Optimal Outlooks

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Disclaimer and Acknowledgements

Disclaimer: I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

- A policymaker needs to make a decision today.

- The *current* decision results in random *future* net benefits to society.

- Hence, the policymaker's decision depends on the outlook about those net benefits.

Question

What's the appropriate notion of an outlook for this policymaker?

Answer

- The needed outlook is not a statistically motivated **predictive density** ...

- But rather an asset-price-based **risk-neutral probability density** (RNPD).

Main Result

- A policymaker reaches the same ex-ante decision by:
  - maximizing social welfare
  - maximizing *risk-neutral* expected benefits
- Maximizing statistical expectation of benefits is typically different.

Intuition

- To make an ex-ante decision, the policymaker weighs social benefits in different future states against each other.

- To maximize social welfare: relevant weights are households' ex-ante relative marginal valuations of resources in those states.

- RNPDs are derived from financial market *prices*.

- Those prices reflect households' ex-ante relative marginal valuations of resources in different future states.

- Hence: the risk-neutral expectation also weighs benefits in different states according to households' ex-ante relative marginal values of resources.

Outline

1. General Policy Problem
2. Risk-Neutral Probabilities
3. Equivalence
4. Possible Concerns
5. Conclusions

GENERAL POLICY PROBLEM

Random Outcomes

- Policymaker chooses an action a today.

- The result of the action next period depends on the realization of x.
  - The random variable x has realizations $\{x_n\}_{n=1}^{N}$.

- The outcome (a, x) results in a benefit of $B(a, x)$.
  - The benefit $B(a, x)$ may be positive or negative.

Examples of B

- **Inflation targeting**: $B(a, x) = -(a + x - \pi^*)^2$
  - a is accommodation
  - x is inflation shock
- **Financial instability**: $B(a, x)$
  - a is bank dividends
  - x is financial stress

Social Welfare

- If realization x_n occurs, households consume $(y(x_n) + B(a, x_n))$.

- Households' ex-ante (subjective) expected utility is:

$$\sum_{n=1}^{N} \pi_n U(y(x_n) + B(a, x_n), x_n)$$

- The households' utility function U is possibly state-dependent.

- Also: π_n are subjective probabilities, not "true" probabilities.

Optimal Choice

- Chain rule: optimal choice of a satisfies FOC:

$$\sum_{n=1}^{N} \pi_n MUC_n(a^*) \frac{\partial B}{\partial a}(a^*, x_n) = 0$$

where $MUC_n(a^*)$ is the marginal utility of consumption in state n:

$$MUC_n(a^*) \equiv U_c(y(x_n) + B(a^*, x_n), x_n)$$

Missing Information

- Policymaker needs to know:
  - State-dependent marginal utility: $MUC_n(a^*)$
  - Household subjective probabilities: π_n.
- No good data on these!
- But we will see:

Relevant information is encoded in *risk-neutral probability density*.

RISK-NEUTRAL PROBABILITIES

RNPD

- Suppose households trade assets *before* policymaker chooses a.
- Let q_n represent the (implied) price of goods in state n.
- Define $q^* = (q_n^*)_{n=1}^N$ to be:

$$q_n^* = \frac{q_n}{\sum_{n=1}^N q_n}$$

- q^* is called the *risk-neutral probability density (RNPD).*
  - probability means: q_n^* is positive and q_n^*'s sum to 1.

RNPD in Equilibrium

- Households treat a^* as given when trading assets.

- In equilibrium, there is a constant $\xi > 0$ such that:

$$q_n = \xi \pi_n MUC_n(a^*)$$

- Hence:

$$q_n^* = \frac{\pi_n MUC_n(a^*)}{\sum_{m=1}^{N} \pi_m MUC_m(a^*)}$$

Risk-Neutral and "True" Probabilities

- The RNPD q^* is not the same as the "true" probability density of x.
- q^* reflects households' marginal utilities.
- And q^* reflects households' subjective probabilities.

$\mathbf{E}^*$

- For any random variable ϕ, define:

$$E^*(\phi) = \sum_{n=1}^{N} q_n^* \phi_n$$

- Define risk-neutral expected benefits:

$$E^*(B(a,x)) = \sum_{n=1}^{N} q_n^* B(a, x_n)$$

EQUIVALENCE

Maximizing E*(Benefits)

- Suppose policymaker chooses a so as to maximize E^*(Benefits).

- Then, $\widehat{a}$ satisfies FOC:

$$E^*\{\frac{\partial B}{\partial a}(\widehat{a}, x)\} = 0$$

Result - Setup

$$\begin{aligned} 0 &= E^*\{\frac{\partial B}{\partial a}(\widehat{a}, x)\} \\ &= \sum_{n=1}^{N} q_n^* \{\frac{\partial B}{\partial a}(\widehat{a}, x_n)\} \end{aligned}$$

- But we know that for some constant $\xi > 0$:

$$q_n^* = \xi \pi_n MUC_n(\widehat{a})$$

Result - Conclusion

- It follows that $\widehat{a}$ also satisfies:

$$0 = \sum_{n=1}^{N} \pi_n MUC_n(\widehat{a}) \frac{\partial B}{\partial a}(\widehat{a}, x_n)$$

- This is the same FOC that characterized a^*.

- Thus: maximizing E^*(Benefits) is the same as maximizing social welfare.
  - But: maximizing E^* only requires knowledge of RNPD.

Verbal Summary

- Standard: Policymaker's optimal choice sets the *outlook* for marginal net benefits equal to zero.

- **Novel: The appropriate notion of the outlook is given by E^*.**

- Policymaker should balance benefits across states of the world using households' relative marginal valuations of resources in different states.

- The relative marginal valuations are given by RNPD, not statistical density.

CONCERNS

Lack of Predictive Power

Concern: RNPDs predict poorly.

Response: This is true but irrelevant.

- Policymakers' decisions should be based on households' relative valuations of resources in different states.

- These aren't predictive: they incorporate subjective probabilities and marginal utilities.

Heterogeneity

Concern: Households aren't the same.

Response: The basic equivalence result extends as long as ...

- Redistributions of resources generated by choice of a can be offset using transfers.

- Similar to: "expanding the pie" argument for free trade.

Costly Information Acquisition

Concern: Possible loss of private incentives to acquire information.

- If policy is set so as to keep an asset's current price constant ...
- Investors have no incentive to get information about its future payoffs.
- Consequence: policy choice does not adequately reflect available information.
- See Bernanke-Woodford (1997) for elegant exposition.

Response

- This concern is mitigated by existence of options with varying strikes.

- With options, investors value information about each outcome of x even if the policymaker ensures that $E^*(\frac{\partial B}{\partial a}(x, a^*))$ always equals zero.

- Note: In constructing RNPDs, we need data on prices from many options with distinct strikes.

Incompleteness of Observed Assets

Concern: Given observed assets, there may be multiple RNPDs.

Response: The basic equivalence result extends as long as ...

- For any action a, the benefit $B(a, x)$ is spanned by the payoffs of observed assets.

- Even without spanning: we can find upper and lower bounds to $B(a, x)$ consistent with absence of arbitrage.

Limited Participation

Concern: Few households trade in option mkts used to construct RNPDs.

Response: This is a problem if they're barred from participating.

- However, I find it more plausible that they are *choosing* not to participate.

- That decision suggests that their relative marginal valuations of resources in various states are similar to that implied by option markets.

Illiquidity

Concern: Asset prices could differ because of liquidity, not risk, differences.

Response: This is a potential issue.

- Specifically: options with similar strikes might have very different prices.
- Right response: appropriate attention to robustness.
- Wrong response: abandon RNPDs completely.

CONCLUSIONS

- Policy decisions often impact the economy a lag.
- Hence, policymakers need some way to gauge the relative likelihoods of future events.
- Monetary: How likely is deflation? How likely is high inflation?
- Financial regulation: How likely is significant financial instability?

- Typical approach: attempt to figure out "true" probability of future events.

- Point of this talk: For policymakers that care about social welfare, the relevant probability is a risk-neutral probability.

- RNPDs encode households' ex-ante marginal valuations of resources in different states.

- Good policymaking should be based on these relative valuations.

- Thus, the risk-neutral probability of deflation could rise because:
  - Households view that outcome as more likely
  - Households' marginal utility of resources in that outcome has risen.

- Both of these changes should matter for a monetary policymaker who can influence the likelihood of deflation.

Implementation Challenges

- Decision-making using RNPDs is not necessarily easy.
  - Need to determine appropriate financial proxy for relevant event.
  - Even then: Available options may not cover longer horizons or extreme tail events.
- Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.

Ninth District Activities

- Minneapolis Fed's Banking Group uses options data to compute RNPDs.
- They report the results on the public website for a wide range of assets.
  - Gold, silver, wheat, S&P 500, exchange rates, etc.
- They report and archive the results on a biweekly basis.
- See http://www.minneapolisfed.org/banking/rnpd.