	3.27
Healthcare	0.85%	1.77%	0.89%	1.73%	0.15	0.23	0.16	0.26	5.30	6.52	5.62	6.55
Industrials	0.94%	1.84%	0.97%	1.84%	0.22	0.46	0.21	0.38	4.85	3.62	4.86	2.94
Materials	1.02%	1.94%	1.05%	1.97%	0.31	0.61	0.26	0.52	3.97	3.97	4.32	3.43
Technology	1.27%	2.10%	1.35%	2.12%	0.21	0.43	0.18	0.35	6.02	4.93	6.23	4.23
Utilities	1.00%	1.71%	1.03%	1.76%	0.21	0.36	0.20	0.37	11.33	4.22	11.91	4.31
Average	**1.03%**	**2.27%**	**1.07%**	**2.33%**	**0.20**	**0.44**	**0.19**	**0.36**	**6.49**	**4.19**	**6.25**	**3.78**

Appendix B (continued): Selected TGARCH-DCC Domestic Equity Hedges

Summary Analysis for Equity Sectors Hedged With Financials, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.45%	1.60%	0.52%	1.72%	0.43	0.45	0.37	0.56	5.36	6.99	10.28	5.56
Cons. Discretionary	0.71%	1.88%	0.74%	2.07%	0.45	0.47	0.40	0.60	6.08	5.38	6.13	3.68
Consumer Staples	0.61%	1.38%	0.62%	1.47%	0.25	0.20	0.21	0.30	6.64	3.62	6.43	3.24
Energy	1.39%	3.84%	1.43%	3.71%	0.33	0.33	0.19	0.41	4.13	4.70	4.30	4.57
Healthcare	0.69%	1.84%	0.74%	2.06%	0.34	0.21	0.31	0.40	6.37	10.63	7.85	6.50
Industrials	0.68%	1.89%	0.72%	2.06%	0.43	0.37	0.36	0.52	4.78	5.54	5.86	4.48
Materials	1.00%	2.66%	1.06%	2.59%	0.46	0.33	0.32	0.49	5.56	5.80	5.68	4.53
Technology	0.98%	2.01%	1.08%	2.17%	0.47	0.37	0.45	0.60	6.68	5.89	7.67	4.57
Utilities	0.95%	2.33%	1.02%	2.31%	0.30	0.20	0.23	0.35	9.74	6.55	11.11	5.63
Average	**0.86%**	**2.21%**	**0.91%**	**2.30%**	**0.39**	**0.33**	**0.32**	**0.47**	**5.75**	**5.94**	**6.27**	**4.51**

Summary Analysis for Equity Sectors Hedged With Healthcare, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.64%	2.01%	0.66%	2.19%	0.48	1.28	0.42	0.79	5.83	4.18	5.54	3.85
Cons. Discretionary	0.90%	2.33%	0.91%	2.48%	0.49	1.33	0.45	0.81	5.99	3.24	5.77	2.84
Consumer Staples	0.62%	1.28%	0.64%	1.38%	0.29	0.64	0.21	0.41	5.86	6.07	5.51	5.18
Energy	1.41%	3.74%	1.44%	4.16%	0.37	1.06	0.22	0.56	4.02	3.76	3.95	3.78
Financials	1.10%	4.64%	1.15%	5.09%	0.54	2.20	0.45	0.92	16.49	4.22	15.91	3.62
Industrials	0.82%	1.93%	0.82%	1.98%	0.47	1.17	0.41	0.75	5.44	4.08	5.36	3.28
Materials	1.10%	2.61%	1.13%	2.76%	0.49	1.06	0.33	0.68	5.27	3.56	5.22	3.03
Technology	1.14%	2.33%	1.15%	2.24%	0.52	1.12	0.60	0.97	7.72	4.50	8.60	3.78
Utilities	1.00%	1.95%	1.04%	2.18%	0.32	0.73	0.23	0.50	11.06	7.14	11.58	6.10
Average	**1.01%**	**2.70%**	**1.03%**	**2.87%**	**0.45**	**1.23**	**0.38**	**0.73**	**7.25**	**4.20**	**7.19**	**3.64**

Summary Analysis for Equity Sectors Hedged With Industrials, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.45%	1.38%	0.47%	1.51%	0.47	0.94	0.45	0.81	5.24	5.62	4.96	5.69
Cons. Discretionary	0.72%	1.64%	0.72%	1.69%	0.51	0.94	0.50	0.86	7.13	4.29	7.22	3.80
Consumer Staples	0.61%	1.29%	0.61%	1.26%	0.27	0.42	0.23	0.42	5.99	3.17	5.93	3.31
Energy	1.30%	3.00%	1.35%	3.51%	0.44	0.88	0.28	0.68	3.82	5.24	3.85	4.83
Financials	0.91%	3.90%	0.96%	4.23%	0.53	1.60	0.50	1.00	13.85	4.90	16.22	3.96
Healthcare	0.69%	1.41%	0.70%	1.46%	0.38	0.49	0.39	0.63	5.39	4.76	5.63	4.84
Materials	0.83%	1.83%	0.86%	1.87%	0.59	0.86	0.47	0.87	4.96	3.57	4.70	3.41
Technology	0.95%	1.72%	0.96%	1.64%	0.55	0.82	0.62	0.98	7.24	4.53	6.82	4.27
Utilities	0.96%	1.94%	1.00%	2.05%	0.32	0.48	0.25	0.50	8.60	4.80	10.87	5.02
Average	**0.86%**	**2.12%**	**0.88%**	**2.24%**	**0.47**	**0.86**	**0.43**	**0.78**	**6.91**	**4.35**	**7.20**	**4.06**

Appendix B (continued): Selected TGARCH-DCC Domestic Equity Hedges

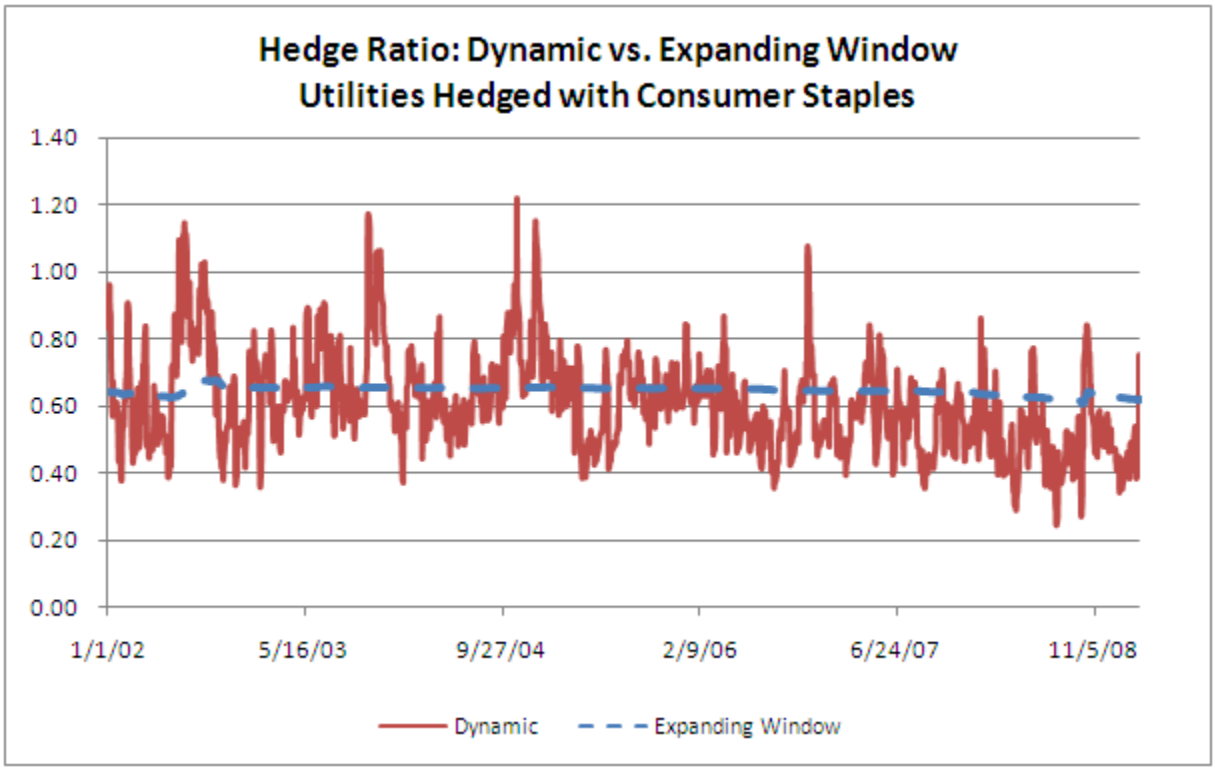
Summary Analysis for Equity Sectors Hedged With Materials, 1/1/02 - 2/28/09, Split at 9/1/08

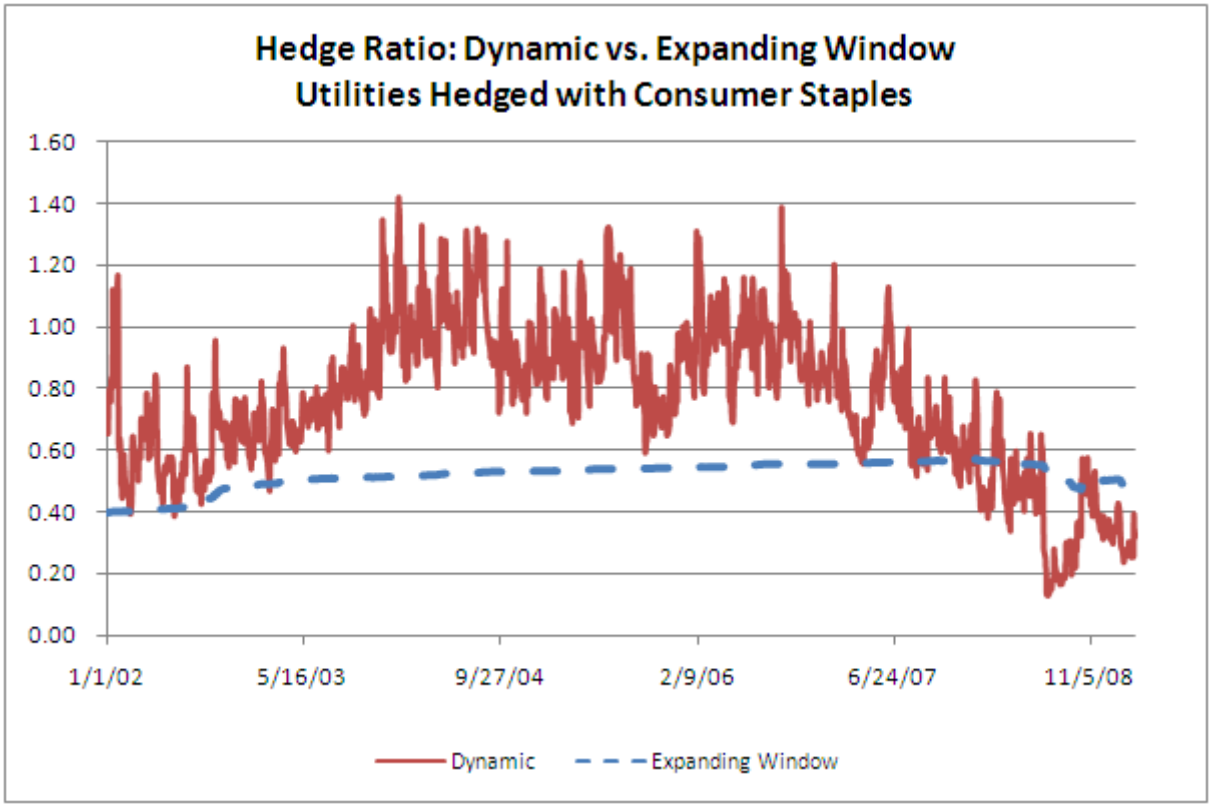
	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.63%	1.66%	0.68%	1.92%	0.32	0.74	0.28	0.54	5.67	3.30	6.08	3.47
Cons. Discretionary	0.89%	2.06%	0.91%	2.09%	0.34	0.68	0.35	0.59	6.53	3.50	6.29	3.07
Consumer Staples	0.66%	1.44%	0.67%	1.40%	0.18	0.30	0.18	0.30	5.18	4.19	5.44	4.07
Energy	1.17%	2.52%	1.24%	3.15%	0.40	0.98	0.26	0.66	3.76	6.57	3.86	5.30
Financials	1.14%	4.20%	1.17%	4.68%	0.36	1.18	0.34	0.68	16.43	3.06	15.86	3.25
Healthcare	0.78%	1.61%	0.80%	1.64%	0.25	0.36	0.24	0.40	5.78	5.35	6.31	5.84
Industrials	0.71%	1.50%	0.72%	1.50%	0.37	0.72	0.36	0.61	6.59	2.45	6.49	2.38
Technology	1.12%	1.93%	1.18%	1.94%	0.35	0.63	0.30	0.56	8.80	4.31	8.20	3.98
Utilities	0.99%	2.03%	1.03%	2.16%	0.24	0.43	0.18	0.38	15.32	5.47	12.72	5.59
Average	**0.93%**	**2.18%**	**0.96%**	**2.34%**	**0.32**	**0.69**	**0.29**	**0.54**	**7.58**	**4.20**	**7.49**	**3.99**

Summary Analysis for Equity Sectors Hedged With Utilities, 1/1/02 - 2/28/09, Split at 9/1/08

	Standard Deviation				Mean β				Kurtosis			
	Dynamic		Expanding Window		Dynamic		Expanding Window		Dynamic		Expanding Window	
Index	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
S&P 500	0.81%	1.89%	0.81%	2.21%	0.33	1.04	0.28	0.58	6.09	2.96	5.76	3.14
Cons. Discretionary	1.07%	2.51%	1.07%	2.67%	0.31	0.96	0.26	0.52	5.76	3.43	5.61	3.04
Consumer Staples	0.71%	1.26%	0.70%	1.27%	0.21	0.54	0.21	0.37	5.24	3.83	5.53	3.52
Energy	1.33%	2.97%	1.36%	3.55%	0.44	1.28	0.27	0.69	3.80	3.39	3.81	3.57
Financials	1.30%	4.81%	1.28%	5.19%	0.38	1.60	0.33	0.70	20.98	3.40	15.89	3.55
Healthcare	0.84%	1.60%	0.84%	1.66%	0.26	0.56	0.23	0.44	5.67	5.38	5.64	6.59
Industrials	0.96%	2.07%	0.96%	2.17%	0.32	0.89	0.27	0.52	5.66	3.09	5.50	2.93
Materials	1.15%	2.66%	1.16%	2.81%	0.39	0.97	0.25	0.57	4.97	3.36	5.12	3.12
Technology	1.26%	2.13%	1.28%	2.23%	0.34	0.90	0.29	0.57	6.60	3.39	6.85	3.48
Average	**1.08%**	**2.50%**	**1.08%**	**2.69%**	**0.33**	**0.96**	**0.26**	**0.55**	**7.34**	**3.66**	**6.74**	**3.72**

Appendix C: Selected GARCH-DCC Hedge Ratio Times Series for Domestic Equities





Appendix D: GARCH-DCC Domestic Equities Diebold-Mariano Summary

Row is Hedged with Column: 1/2/02 - 2/27/09

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		***	***	***	***	**	**	***	***	***
Discretionary			***		**	**			***	
Staples		*			**	***			***	
Energy	***	***	***		*	***	***	***	***	***
Financials	**	***	***	***		**	*	***	***	**
Healthcare	*		***	***	***			*		
Industrials		**	***		***				***	
Materials	***	**	***	***	**	***	***		***	**
Technology	***	*	***	***	***			***		***
Utilities	***	***	***	**	***	***	***	***	***	

Row is Hedged with Column: 1/2/02 - 8/31/08

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		***	***	***	***	**	***	***	***	
Discretionary			***	***	**	**		**	***	
Staples		***		***	***	***	**		***	
Energy	***	***	***		***	***	***	***	***	**
Financials	**	***	***	***		***	**		***	
Healthcare			***	***	***			**		
Industrials		**	***	***	***				***	
Materials	***	***	***	***	***	***	***		***	**
Technology	***	***	***	***	***			***		**
Utilities	***	***	***	**	***	***	***	***	***	

Row is Hedged with Column: 9/1/08 - 2/27/09

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		**	***	**		**	*	***	***	***
Discretionary			***			*			***	**
Staples						***			***	
Energy	***	***	***			***	***	***	***	***
Financials	*	***	***	***		**		***	***	***
Healthcare	*				**					
Industrials			***		*				***	*
Materials			***			**			***	**
Technology	***		***		*					*
Utilities		**	***			***	**	*	***	

Appendix E: TGARCH-DCC Domestic Equities Diebold-Mariano Summary

Row is Hedged with Column: 1/2/02 - 2/27/09

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		***	***	***	***	***	***	***	***	***
Discretionary			***	***	**	**		***	***	**
Staples		***			*	***			***	
Energy	***	***	***			***	***	***	***	***
Financials	***		***	***		**	**	***	***	**
Healthcare	*		***	***	***		**	**		
Industrials		***	***	***	***			**	***	
Materials	***	*	***	***	**	***	***		***	***
Technology	***	***	***	***	***			***		***
Utilities	**	***	***	***	**	***	***	***	***	

Row is Hedged with Column: 1/2/02 - 8/31/08

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		***	***	***	***	***	***	***	***	**
Discretionary			***	***	**	**		***	***	
Staples		***		***	**	***		**	***	
Energy	***	***	***		***	***	***	***	***	***
Financials	**	***	***	***		***	***	**	***	
Healthcare	***	*	***	***	***		**	***		
Industrials		*	***	***	***			**	***	
Materials	***	***	***	***	***	***	***		***	**
Technology	***	***	***	***	***			***		**
Utilities	***	***	***	**	***	***	***	**	***	

Row is Hedged with Column: 9/1/08 - 2/27/09

	S&P 500	Discretionary	Staples	Energy	Financials	Healthcare	Industrials	Materials	Technology	Utilities
S&P 500		***	***	***	*	***	**	***	***	**
Discretionary			***		**	**	*		***	***
Staples						***			***	
Energy	***	***	***			***	***	***	***	***
Financials	**	***	***	***		**	*	***	***	***
Healthcare					**					
Industrials			***		*				***	**
Materials			***			**			***	**
Technology	***		***							**
Utilities		*	***	**		***	*	**	***	

Note: 1, 2, and 3asterisks represent significance at the 0.10, 0.05 and 0.01 levels, respectively

Appendix F: Artificial Correlation Increases Hedging with S&P 500 (TGARCH-DCC)

Discretionary

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	-1.247E-06	-1.313E-06	-1.344E-06	-1.341E-06	-1.304E-06	-1.232E-06
p-value	0.165	0.152	0.145	0.145	0.151	0.165
1/2/2002 - 8/31/2008	1.734E-07	1.367E-07	1.217E-07	1.285E-07	1.570E-07	2.072E-07
p-value	0.619	0.592	0.580	0.583	0.598	0.624
9/1/2008 - 2/27/2009	-2.023E-05	-2.069E-05	-2.094E-05	-2.099E-05	-2.083E-05	-2.047E-05
p-value	0.155	0.145	0.138	0.134	0.132	0.132

Staples

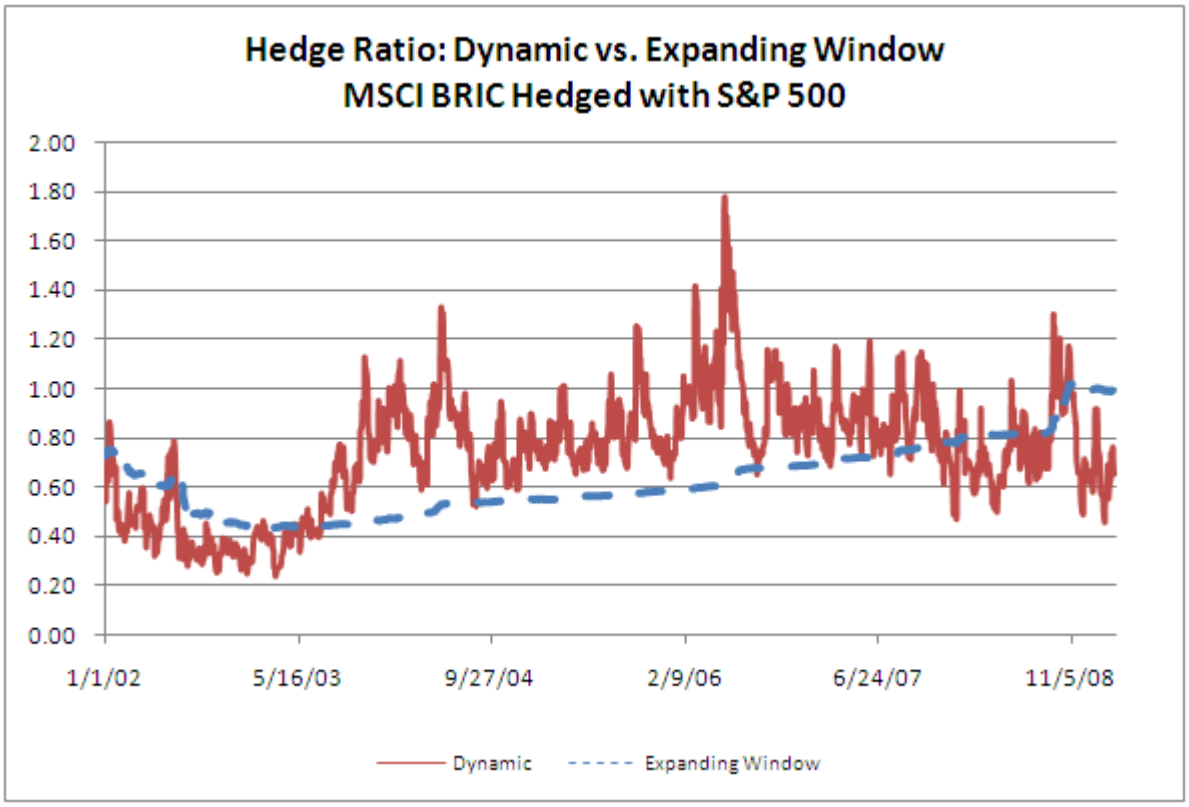
		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	8.929E-07	8.408E-07	7.975E-07	7.630E-07	7.373E-07	7.204E-07
p-value	0.883	0.874	0.866	0.861	0.857	0.855
1/2/2002 - 8/31/2008	2.165E-07	1.896E-07	1.687E-07	1.539E-07	1.452E-07	1.427E-07
p-value	0.730	0.699	0.673	0.654	0.641	0.634
9/1/2008 - 2/27/2009	9.935E-06	9.548E-06	9.205E-06	8.907E-06	8.653E-06	8.445E-06
p-value	0.838	0.837	0.837	0.839	0.841	0.844

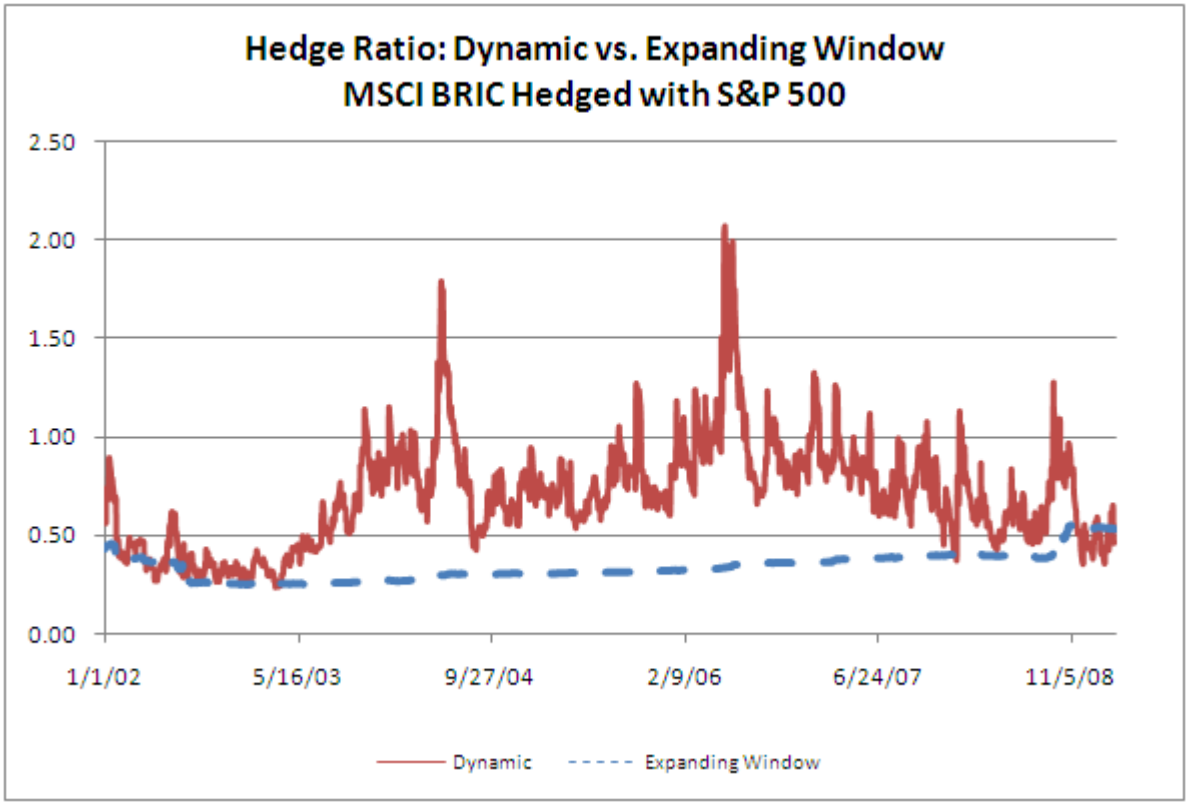
Industrials

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	-1.765E-07	-2.311E-07	-2.561E-07	-2.512E-07	-2.166E-07	-1.523E-07
p-value	0.454	0.438	0.429	0.428	0.436	0.453
1/2/2002 - 8/31/2008	1.465E-07	1.285E-07	1.406E-07	1.675E-07	2.144E-07	2.812E-07
p-value	0.809	0.791	0.804	0.813	0.827	0.846
9/1/2008 - 2/27/2009	-4.494E-06	-5.107E-06	-5.559E-06	-5.850E-06	-5.979E-06	-5.947E-06
p-value	0.416	0.401	0.388	0.378	0.371	0.366

Note: For each time interval, the test statistic is the first row and p-value is the bottom row

Appendix G: Selected DCC-GARCH Hedge Ratio Times Series for International Equities





Appendix H: GARCH-DCC International Equities Diebold-Mariano Summary

Hedge MSCI Indices with S&P500, 1/2/02 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.886E-05	4.579E-06	-4.119	0.000
Belgium	-4.371E-06	2.881E-06	-1.517	0.065
BRIC	-6.228E-07	6.164E-06	-0.101	0.460
Canada	-5.950E-07	2.193E-06	-0.271	0.393
EAFE	-3.750E-06	1.884E-06	-1.990	0.023
EM	-5.368E-07	2.049E-06	-0.262	0.397
Germany	-1.060E-06	1.726E-06	-0.614	0.270
HK	-6.269E-06	2.793E-06	-2.245	0.012
Italy	-1.749E-06	1.337E-06	-1.308	0.096
Japan	-8.460E-06	2.739E-06	-3.089	0.001
Mexico	-1.169E-05	5.100E-06	-2.292	0.011
Netherlands	1.268E-06	1.508E-06	0.841	0.800
Singapore	-7.887E-06	4.081E-06	-1.933	0.027
Spain	-3.896E-06	1.632E-06	-2.388	0.009
Sweden	-1.885E-06	2.516E-06	-0.749	0.227
Switzerland	1.481E-06	1.499E-06	0.988	0.838
UK	-4.246E-06	2.563E-06	-1.657	0.049
World	1.181E-06	4.026E-07	2.933	0.998

Hedge MSCI Indices with MSCI World, 1/2/02 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.809E-05	3.397E-06	-5.326	0.000
Belgium	-1.288E-06	2.521E-06	-0.511	0.305
BRIC	-5.616E-06	2.921E-06	-1.922	0.027
Canada	4.914E-07	1.602E-06	0.307	0.620
EAFE	1.296E-06	7.894E-07	1.641	0.950
EM	-5.670E-06	2.151E-06	-2.636	0.004
Germany	1.544E-06	2.097E-06	0.736	0.769
HK	9.429E-07	3.414E-06	0.276	0.609
Italy	1.920E-06	1.152E-06	1.667	0.952
Japan	-4.954E-06	3.089E-06	-1.604	0.054
Mexico	-1.600E-06	3.569E-06	-0.448	0.327
Netherlands	2.562E-06	1.513E-06	1.693	0.955
Singapore	1.470E-07	3.390E-06	0.043	0.517
Spain	1.909E-06	1.400E-06	1.363	0.914
Sweden	1.162E-06	2.555E-06	0.455	0.675
Switzerland	4.961E-06	1.505E-06	3.297	1.000
UK	1.410E-06	1.183E-06	1.192	0.883

Hedge MSCI Indices with S&P500, 1/2/02 - 8/31/08

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.023E-05	2.780E-06	-3.678	0.000
Belgium	-2.952E-06	2.604E-06	-1.134	0.129
BRIC	2.040E-06	2.611E-06	0.781	0.783
Canada	1.047E-06	1.591E-06	0.658	0.745
EAFE	-9.584E-07	6.674E-07	-1.436	0.076
EM	-7.126E-07	9.646E-07	-0.739	0.230
Germany	-2.476E-06	1.118E-06	-2.215	0.013
HK	-6.798E-06	2.700E-06	-2.518	0.006
Italy	-8.597E-07	9.213E-07	-0.933	0.175
Japan	-7.246E-06	2.275E-06	-3.185	0.001
Mexico	-1.606E-05	4.602E-06	-3.489	0.000
Netherlands	4.224E-07	1.117E-06	0.378	0.647
Singapore	-8.486E-06	3.696E-06	-2.296	0.011
Spain	-2.546E-06	1.350E-06	-1.886	0.030
Sweden	-1.283E-06	2.026E-06	-0.633	0.263
Switzerland	6.936E-07	1.275E-06	0.544	0.707
UK	-1.229E-06	1.085E-06	-1.132	0.129
World	3.194E-07	2.313E-07	1.381	0.916

Hedge MSCI Indices with MSCI World, 1/2/02 - 8/31/08

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.379E-05	2.807E-06	-4.914	0.000
Belgium	-2.368E-06	2.525E-06	-0.938	0.174
BRIC	-7.202E-06	2.140E-06	-3.365	0.000
Canada	2.539E-08	1.505E-06	0.017	0.507
EAFE	1.347E-07	3.178E-07	0.424	0.664
EM	-6.091E-06	1.788E-06	-3.407	0.000
Germany	-1.022E-06	9.057E-07	-1.128	0.130
HK	-4.278E-06	2.756E-06	-1.553	0.060
Italy	1.328E-06	7.793E-07	1.704	0.956
Japan	-7.989E-06	2.797E-06	-2.857	0.002
Mexico	-6.822E-06	2.852E-06	-2.392	0.008
Netherlands	7.393E-07	9.246E-07	0.800	0.788
Singapore	-4.448E-06	2.613E-06	-1.703	0.044
Spain	6.326E-07	8.870E-07	0.713	0.762
Sweden	1.158E-06	1.851E-06	0.626	0.734
Switzerland	2.577E-06	1.147E-06	2.247	0.988
UK	3.893E-07	7.775E-07	0.501	0.692

Appendix H (cont.): GARCH-DCC International Equities Diebold-Mariano Summary

Hedge MSCI Indices with S&P500, 9/1/08 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.343E-04	4.390E-05	-3.060	0.001
Belgium	-2.333E-05	2.095E-05	-1.114	0.134
BRIC	-3.622E-05	8.117E-05	-0.446	0.328
Canada	-2.254E-05	2.176E-05	-1.036	0.151
EAFE	-4.107E-05	2.065E-05	-1.989	0.024
EM	1.813E-06	2.575E-05	0.070	0.528
Germany	1.787E-05	1.920E-05	0.931	0.823
HK	8.155E-07	1.638E-05	0.050	0.520
Italy	-1.363E-05	1.493E-05	-0.913	0.181
Japan	-2.470E-05	2.232E-05	-1.107	0.135
Mexico	4.668E-05	3.963E-05	1.178	0.880
Netherlands	1.258E-05	1.592E-05	0.790	0.785
Singapore	1.238E-07	3.252E-05	0.004	0.502
Spain	-2.194E-05	1.345E-05	-1.632	0.053
Sweden	-9.937E-06	2.355E-05	-0.422	0.337
Switzerland	1.200E-05	1.299E-05	0.924	0.821
UK	-4.459E-05	2.838E-05	-1.571	0.059
World	1.270E-05	4.062E-06	3.127	0.999

Hedge MSCI Indices with MSCI World, 9/1/08 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-7.555E-05	2.570E-05	-2.939	0.002
Belgium	1.315E-05	1.334E-05	0.985	0.837
BRIC	1.558E-05	3.475E-05	0.448	0.673
Canada	6.722E-06	1.216E-05	0.553	0.709
EAFE	1.682E-05	9.838E-06	1.709	0.955
EM	-3.707E-08	2.178E-05	-0.002	0.499
Germany	3.585E-05	2.914E-05	1.230	0.890
HK	7.075E-05	2.754E-05	2.569	0.994
Italy	9.836E-06	1.242E-05	0.792	0.785
Japan	3.562E-05	2.196E-05	1.622	0.946
Mexico	6.821E-05	3.069E-05	2.223	0.986
Netherlands	2.693E-05	1.628E-05	1.655	0.950
Singapore	6.158E-05	2.833E-05	2.174	0.984
Spain	1.898E-05	1.627E-05	1.167	0.877
Sweden	1.210E-06	2.883E-05	0.042	0.517
Switzerland	3.684E-05	1.305E-05	2.823	0.997
UK	1.506E-05	1.269E-05	1.187	0.881

Appendix I: TGARCH-DCC International Equities Diebold-Mariano Summary

Hedge MSCI Indices with S&P500, 1/2/02 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.241E-05	3.020E-06	-4.111	0.000
Belgium	-4.678E-06	2.590E-06	-1.806	0.036
BRIC	-5.590E-07	1.637E-06	-0.342	0.366
Canada	-3.955E-07	1.329E-06	-0.298	0.383
EAFE	-1.640E-06	1.133E-06	-1.448	0.074
EM	-1.640E-06	1.133E-06	-1.448	0.074
Germany	1.420E-06	2.543E-06	0.558	0.712
HK	-3.894E-06	3.212E-06	-1.213	0.113
Italy	5.439E-07	1.431E-06	0.380	0.648
Japan	-4.548E-06	2.881E-06	-1.579	0.057
Mexico	-1.580E-05	4.080E-06	-3.874	0.000
Netherlands	1.433E-06	1.786E-06	0.802	0.789
Singapore	-6.843E-06	2.878E-06	-2.378	0.009
Spain	-1.621E-06	1.827E-06	-0.887	0.188
Sweden	2.265E-06	2.886E-06	0.785	0.784
Switzerland	3.270E-06	1.649E-06	1.983	0.976
UK	-1.707E-06	1.798E-06	-0.949	0.171
World	9.179E-07	3.681E-07	2.493	0.994

Hedge MSCI Indices with MSCI World, 1/2/02 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.486E-05	2.851E-06	-5.210	0.000
Belgium	-1.729E-06	2.330E-06	-0.742	0.229
BRIC	1.462E-07	3.592E-06	0.041	0.516
Canada	7.779E-07	1.511E-06	0.515	0.697
EAFE	1.159E-06	7.947E-07	1.458	0.928
EM	-5.262E-06	2.198E-06	-2.394	0.008
Germany	2.302E-06	2.254E-06	1.021	0.846
HK	-6.589E-08	3.538E-06	-0.019	0.493
Italy	3.846E-06	1.726E-06	2.229	0.987
Japan	-4.682E-06	3.930E-06	-1.191	0.117
Mexico	-5.012E-06	3.153E-06	-1.590	0.056
Netherlands	1.389E-06	1.502E-06	0.925	0.822
Singapore	-4.076E-06	2.732E-06	-1.492	0.068
Spain	2.734E-06	1.805E-06	1.515	0.935
Sweden	2.436E-06	2.886E-06	0.844	0.801
Switzerland	4.366E-06	1.441E-06	3.031	0.999
UK	2.160E-06	1.206E-06	1.791	0.963

Hedge MSCI Indices with S&P500, 1/2/02 - 8/31/08

	Coefficient	Std. Error	t-Stat	p-value
Australia	-8.934E-06	2.426E-06	-3.682	0.000
Belgium	-3.434E-06	2.422E-06	-1.418	0.078
BRIC	2.758E-06	2.719E-06	1.014	0.845
Canada	-1.293E-07	1.308E-06	-0.099	0.461
EAFE	-7.797E-07	6.606E-07	-1.180	0.119
EM	-4.904E-07	9.701E-07	-0.505	0.307
Germany	-2.266E-06	1.113E-06	-2.036	0.021
HK	-6.543E-06	2.958E-06	-2.212	0.014
Italy	-5.699E-07	9.258E-07	-0.616	0.269
Japan	-8.406E-06	1.960E-06	-4.288	0.000
Mexico	-1.860E-05	4.082E-06	-4.557	0.000
Netherlands	-1.255E-07	1.338E-06	-0.094	0.463
Singapore	-9.378E-06	2.734E-06	-3.430	0.000
Spain	-2.347E-06	1.299E-06	-1.807	0.035
Sweden	5.783E-07	2.074E-06	0.279	0.610
Switzerland	2.386E-07	8.682E-07	0.275	0.608
UK	-1.008E-06	1.052E-06	-0.958	0.169
World	2.881E-07	2.142E-07	1.345	0.911

Hedge MSCI Indices with MSCI World, 1/2/02 - 8/31/08

	Coefficient	Std. Error	t-Stat	p-value
Australia	-1.289E-05	2.526E-06	-5.101	0.000
Belgium	-1.734E-06	2.146E-06	-0.808	0.210
BRIC	-5.739E-06	1.765E-06	-3.251	0.001
Canada	-9.796E-07	1.272E-06	-0.770	0.221
EAFE	4.330E-08	2.938E-07	0.147	0.559
EM	-5.571E-06	1.744E-06	-3.195	0.001
Germany	-8.635E-07	1.004E-06	-0.860	0.195
HK	-4.654E-06	2.640E-06	-1.763	0.039
Italy	1.554E-06	9.251E-07	1.680	0.953
Japan	-9.890E-06	2.245E-06	-4.406	0.000
Mexico	-1.067E-05	2.456E-06	-4.343	0.000
Netherlands	5.820E-08	1.135E-06	0.051	0.520
Singapore	-6.200E-06	2.149E-06	-2.885	0.002
Spain	6.436E-07	9.269E-07	0.694	0.756
Sweden	1.868E-06	2.045E-06	0.913	0.819
Switzerland	1.391E-06	6.781E-07	2.052	0.980
UK	6.735E-07	7.509E-07	0.897	0.815

Appendix I (cont.): TGARCH-DCC International Equities Diebold-Mariano Summary

Hedge MSCI Indices with S&P500, 9/1/08 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-5.929E-05	2.776E-05	-2.135	0.017
Belgium	-2.143E-05	1.696E-05	-1.263	0.104
BRIC	-5.983E-05	6.502E-05	-0.920	0.180
Canada	-3.982E-06	7.742E-06	-0.514	0.304
EAFE	-1.324E-05	1.207E-05	-1.096	0.137
EM	-1.484E-06	1.940E-05	-0.076	0.470
Germany	5.108E-05	3.010E-05	1.697	0.954
HK	3.180E-05	2.160E-05	1.472	0.928
Italy	1.555E-05	1.572E-05	0.989	0.838
Japan	4.743E-05	3.023E-05	1.569	0.940
Mexico	2.188E-05	1.962E-05	1.115	0.867
Netherlands	2.242E-05	1.778E-05	1.261	0.895
Singapore	2.731E-05	1.770E-05	1.543	0.937
Spain	8.160E-06	1.804E-05	0.452	0.674
Sweden	2.499E-05	2.841E-05	0.880	0.810
Switzerland	4.411E-05	1.654E-05	2.667	0.996
UK	-1.112E-05	1.955E-05	-0.569	0.285
World	9.403E-06	3.896E-06	2.414	0.991

Hedge MSCI Indices with MSCI World, 9/1/08 - 2/27/09

	Coefficient	Std. Error	t-Stat	p-value
Australia	-4.140E-05	2.135E-05	-1.939	0.027
Belgium	-1.665E-06	1.786E-05	-0.093	0.463
BRIC	7.943E-05	4.450E-05	1.785	0.962
Canada	2.446E-05	1.079E-05	2.266	0.987
EAFE	1.619E-05	9.542E-06	1.697	0.954
EM	-1.099E-06	2.256E-05	-0.049	0.481
Germany	4.494E-05	3.124E-05	1.439	0.924
HK	6.175E-05	3.451E-05	1.789	0.962
Italy	3.472E-05	1.951E-05	1.780	0.961
Japan	6.549E-05	4.546E-05	1.440	0.924
Mexico	7.119E-05	2.441E-05	2.916	0.998
Netherlands	1.932E-05	1.484E-05	1.302	0.902
Singapore	2.454E-05	2.680E-05	0.916	0.819
Spain	3.089E-05	2.257E-05	1.369	0.913
Sweden	1.010E-05	3.176E-05	0.318	0.624
Switzerland	4.445E-05	1.666E-05	2.668	0.996
UK	2.218E-05	1.320E-05	1.681	0.952

Appendix J: Artificial Correlation Increases for International Equities (TGARCH-DCC)

Canada/World

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	7.779E-07	5.950E-07	4.350E-07	2.978E-07	1.836E-07	9.213E-08
p-value	0.697	0.655	0.616	0.580	0.549	0.524
1/2/2002 - 8/31/2008	-9.796E-07	-9.202E-07	-8.478E-07	-7.625E-07	-6.642E-07	-5.530E-07
p-value	0.221	0.240	0.263	0.288	0.317	0.350
9/1/2008 - 2/27/2009	2.446E-05	2.101E-05	1.772E-05	1.458E-05	1.161E-05	8.784E-06
p-value	0.987	0.979	0.964	0.937	0.893	0.826

Italy/World

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	3.846E-06	3.661E-06	3.504E-06	3.374E-06	3.271E-06	3.196E-06
p-value	0.987	0.987	0.986	0.986	0.987	0.987
1/2/2002 - 8/31/2008	1.554E-06	1.669E-06	1.799E-06	1.946E-06	2.109E-06	2.288E-06
p-value	0.953	0.966	0.976	0.985	0.991	0.995
9/1/2008 - 2/27/2009	3.472E-05	3.051E-05	2.647E-05	2.261E-05	1.893E-05	1.542E-05
p-value	0.961	0.948	0.930	0.906	0.875	0.836

Switzerland/S&P500

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	3.270E-06	3.152E-06	3.054E-06	2.976E-06	2.917E-06	2.878E-06
p-value	0.976	0.975	0.974	0.974	0.973	0.973
1/2/2002 - 8/31/2008	2.386E-07	3.257E-07	4.250E-07	5.367E-07	6.608E-07	7.972E-07
p-value	0.608	0.642	0.677	0.712	0.747	0.780
9/1/2008 - 2/27/2009	4.411E-05	4.123E-05	3.848E-05	3.584E-05	3.332E-05	3.091E-05
p-value	0.996	0.995	0.994	0.992	0.990	0.987

World/S&P500

		Increase Correlation				
	Original	1%	2%	3%	4%	5%
1/2/2002 - 2/27/2009	9.179E-07	9.781E-07	1.058E-06	1.158E-06	1.278E-06	1.417E-06
p-value	0.994	0.990	0.993	0.995	0.996	0.998
1/2/2002 - 8/31/2008	2.881E-07	3.245E-07	3.721E-07	4.311E-07	5.015E-07	5.831E-07
p-value	0.911	0.865	0.906	0.940	0.965	0.981
9/1/2008 - 2/27/2009	9.403E-06	9.784E-06	1.030E-05	1.095E-05	1.174E-05	1.266E-05
p-value	0.991	0.981	0.980	0.978	0.978	0.978

References

Bollerslev, T. (1986), "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, 31, 307-327.

Christie, A. (1982), "The stochastic behavior of common stock variances," *Journal of Financial Economics*, 10, 407-432.

Diebold, F.X. and Mariano, R. (1995), "Comparing predictive accuracy," *Journal of Business and Economic Statistics*, 13, 253-265.

Engle, R.F. (1982), "Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation," *Econometrica*, 50, 987–1008.

Engle, R.F. (2002), "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business & Economic Statistics*, 20, 339-350.

Engle, R.F. (2008), "High dimension dynamic correlations," Working Paper.

Engle, R.F. (2009), Anticipating Correlations. Princeton, NJ: Princeton University Press.

Engle, R.F. "Vlab." The Volatility Institute. April 23, 2009. http://vlab.stern.nyu.edu.

Euronext. "Euronext National Indices." NYSE Euronext. 4/27/09. http://www.euronext.com/editorial/wide/editorial-2332-NL.html.

Khandani, A. and Lo, A. (2007), "What happened to the quants in August 2007?" Working Paper.

MSCI Barra. "MSCI Barra Equity Indices Definitions." MSCI Barra. 4/27/09. http://www.mscibarra.com/products/indices/equity/definitions.jsp.

Nelson, D.B. (1991), "Conditional heteroskedasticity in asset returns: A new approach," *Econometrica*, 59, 347-370.

Schwert, G.W. (1989), "Why does stock market volatility change over time?" *Journal of Finance*, 44, 1115-1153.

Standard & Poor's. "S&P 500 Constituent List." Standard & Poor's Financial Services LLC. 4/23/09. http://www2.standardandpoors.com/portal/site/sp/en/us/page.topic/indices_500/2,3,2,2,0,0,0,0,0,2,3,0,0,0,0,0.html.

Zakoian, J. M. (1994), "Threshold heteroskedastic models," *Journal of Economic Dynamics and Control*, 18, 931-955.