Duke University, Fuqua School of Business
University of Pennsylvania, Wharton School of Business

Central Bank Policy Impacts on the Distribution of Future Interest Rates

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Notes for presentation at the Federal Reserve Bank of New York/
New York University Conference on "Risk Neutral Densities"

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I. State Prices and Risk Neutral Densities Implicit in Prices of Interest Rate Caps and Floors

Disadvantages of Many Prior Approaches for Estimating Risk Neutral Densities

- 1. ***Short-term option prices used***.

  Most options mature in 3 months to 18 months, as many markets only have active markets for those maturities. Often there are not options actively traded for a large number of standardized strike prices.

- 2. ***Parametric vs. nonparametric approach.***

  Applications often parameterize option prices with 3 or 4 parameters (mean, variance, skewness, kurtosis) and estimate implied volatility surfaces and entire risk-neutral densities. It is well-known among practitioners that these methods can be off significantly in estimating tail risks.

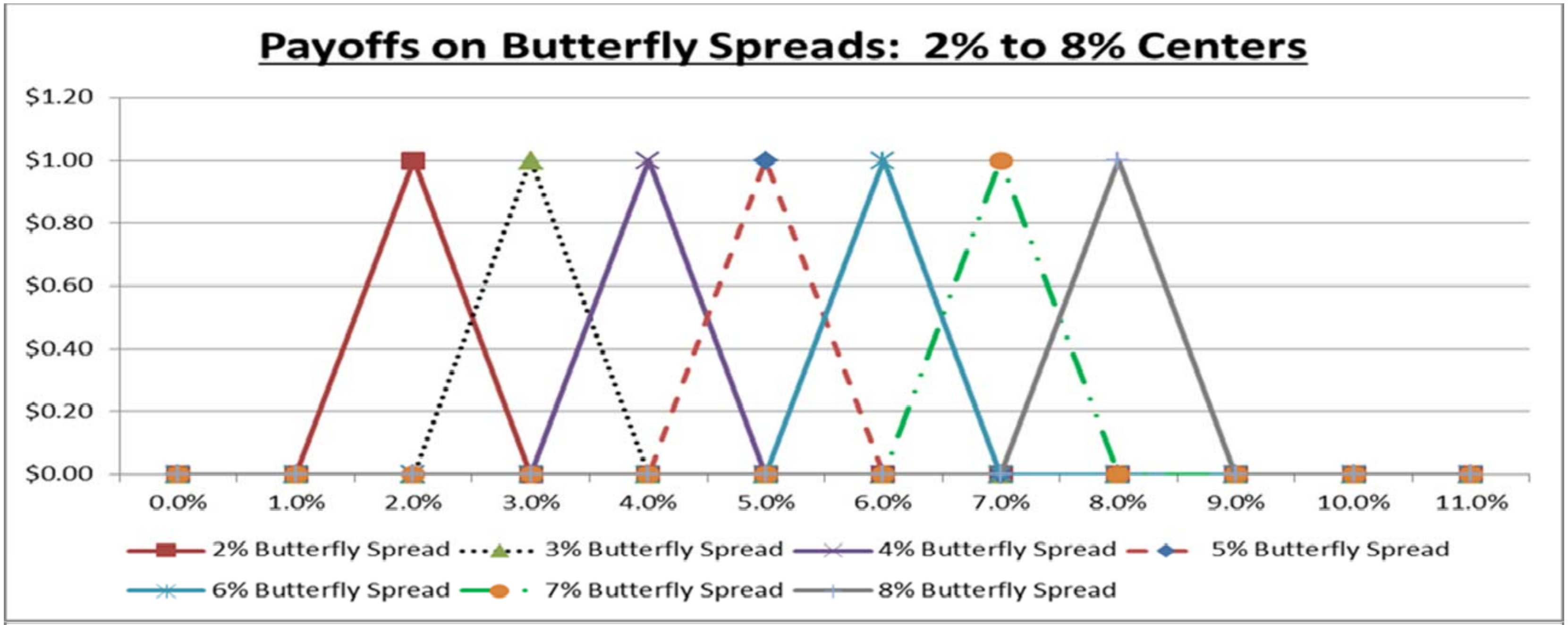
State Prices Implicit in Interest Rate Cap and Floor Prices

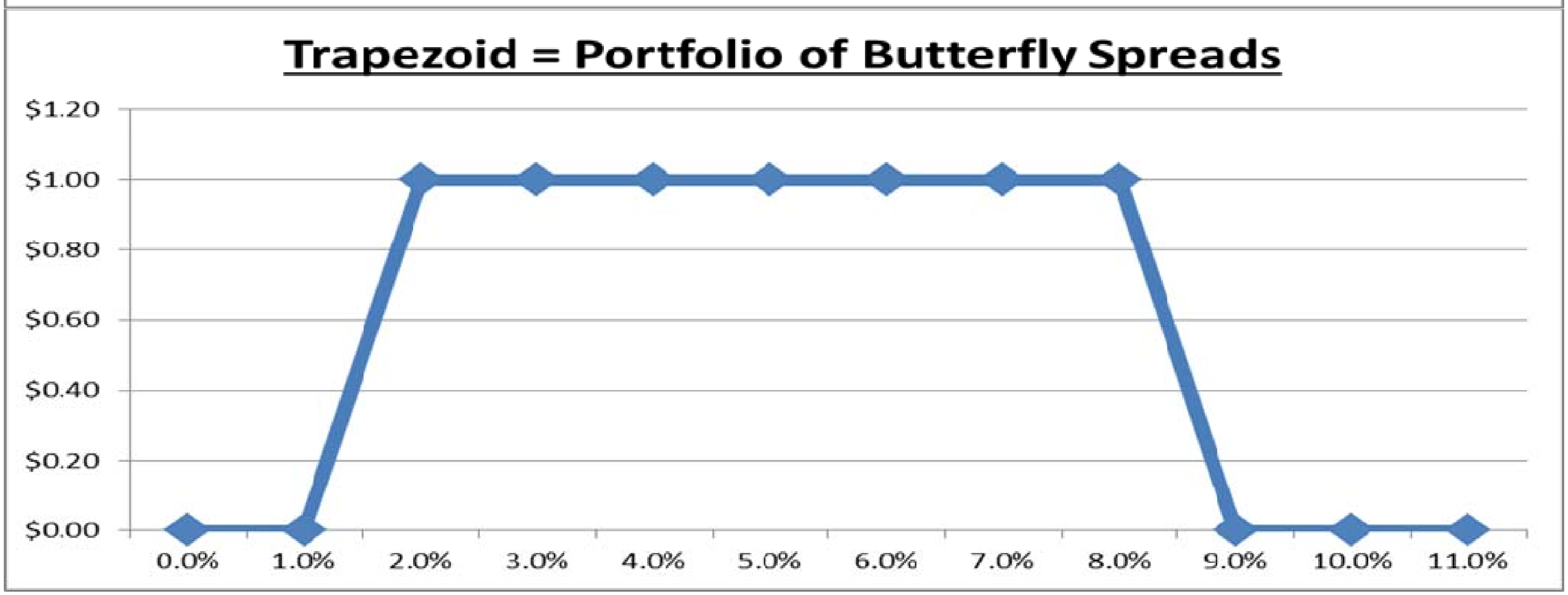
- Interest rate caps and floors are portfolios of long-term put and call options on 3-month LIBOR. No option on first quarter rate, so 19 quarterly options on 5-yr floor, and 11 quarterly options on 3-year floor. Caps and floors are portfolios of long-term options and are traded in large volumes by many portfolio managers and financial institutions to hedge/manage option risk.

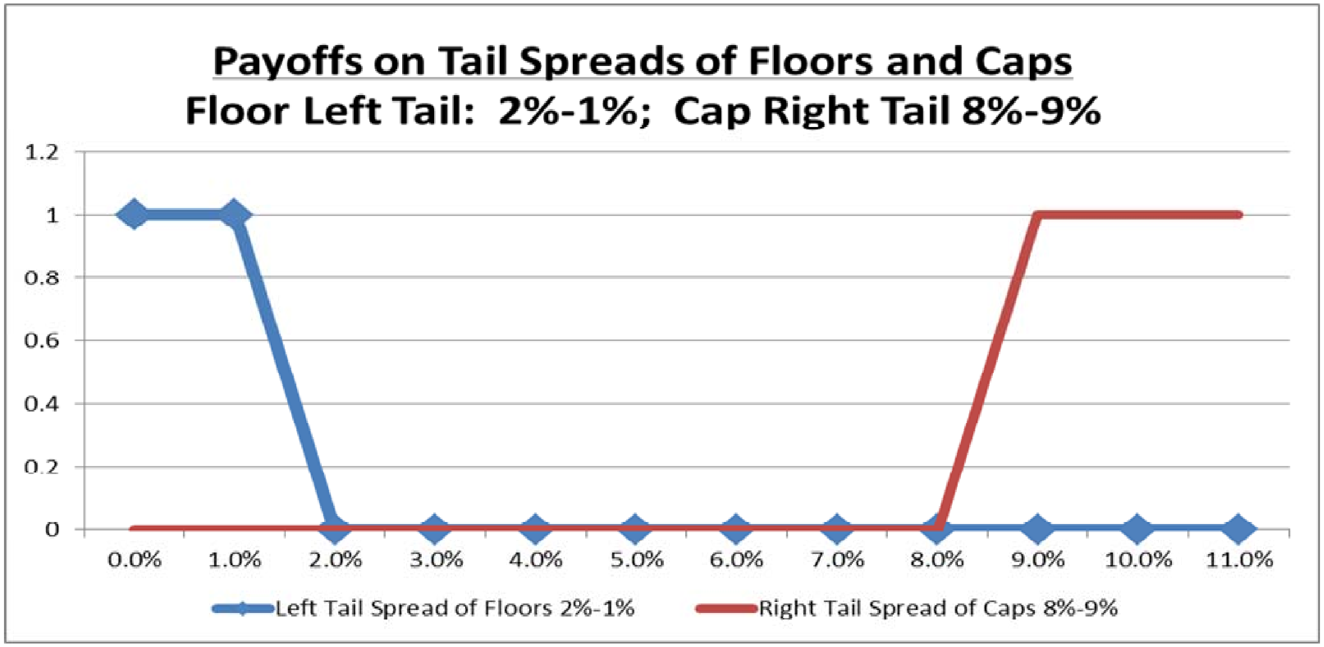
- Difference between 5-yr floor price and 4-year floor price is value of 4 quarterly options on LIBOR in year 5, a "floorlet." Similar for "caplets."

- Approach: Compute butterfly spreads of option prices with various strike rates, per Breeden-Litzenberger 1978, to get prices of (triangles of) state contingent claims, proportional to the "risk neutral density."

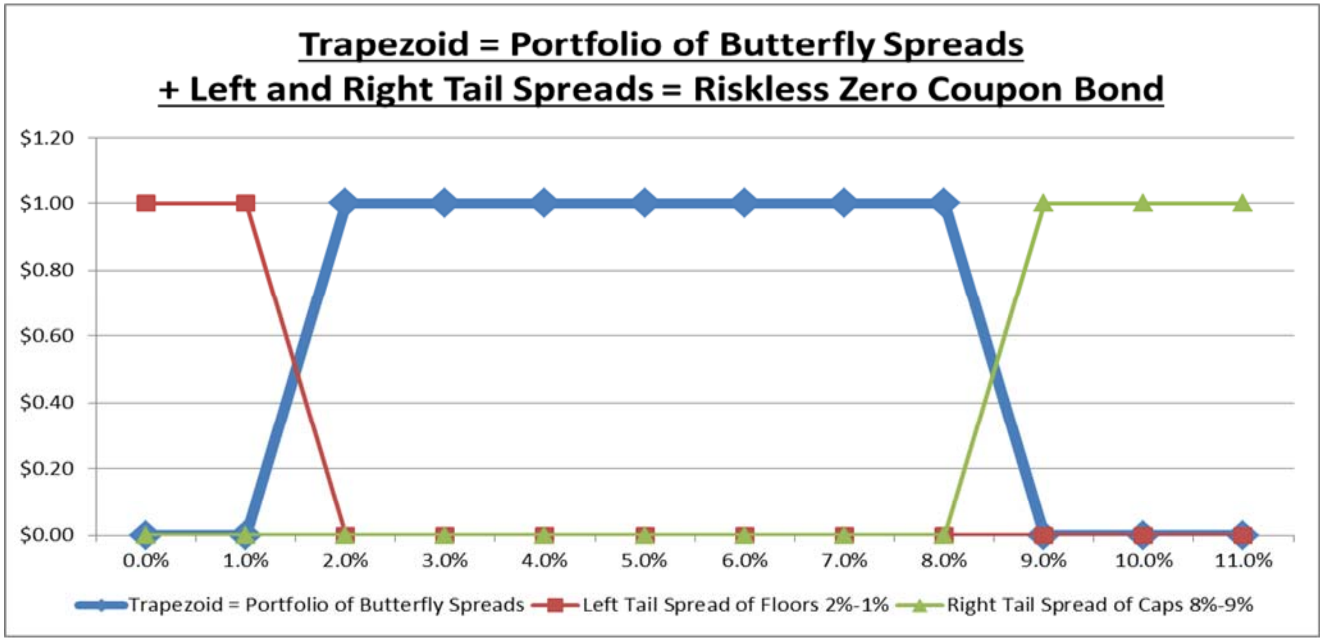
- Example: Long 1 floor with strike rate of 2%, short 2 for 3%, long 1 for 4% gives payoff only between 2% and 4%, peaking at 3%.


Payoffs on Butterfly Spreads: 2% to 8% Centers
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
2% Butterfly Spread
3% Butterfly Spread
4% Butterfly Spread
5% Butterfly Spread
6% Butterfly Spread
7% Butterfly Spread
8% Butterfly Spread


Trapezoid = Portfolio of Butterfly Spreads
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%



Payoffs on Tail Spreads of Floors and Caps
Floor Left Tail: 2%-1%; Cap Right Tail 8%-9%
1.2
1
0.8
0.6
0.4
0.2
0
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
Left Tail Spread of Floors 2%-1%
Right Tail Spread of Caps 8%-9%



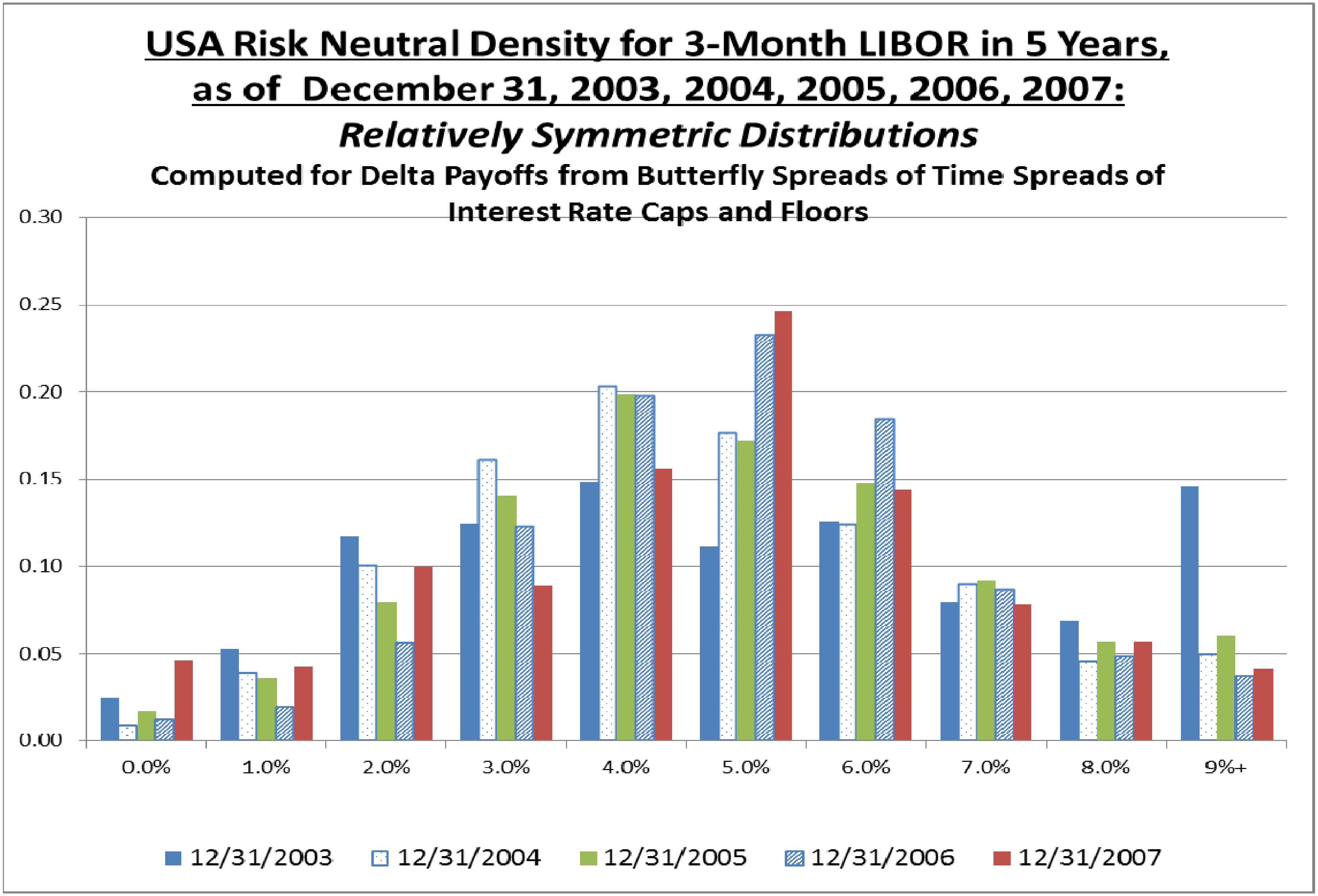
Trapezoid = Portfolio of Butterfly Spreads
+ Left and Right Tail Spreads = Riskless Zero Coupon Bond
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
Trapezoid = Portfolio of Butterfly Spreads
Left Tail Spread of Floors 2%-1%
Right Tail Spread of Caps 8%-9%

Butterfly Spread and Tail Spread Costs and Risk Neutral Probabilites

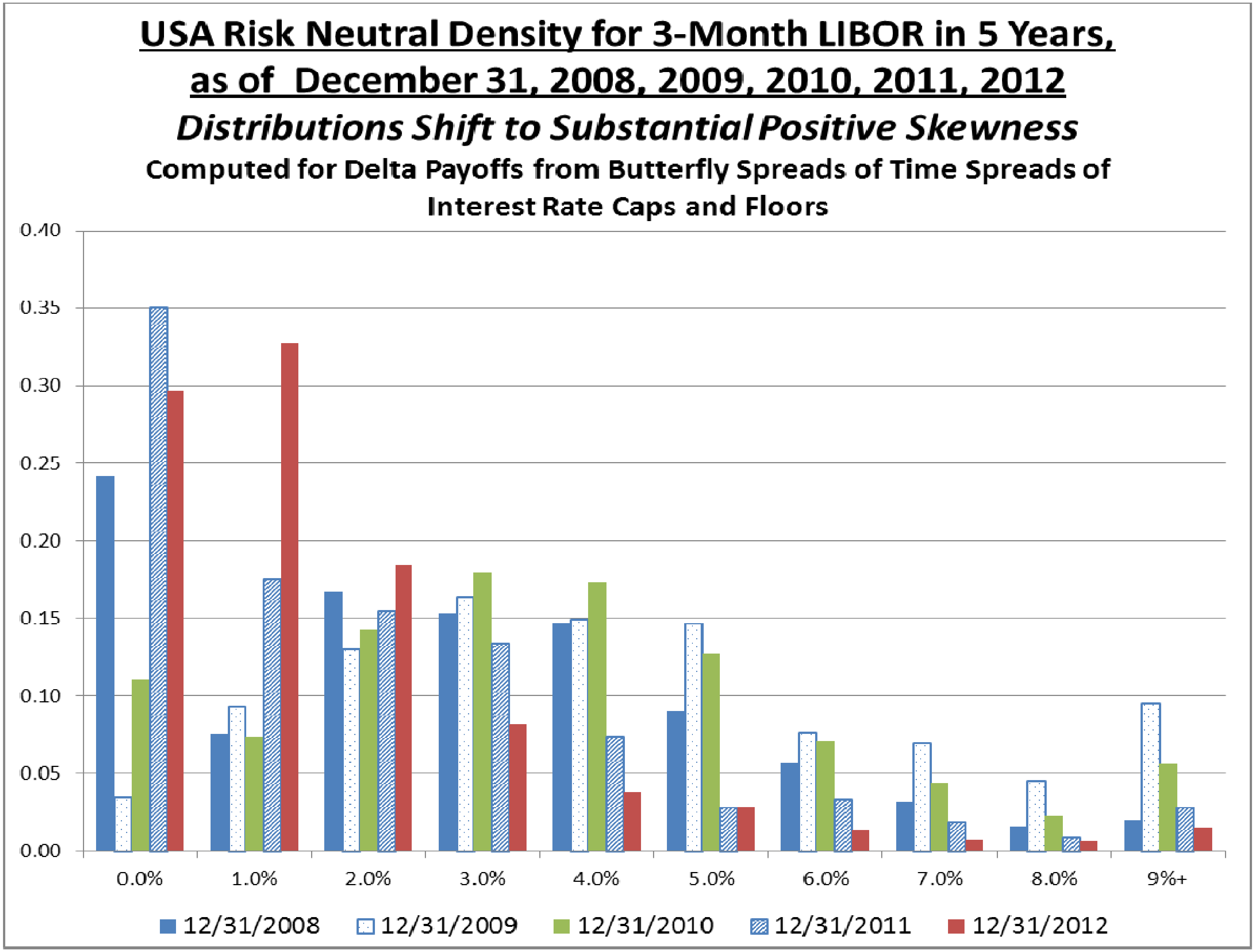
Figure 6F

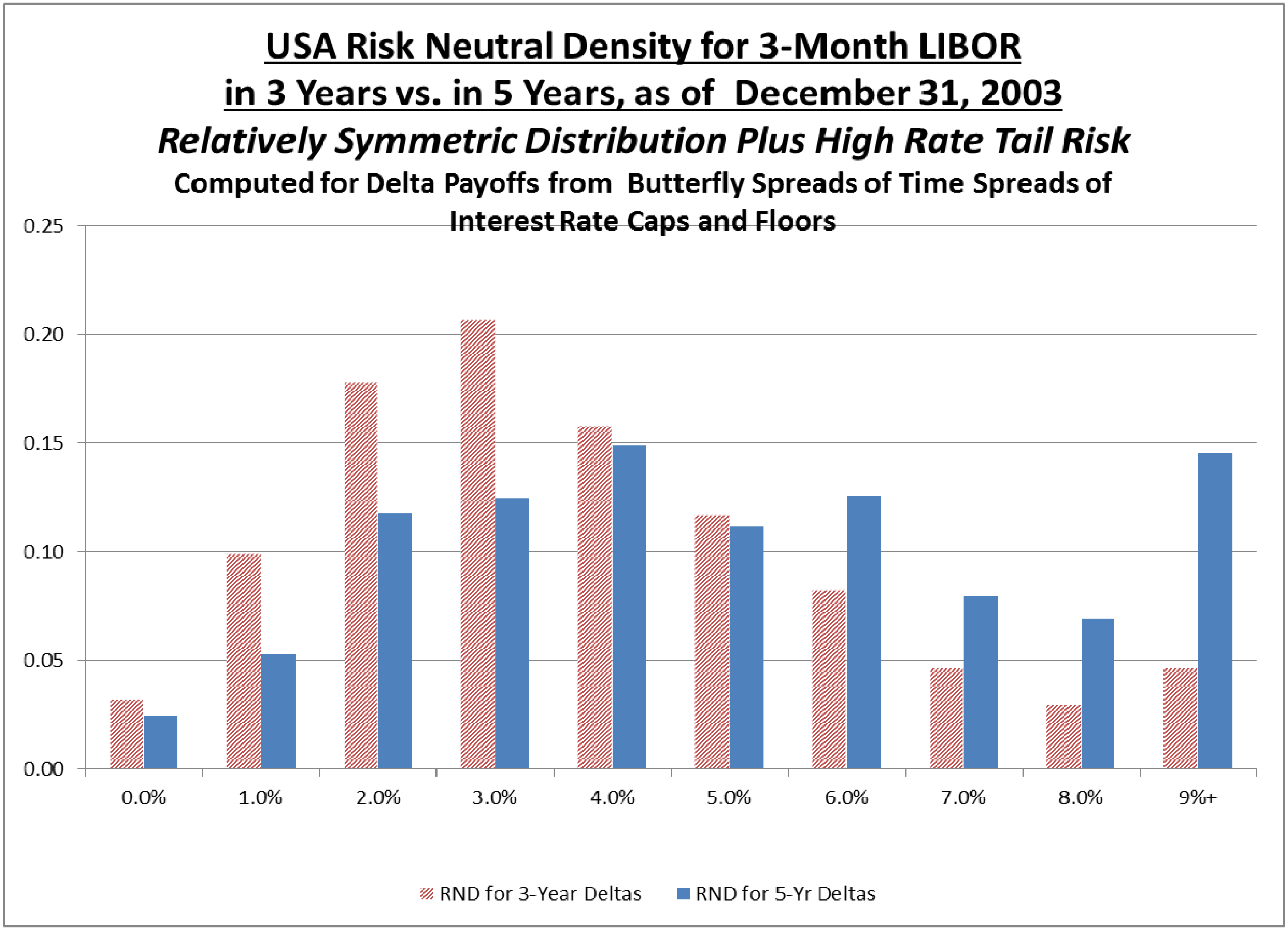
	Spread Cost	"Risk-Neutral Probability"
"0%" = Left tail spread: Long 1%, Short 0% floorlet	$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	$0.180	0.184
3% Butterfly spread	$0.080	0.082
4% Butterfly spread	$0.037	0.038
5% Butterfly spread	$0.028	0.028
6% Butterfly spread	$0.014	0.014
7% Butterfly spread	$0.007	0.007
8% Butterfly spread	$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	$0.015	0.015
Totals	$0.977	1.000

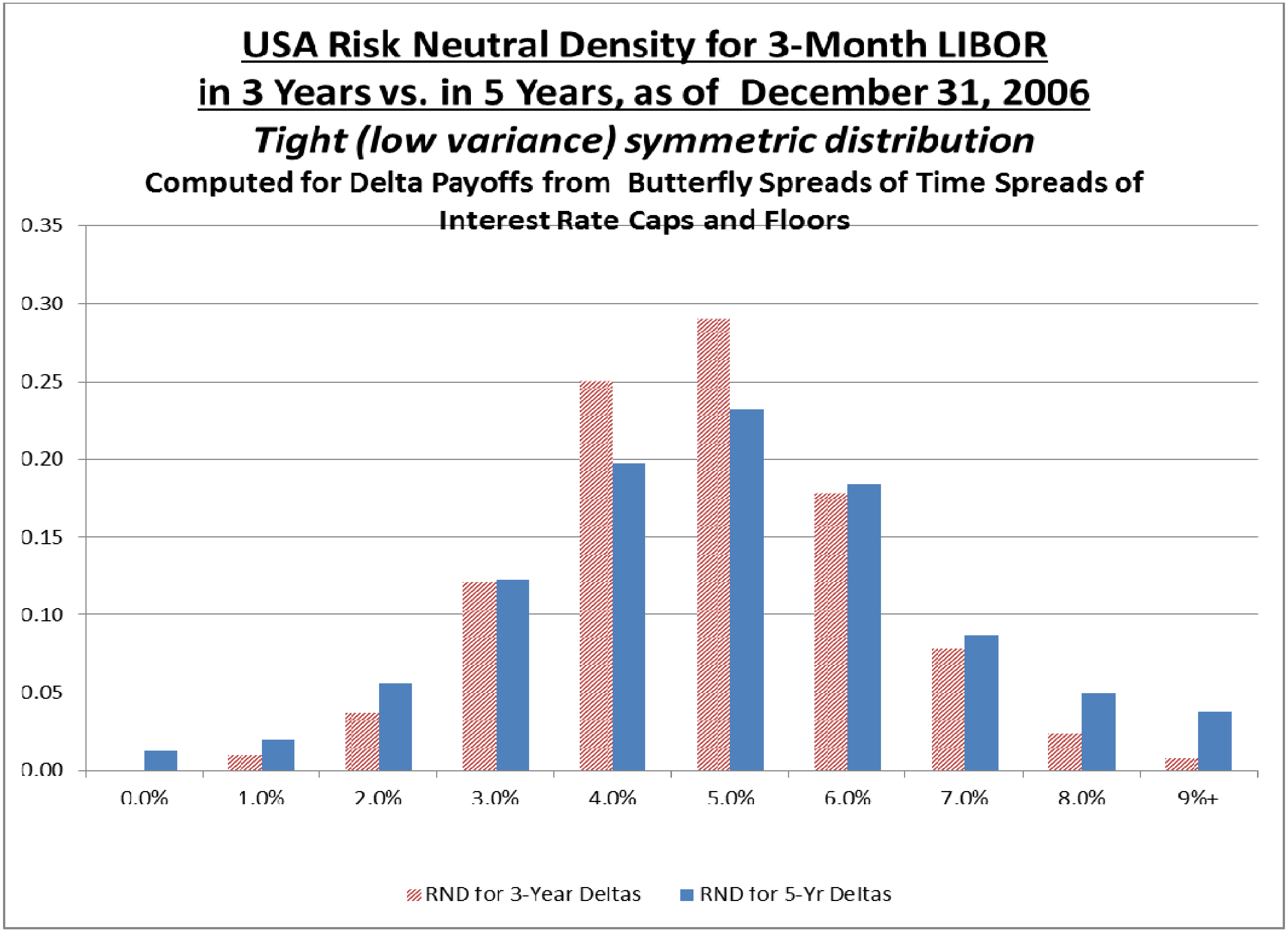
II. Estimates of USA State Prices Implicit in Prices of Interest Rate Caps and Floors, 2003-2012.



USA Risk Neutral Density for 3-Month LIBOR in 5 Years,
as of December 31, 2003, 2004, 2005, 2006, 2007:
Relatively Symmetric Distributions
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
12/31/2003
12/31/2004
12/31/2005
12/31/2006
12/31/2007

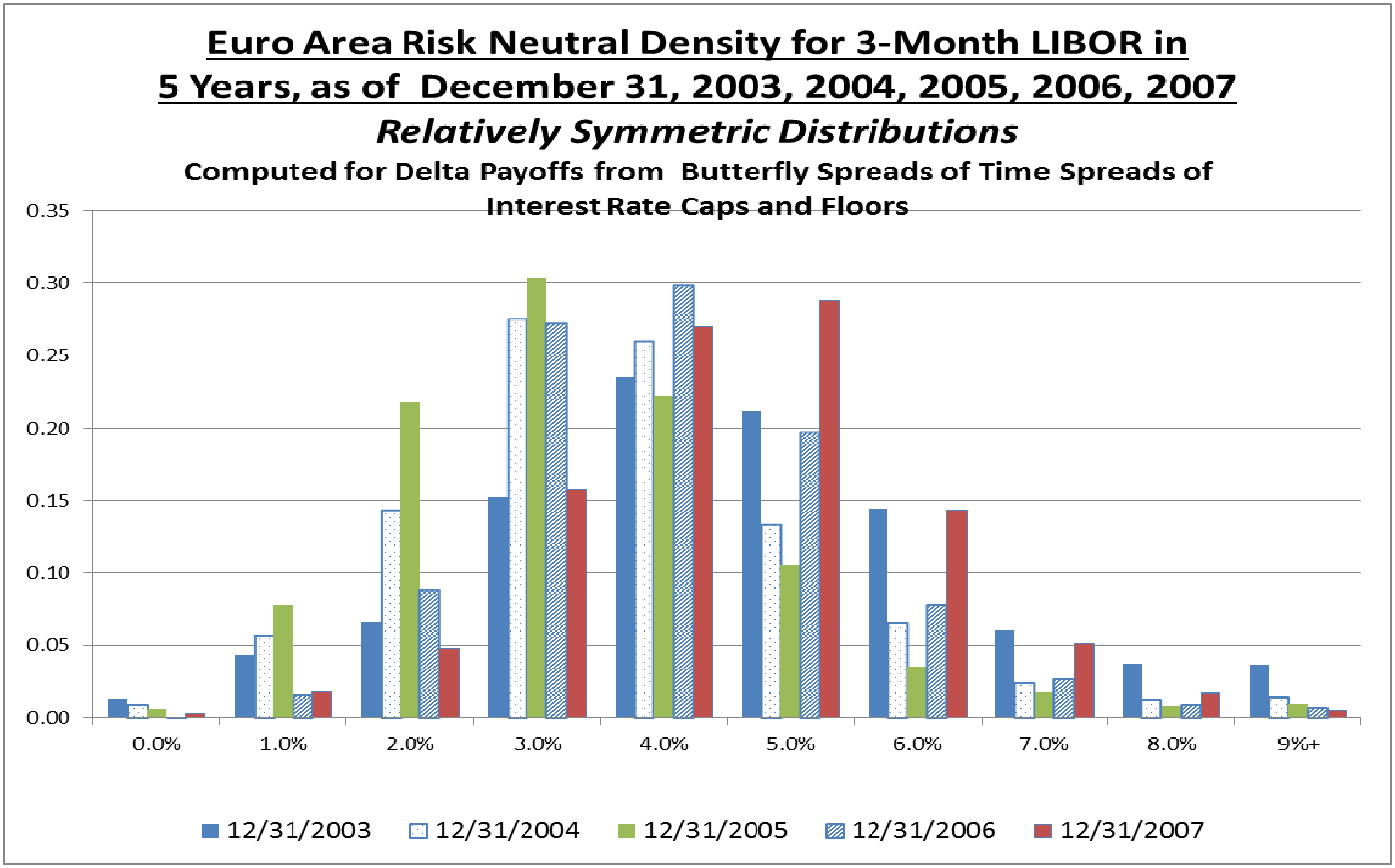

USA Risk Neutral Density for 3-Month LIBOR in 5 Years,
as of December 31, 2008, 2009, 2010, 2011, 2012
Distributions Shift to Substantial Positive Skewness
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
12/31/2008
12/31/2009
12/31/2010
12/31/2011
12/31/2012

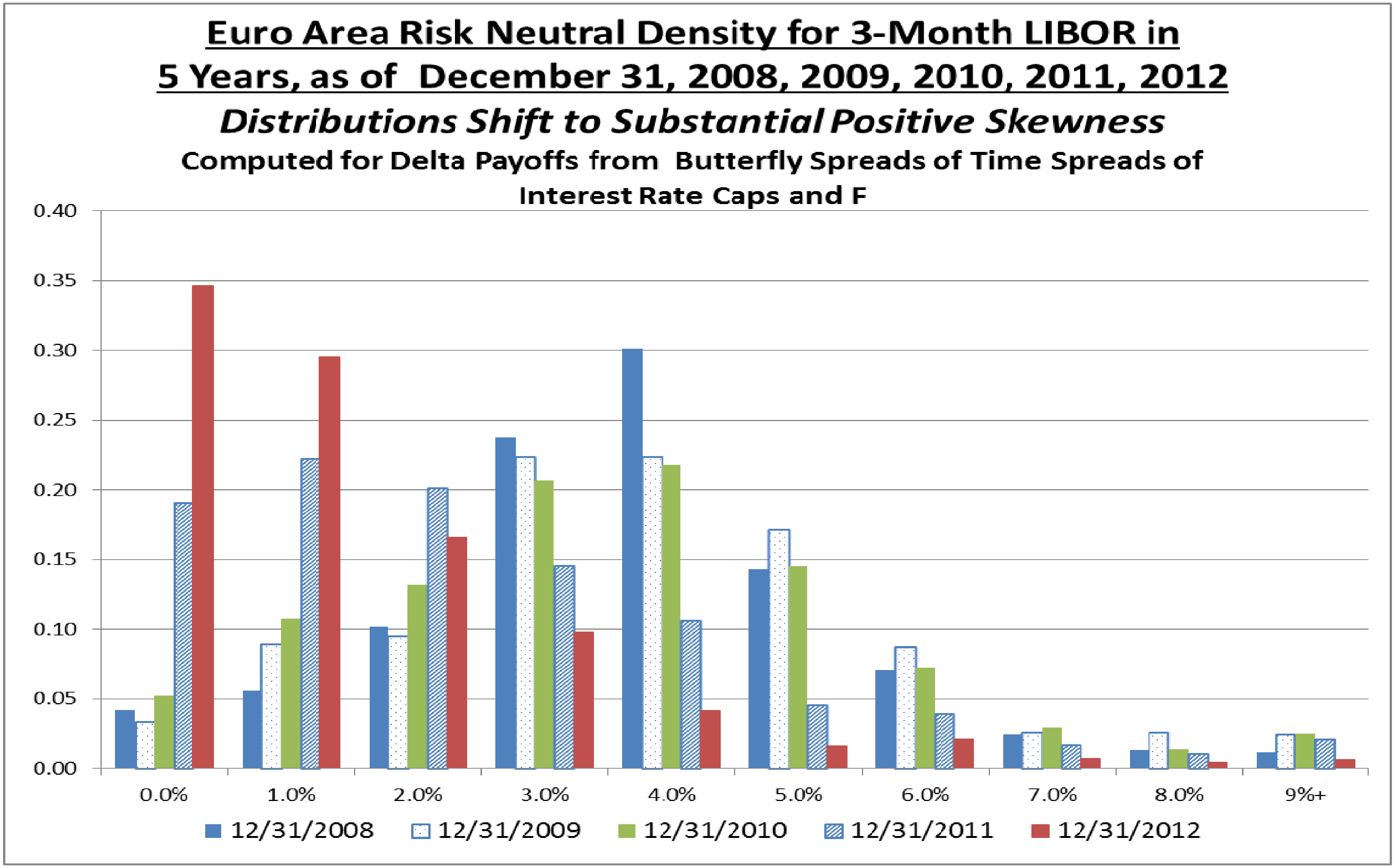

USA Risk Neutral Density for 3-Month LIBOR
in 3 Years vs. in 5 Years, as of December 31, 2003
Relatively Symmetric Distribution Plus High Rate Tail Risk
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of
Interest Rate Caps and Floors
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
RND for 3-Year Deltas
RND for 5-Yr Deltas

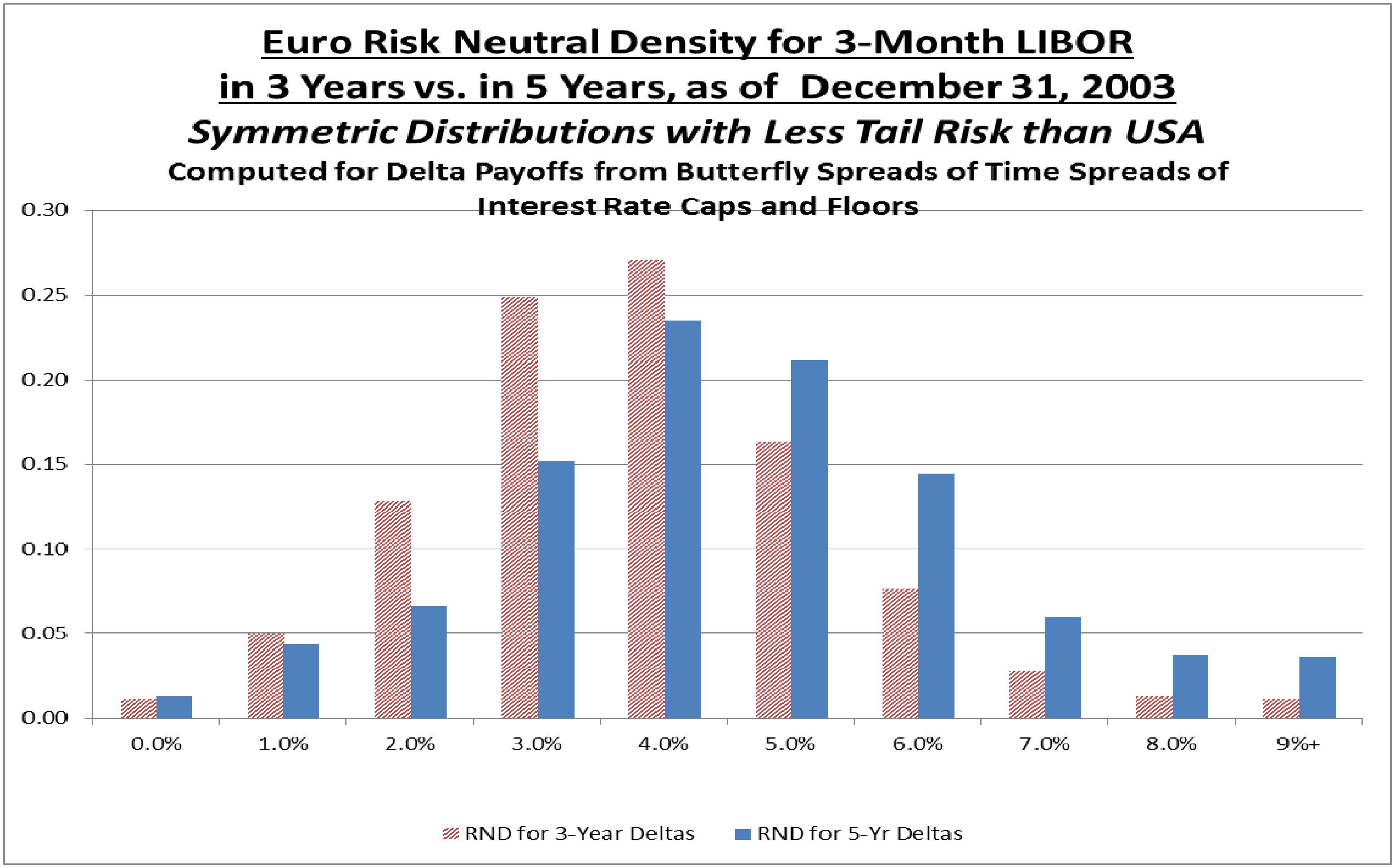


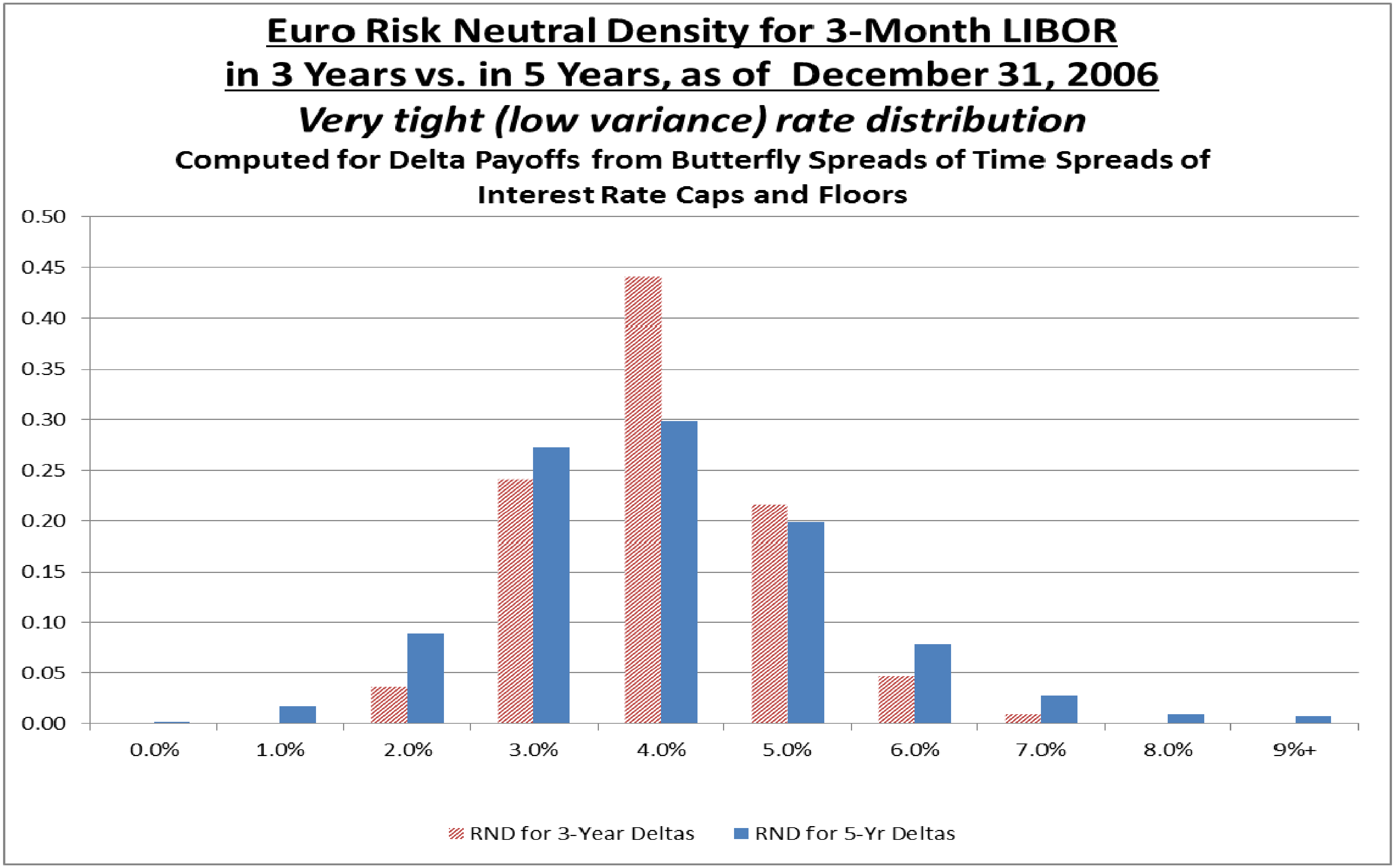
USA Risk Neutral Density for 3-Month LIBOR
in 3 Years vs. in 5 Years, as of December 31, 2006
Tight (low variance) symmetric distribution
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
RND for 3-Year Deltas
RND for 5-Yr Deltas

III. Risk Neutral Densities for 6-Month Euro LIBOR from Dec 2003 to Dec 2012


Euro Area Risk Neutral Density for 3-Month LIBOR in
5 Years, as of December 31, 2003, 2004, 2005, 2006, 2007
Relatively Symmetric Distributions
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of
Interest Rate Caps and Floors
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
12/31/2003
12/31/2004
12/31/2005
12/31/2006
12/31/2007


Euro Area Risk Neutral Density for 3-Month LIBOR in
5 Years, as of December 31, 2008, 2009, 2010, 2011, 2012
Distributions Shift to Substantial Positive Skewness
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of
Interest Rate Caps and F
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
12/31/2008
12/31/2009
12/31/2010
12/31/2011
12/31/2012


Euro Risk Neutral Density for 3-Month LIBOR
in 3 Years vs. in 5 Years, as of December 31, 2003
Symmetric Distributions with Less Tail Risk than USA
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
RND for 3-Year Deltas
RND for 5-Yr Deltas

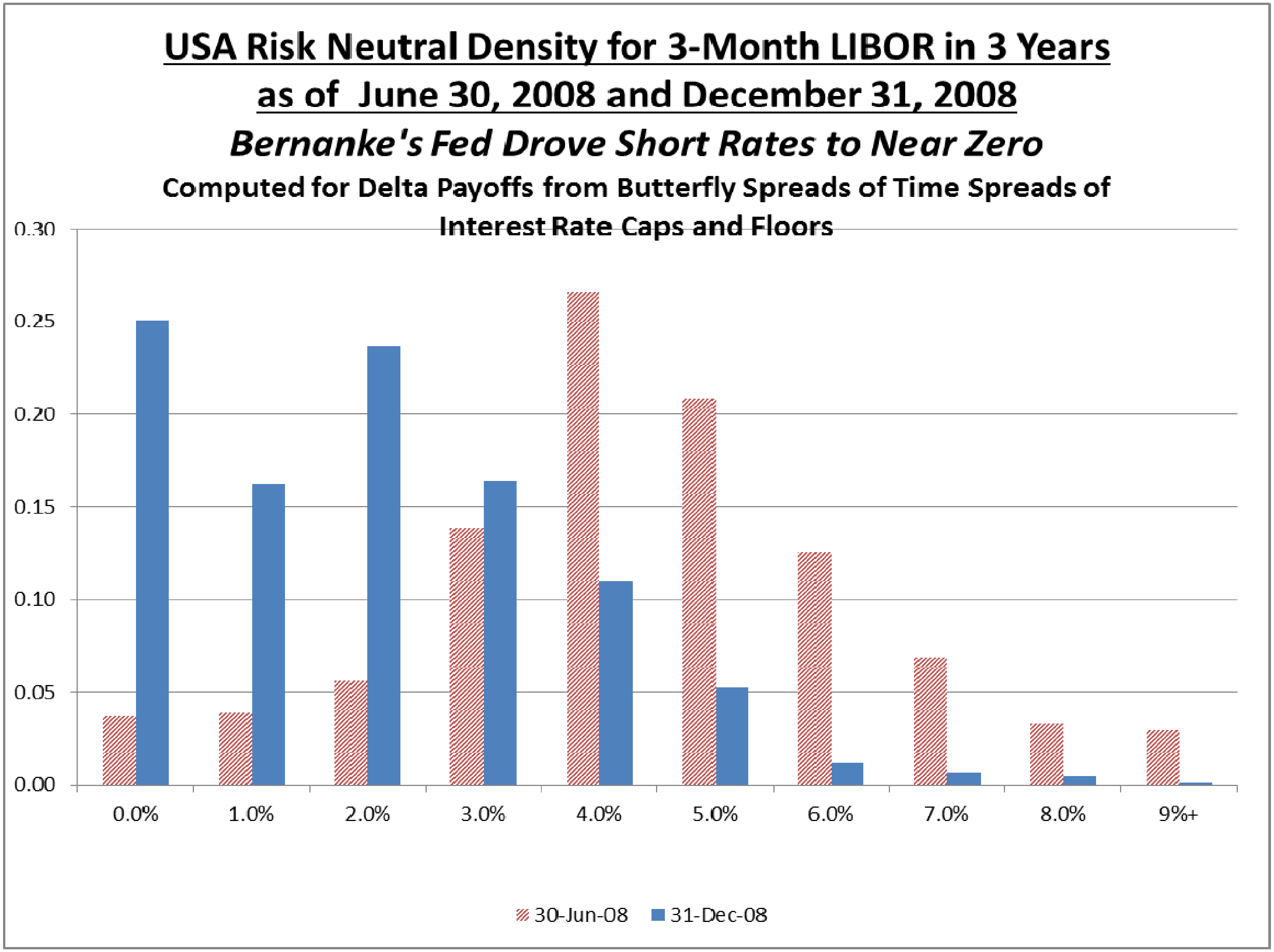


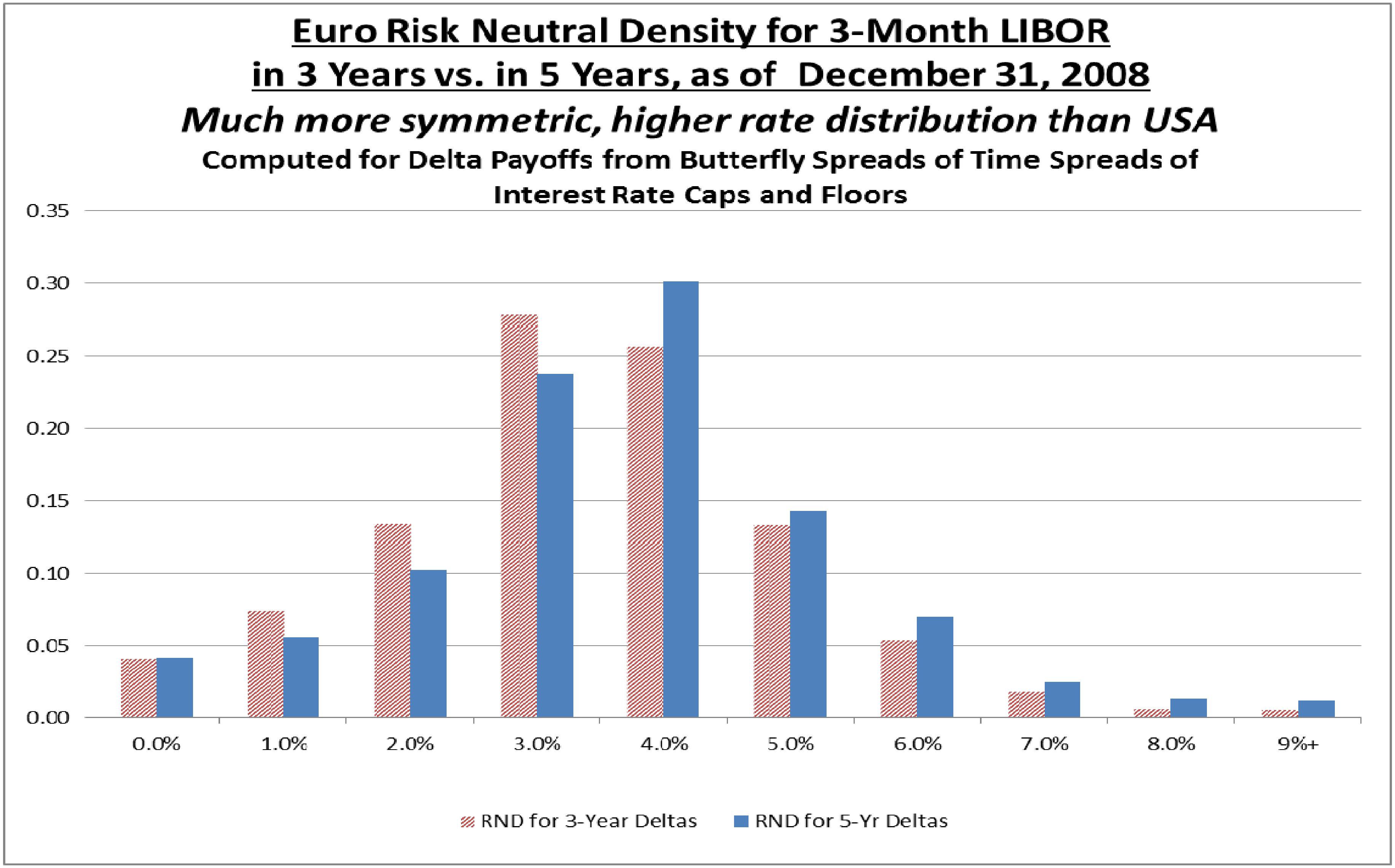
Euro Risk Neutral Density for 3-Month LIBOR
in 3 Years vs. in 5 Years, as of December 31, 2006
Very tight (low variance) rate distribution
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
RND for 3-Year Deltas
RND for 5-Yr Deltas

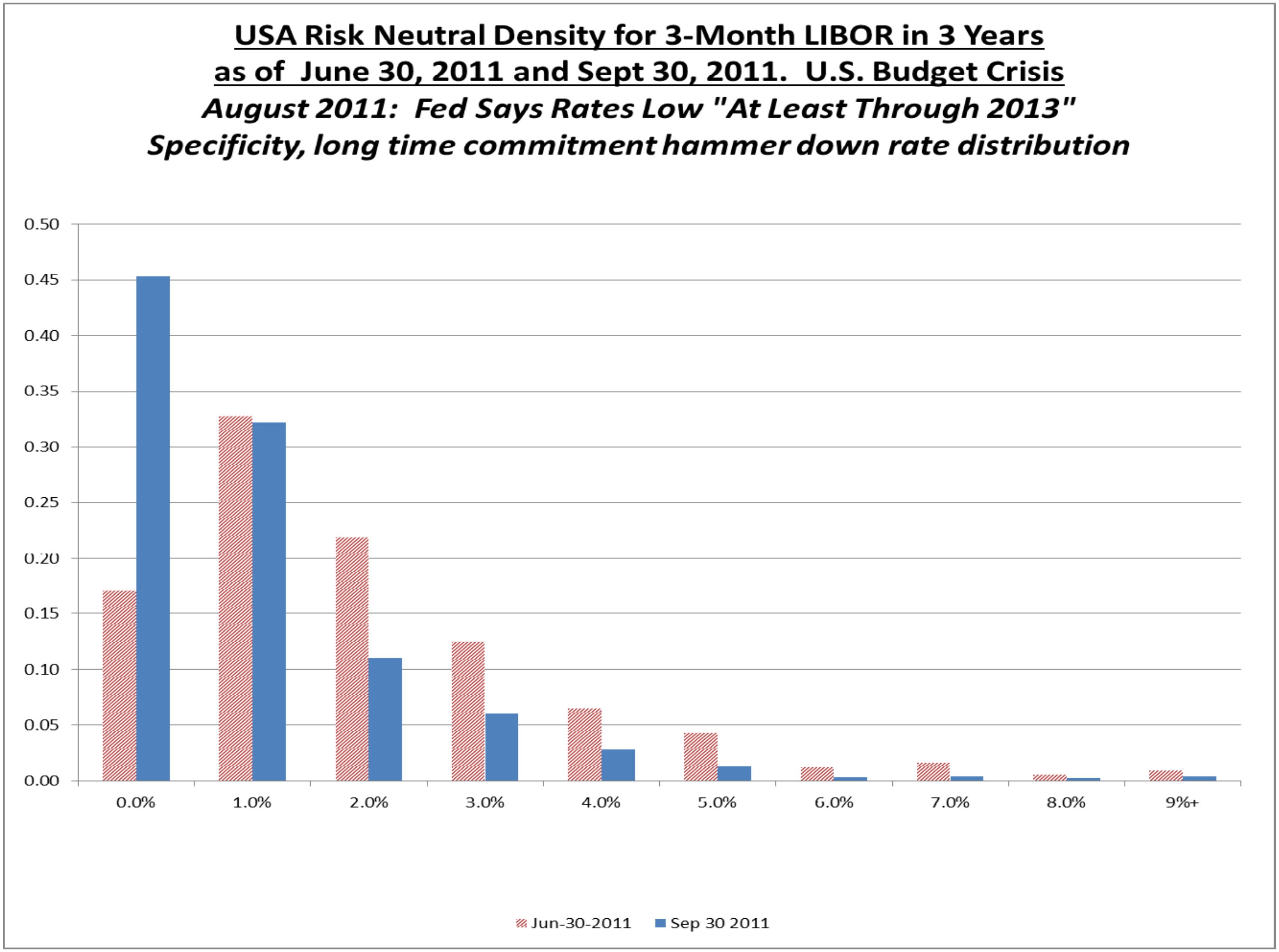
IV. Impact of USA Federal Reserve Policy Announcements on State Prices and Risk Neutral Densities for Future Levels of 3-Month LIBOR

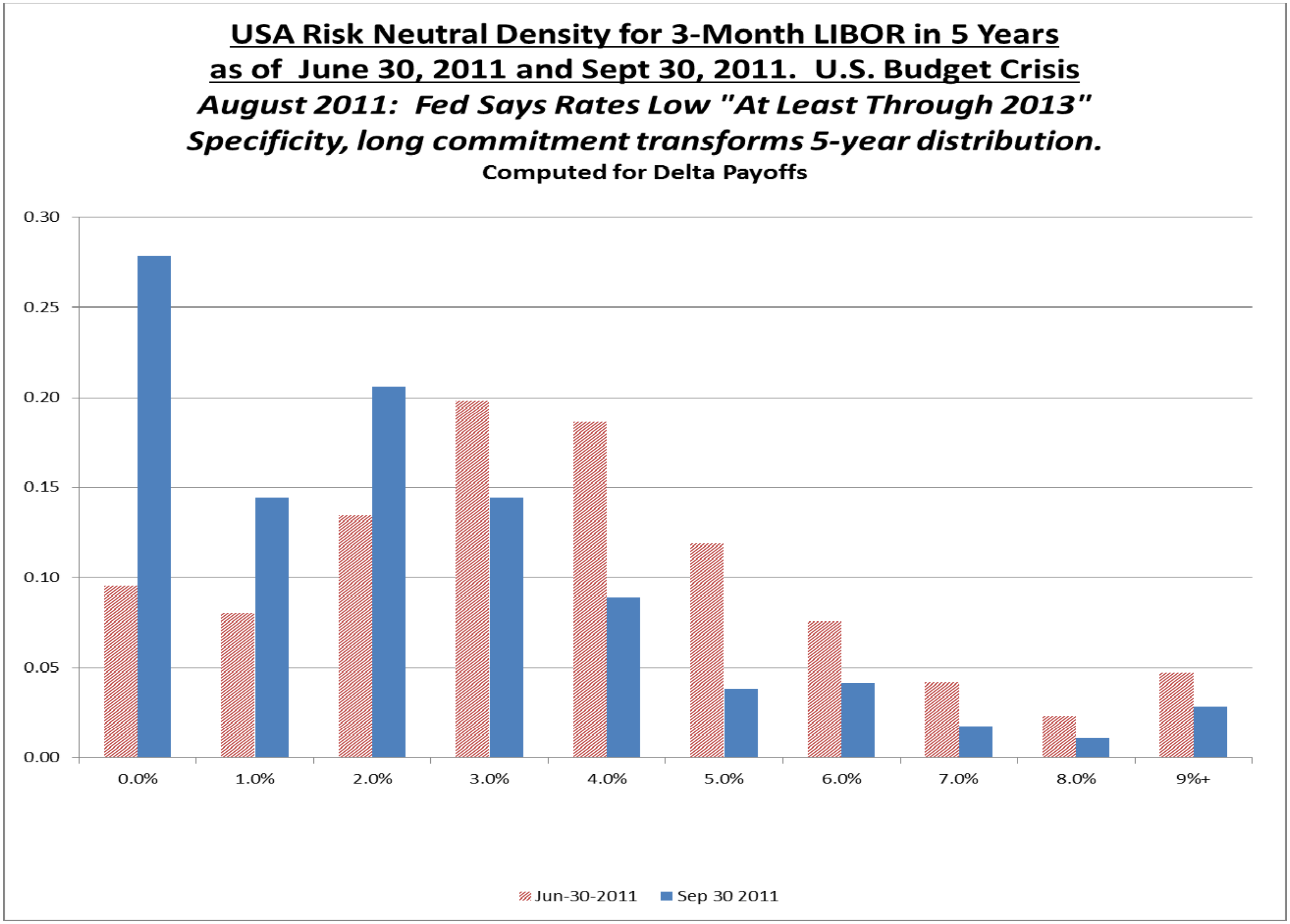
Major Federal Reserve Policy Announcements 2008-2013

- December 2008. Cut rates to record lows in financial panic.
- March 2009: Will keep rates close to zero for "extended period." Stock market bottoms March 9th.
- August 2011: Will keep rates extremely low "at least until 2013."
- September 2012: Low "at least until 2015"
- December 2012: Will tie low rates to range in Unemployment (>6.5%) and Inflation (<2%).
- May/June 2013: **May 22**: Given economic strength, Fed is seriously considering "tapering" asset purchases (QE3). **June 19:** Housing market is strong and supportive; tapering QE3 likely in 2nd half 2013.


USA Risk Neutral Density for 3-Month LIBOR in 3 Years
as of June 30, 2008 and December 31, 2008
Bernanke's Fed Drove Short Rates to Near Zero
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interest Rate Caps and Floors
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
30-Jun-08
31-Dec-08


Euro Risk Neutral Density for 3-Month LIBOR
in 3 Years vs. in 5 Years, as of December 31, 2008
Much more symmetric, higher rate distribution than USA
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of
Interest Rate Caps and Floors
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
RND for 3-Year Deltas
RND for 5-Yr Deltas


USA Risk Neutral Density for 3-Month LIBOR in 3 Years
as of June 30, 2011 and Sept 30, 2011. U.S. Budget Crisis
August 2011: Fed Says Rates Low "At Least Through 2013"
Specificity, long time commitment hammer down rate distribution
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
Jun-30-2011
Sep 30 2011


USA Risk Neutral Density for 3-Month LIBOR in 5 Years
as of June 30, 2011 and Sept 30, 2011. U.S. Budget Crisis
August 2011: Fed Says Rates Low "At Least Through 2013"
Specificity, long commitment transforms 5-year distribution.
Computed for Delta Payoffs
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
Jun-30-2011
Sep 30 2011

V. Risk Neutral Densities for Euro LIBOR During the Sovereign Debt Crisis 2010-2013

Key Events in the European Sovereign Debt Crisis European Central Bank 2010-2012

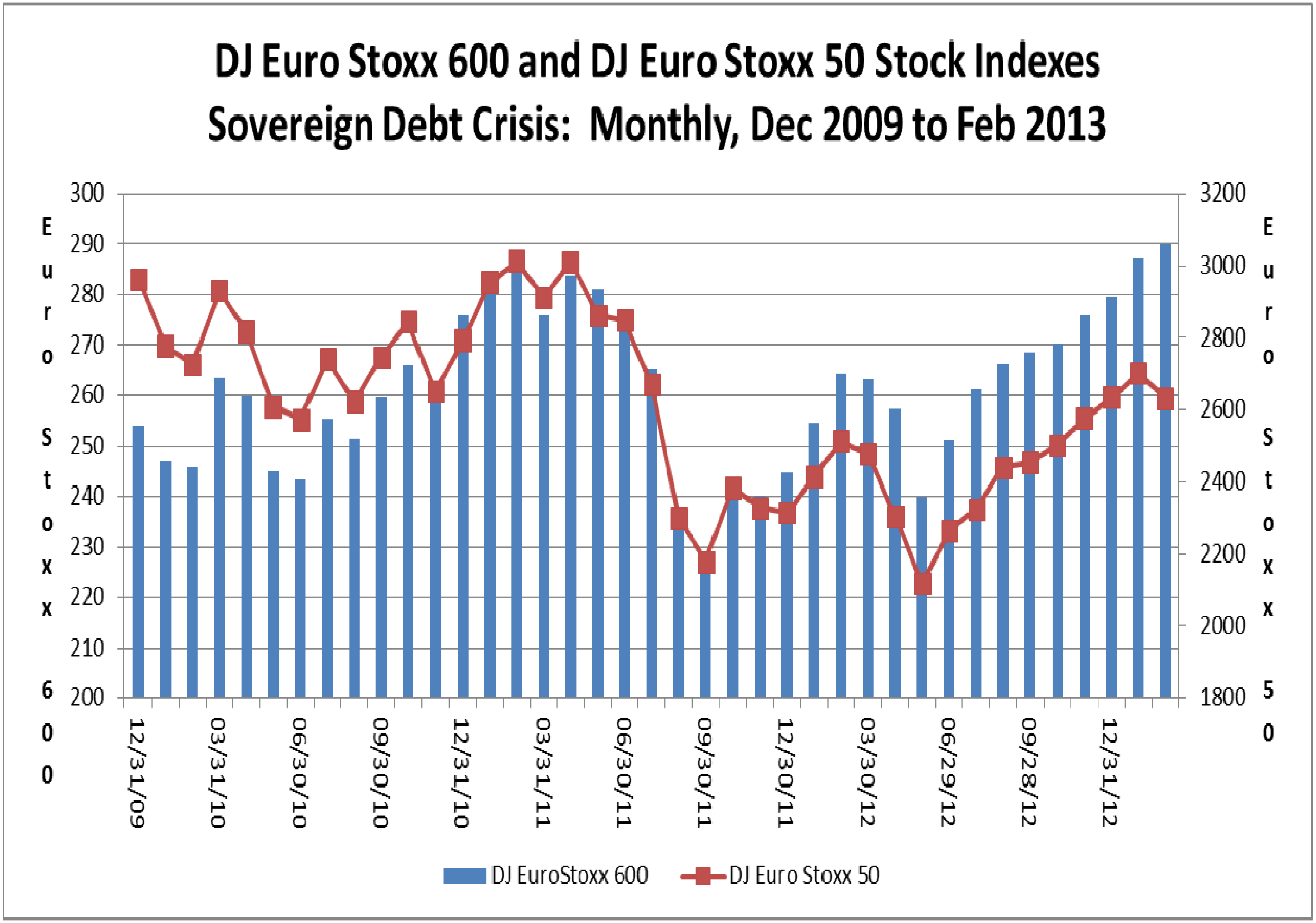
Source: BBC, Reuters

- January 2010. Greek deficit revised upward from 3.7% to 12.7%. "Severe irregularities" in accounting.
- April, May 2010, EU agrees to $30 billion, then $110 billion bailout of Greece. Ireland bailed out in November 2010.

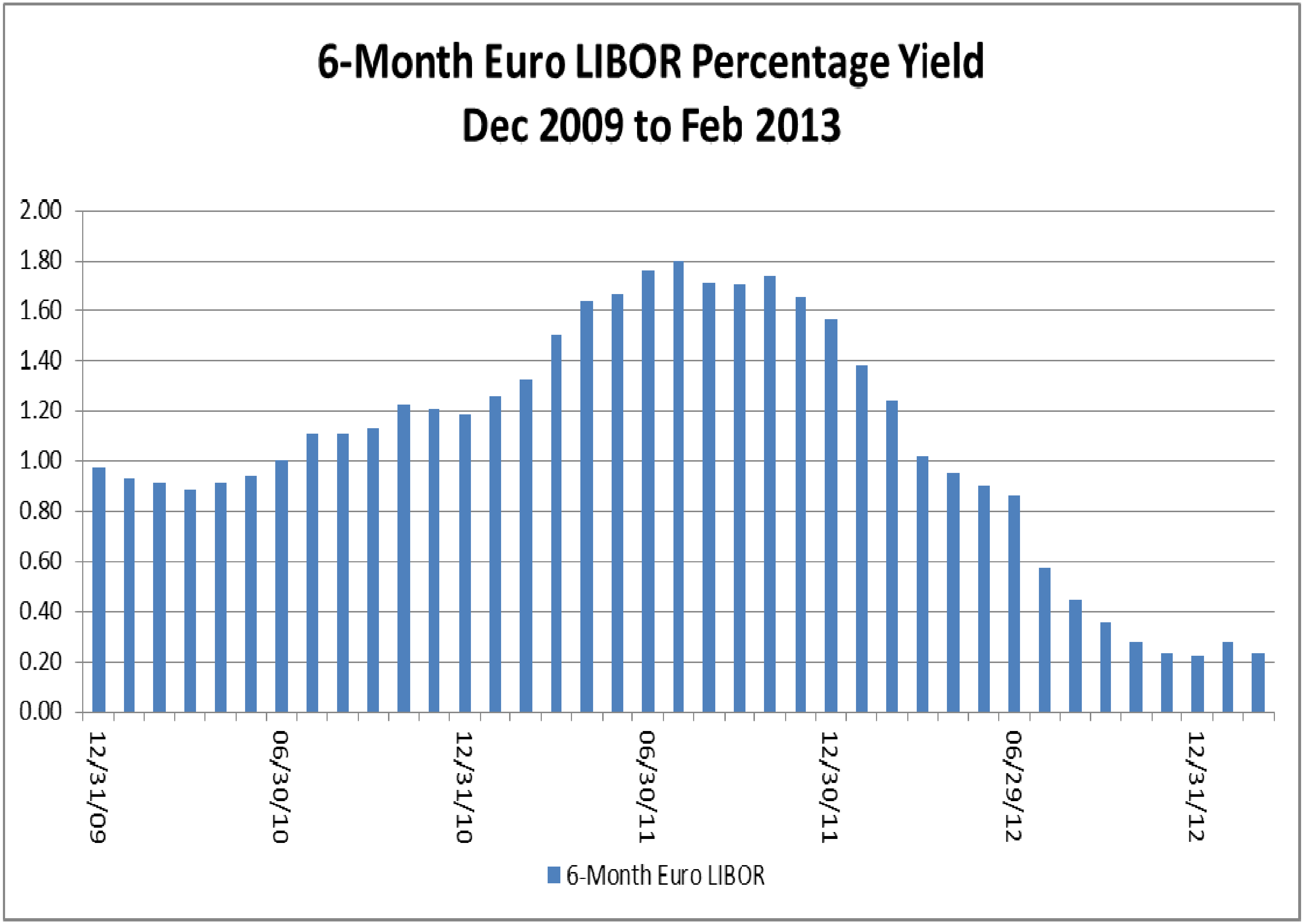
- July 2011: Talk of Greek exit from Euro. Second bailout agreed.
- August 2011: European Commission President Barroso warns sovereign debt crisis spreading. Spain, Italy yields surge.
- November 1, 2011: Mario Draghi takes over European Central Bank from Jean-Claude Trichet. Draghi cuts rates twice quickly.

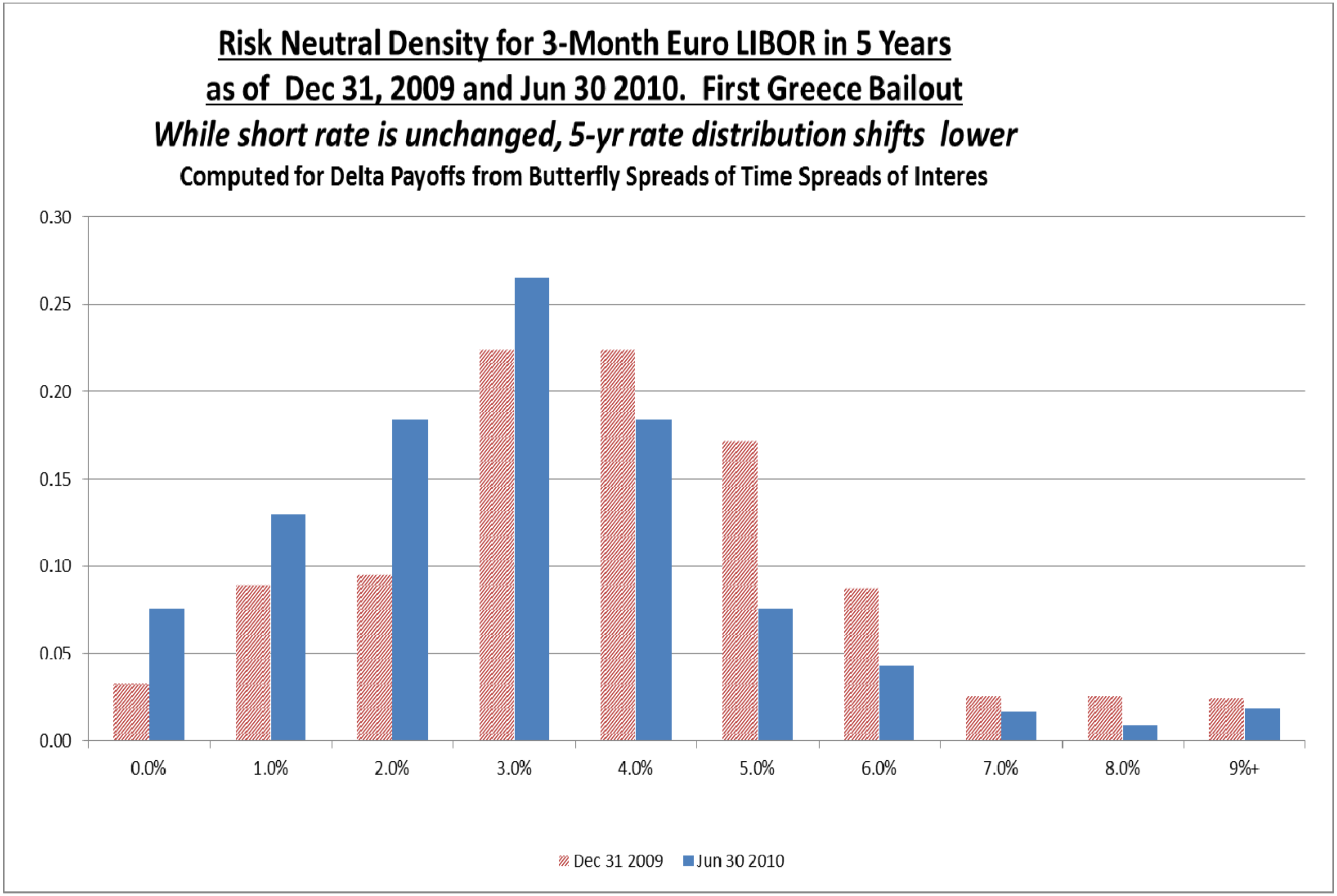
- July, 2012: ECB cuts rates again.
- September, 2012: ECB ready to buy "unlimited amounts" of bonds of weaker member countries. Draghi says ECB will do "whatever it takes to preserve the Euro." "…and believe me, it will be enough."

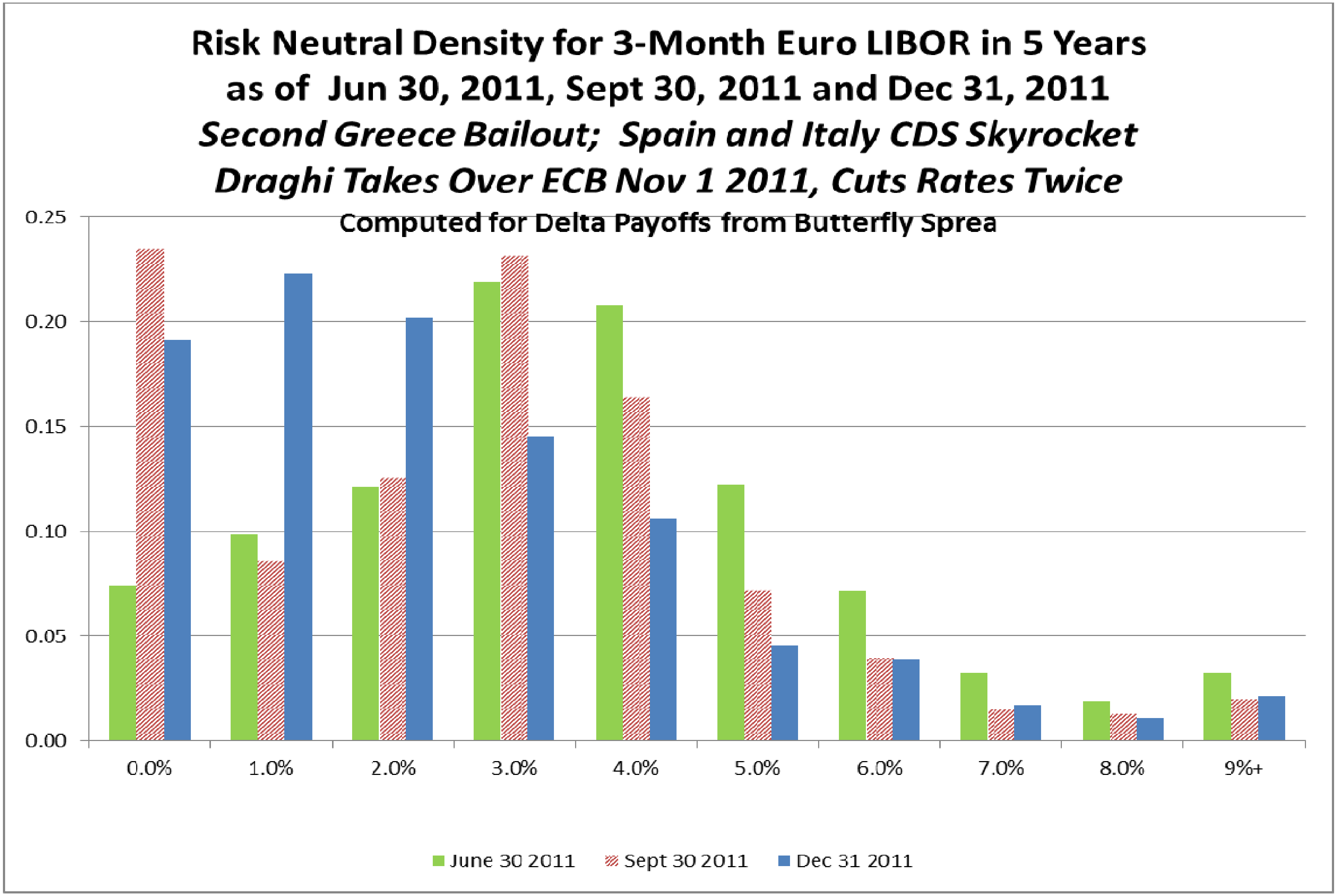
- May/June 2013: U.S.Fed considers "tapering" asset purchases, as economy strengthens. Long term interest rates move up sharply.

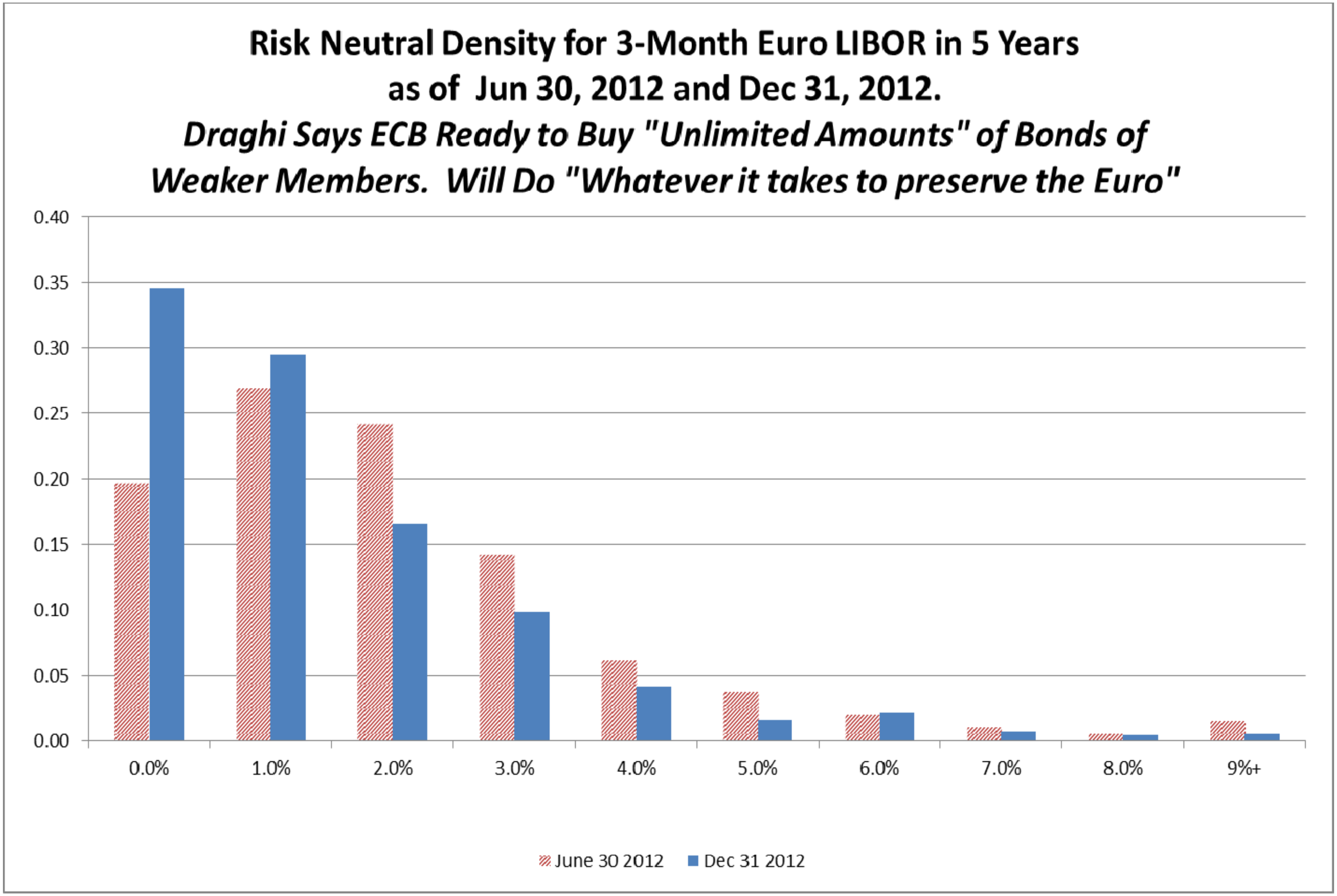


DJ Euro Stoxx 600 and DJ Euro Stoxx 50 Stock Indexes
Sovereign Debt Crisis: Monthly, Dec 2009 to Feb 2013
Euro Stoxx 600
Euro Stoxx 50
300
290
280
270
260
250
240
230
220
210
200
3200
3000
2800
2600
2400
2200
2000
1800
12/31/09
03/31/10
06/30/10
09/30/10
12/31/10
03/31/11
06/30/11
09/30/11
12/30/11
03/30/12
06/29/12
09/28/12
12/31/12
DJ EuroStoxx 600
DJ Euro Stoxx 50


6-Month Euro LIBOR Percentage Yield
Dec 2009 to Feb 2013
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
12/31/09
06/30/10
12/31/10
06/30/11
12/30/11
06/29/12
12/31/12
6-Month Euro LIBOR

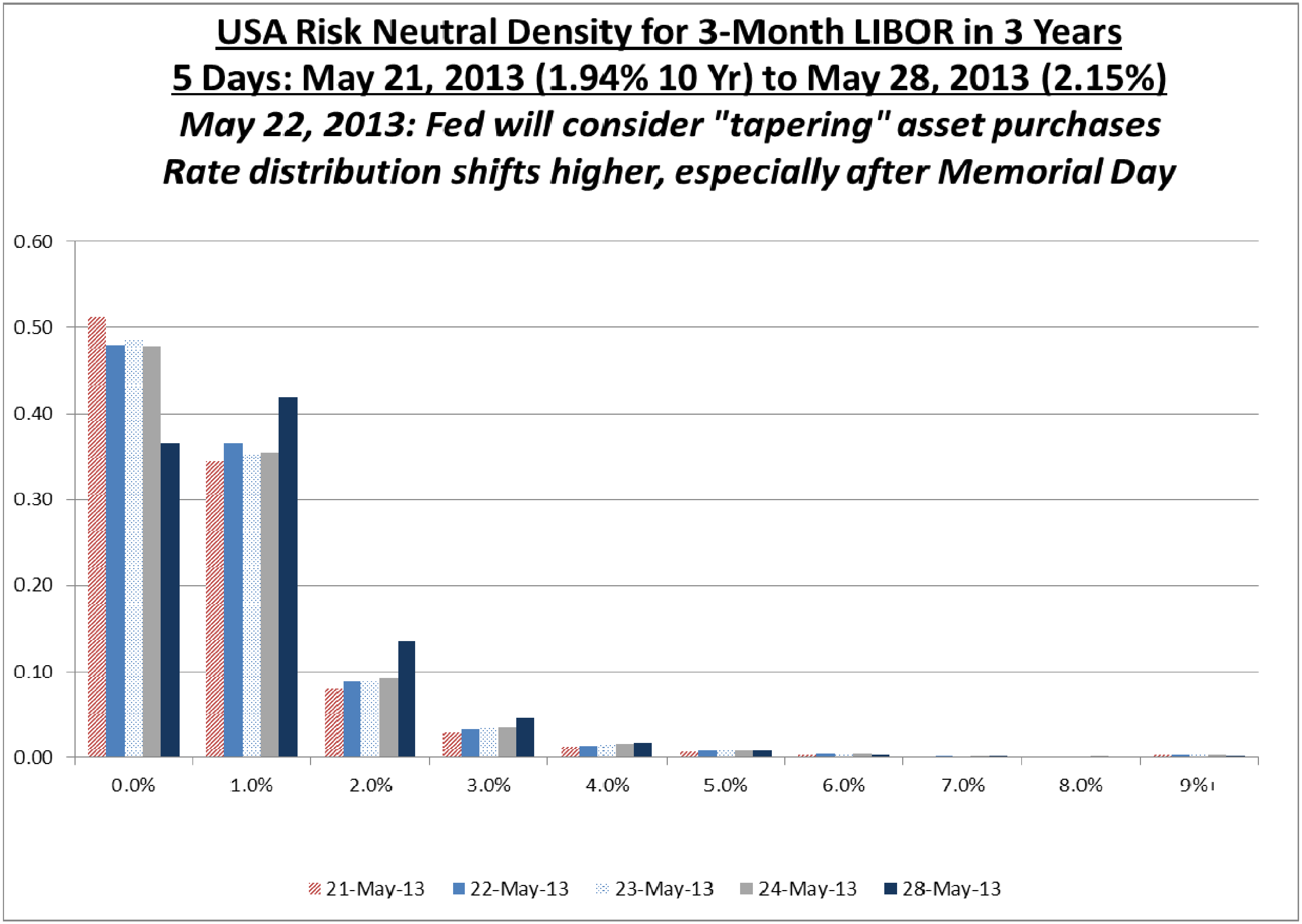

Risk Neutral Density for 3-Month Euro LIBOR in 5 Years
as of Dec 31, 2009 and Jun 30 2010. First Greece Bailout
While short rate is unchanged, 5-yr rate distribution shifts lower
Computed for Delta Payoffs from Butterfly Spreads of Time Spreads of Interes
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
Dec 31 2009
Jun 30 2010


Risk Neutral Density for 3-Month Euro LIBOR in 5 Years
as of Jun 30, 2011, Sept 30, 2011 and Dec 31, 2011
Second Greece Bailout; Spain and Italy CDS Skyrocket
Draghi Takes Over ECB Nov 1 2011, Cuts Rates Twice
Computed for Delta Payoffs from Butterfly Sprea
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 30 2011
Sept 30 2011
Dec 31 2011

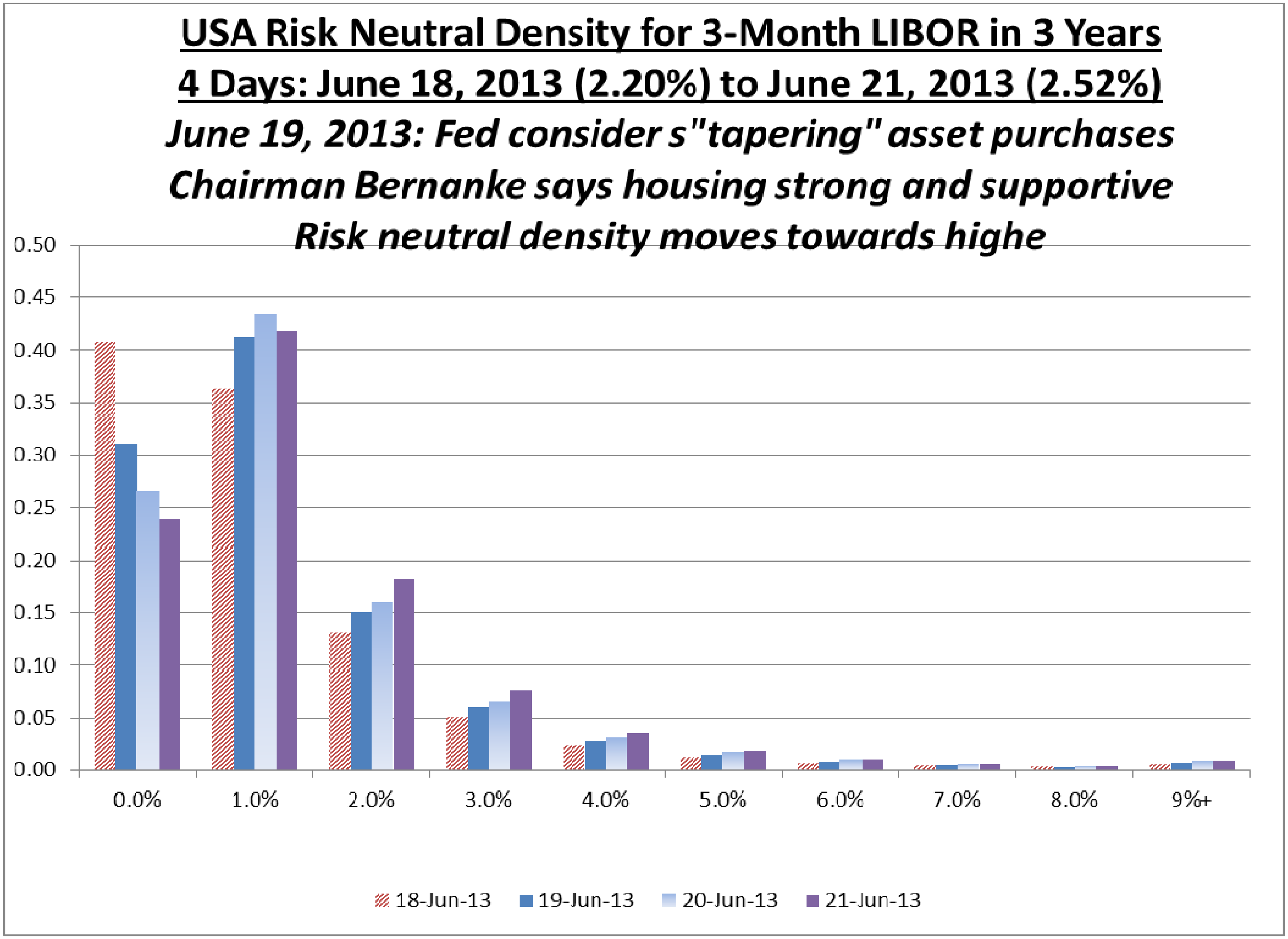

Risk Neutral Density for 3-Month Euro LIBOR in 5 Years
as of Jun 30, 2012 and Dec 31, 2012.
Draghi Says ECB Ready to Buy "Unlimited Amounts" of Bonds of Weaker Members. Will Do "Whatever it takes to preserve the Euro"
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 30 2012
Dec 31 2012

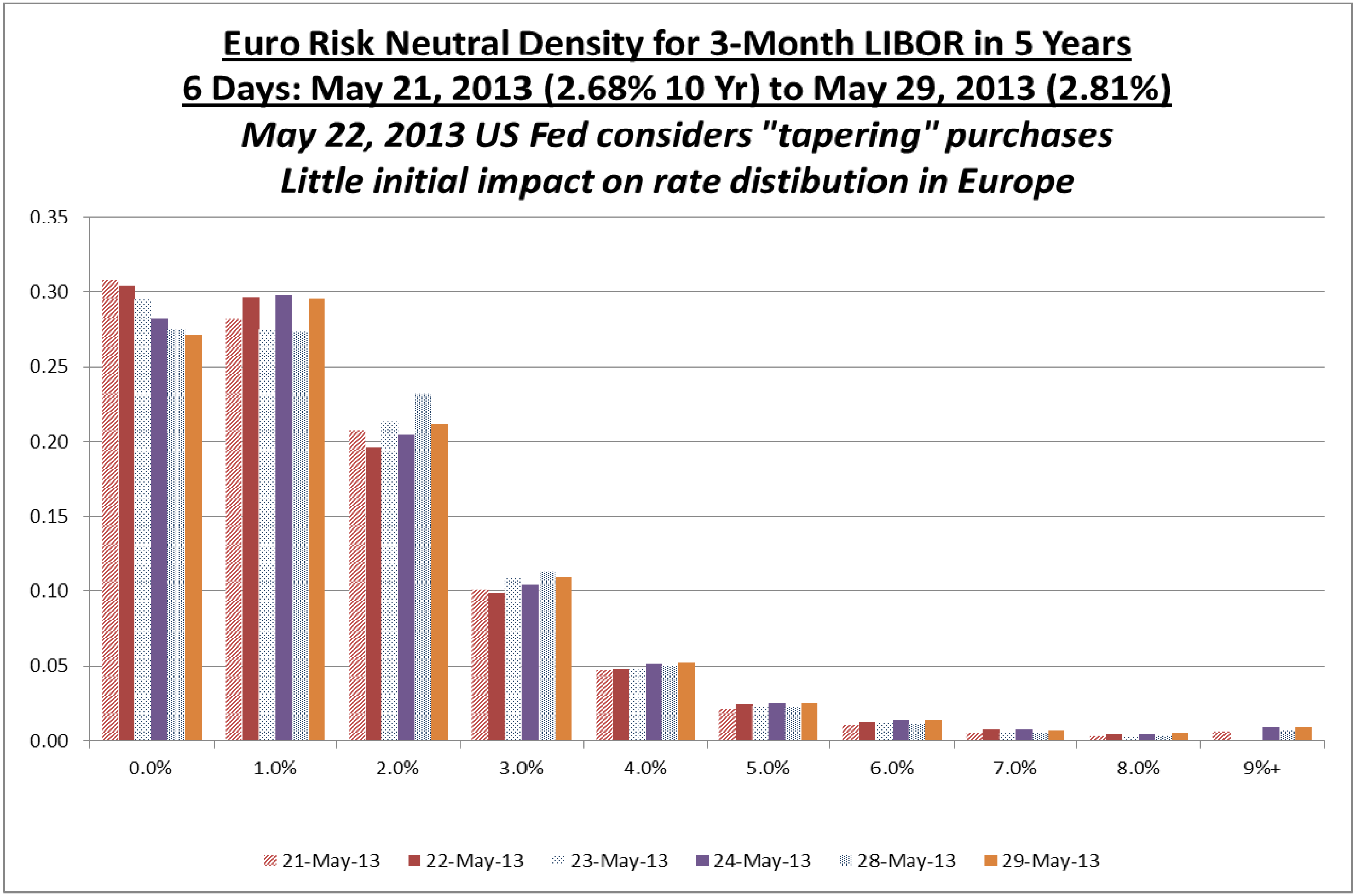
VI. May-September 2013

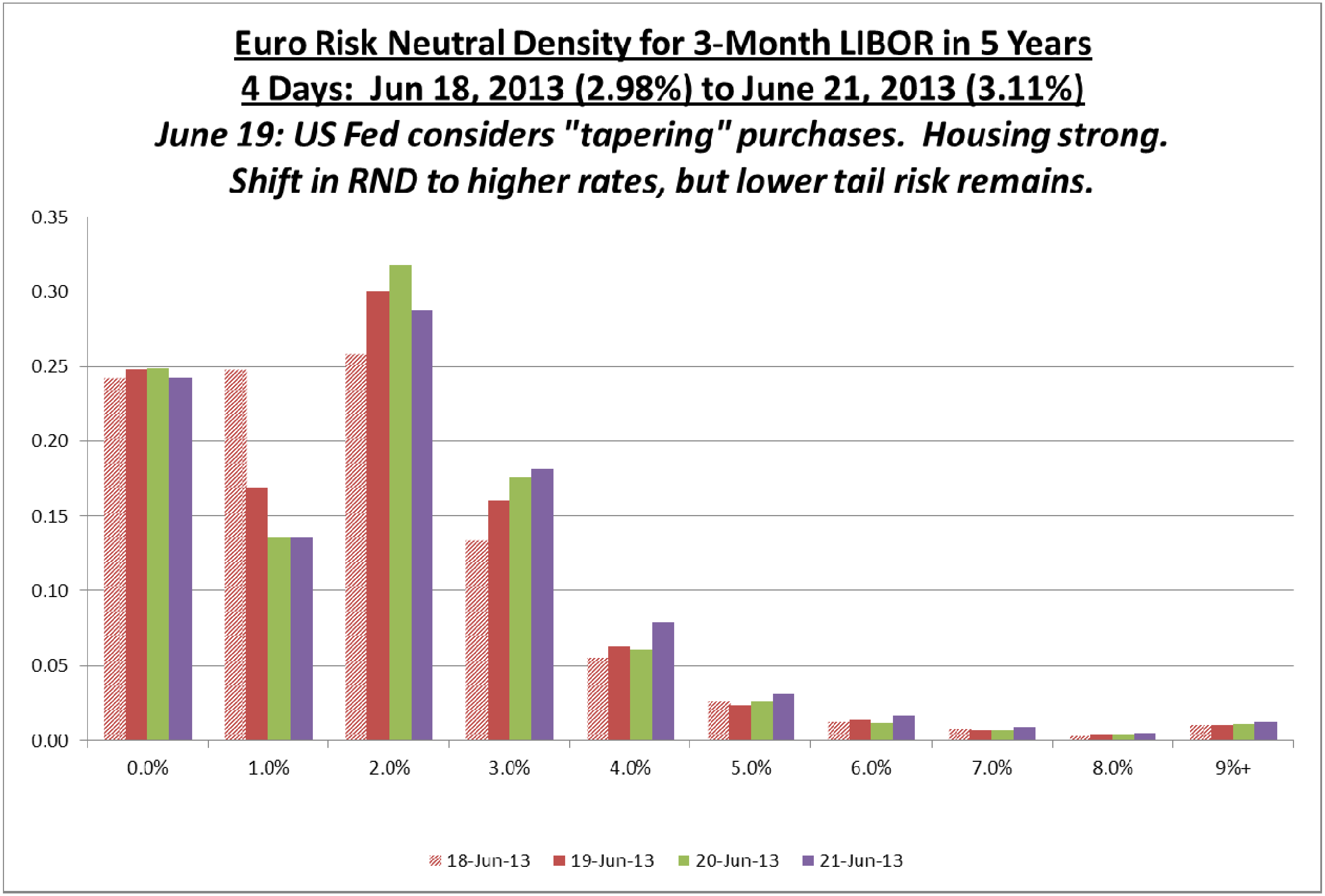
U.S. Federal Reserve Considers "Tapering" Asset Purchases

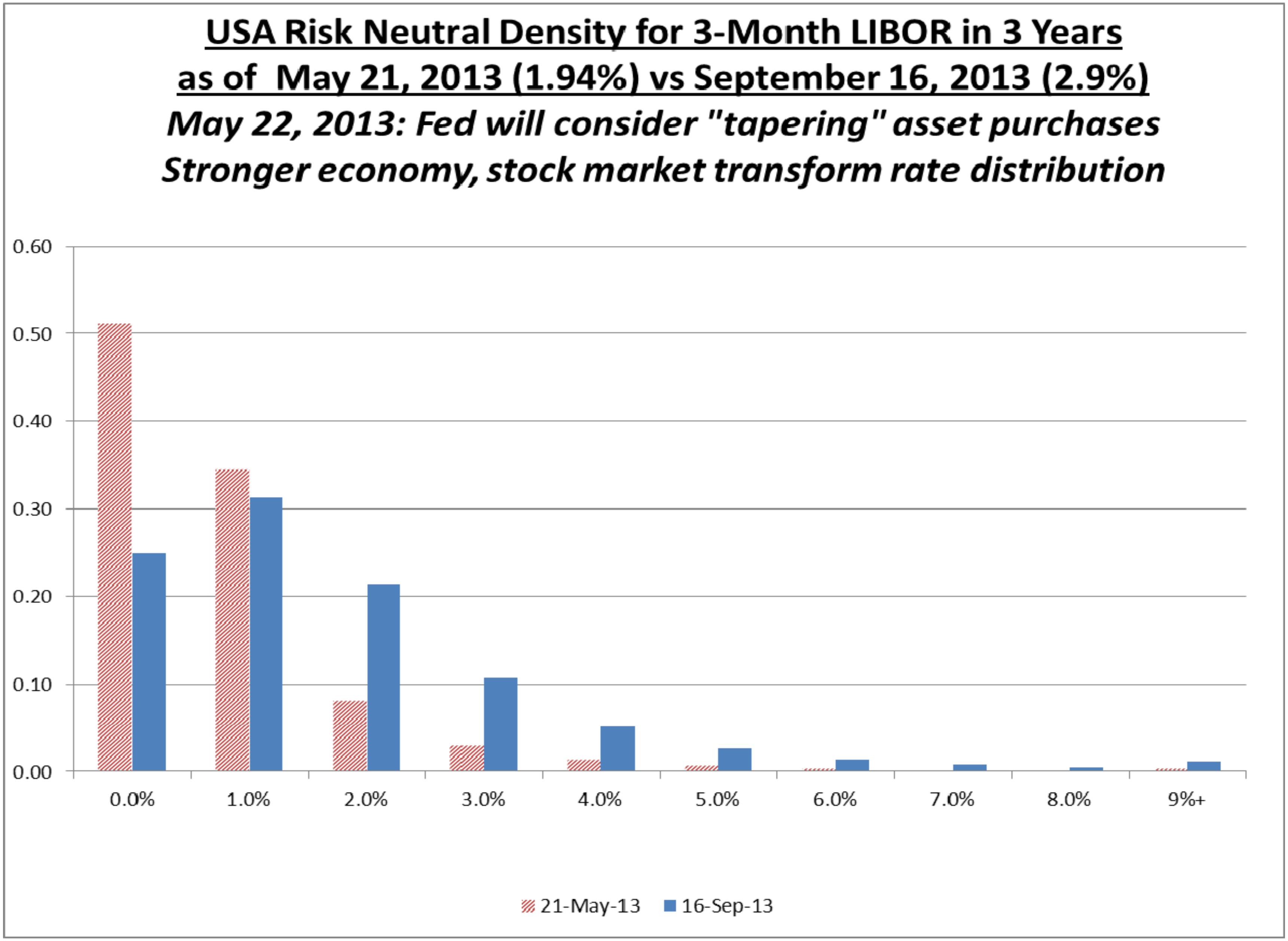


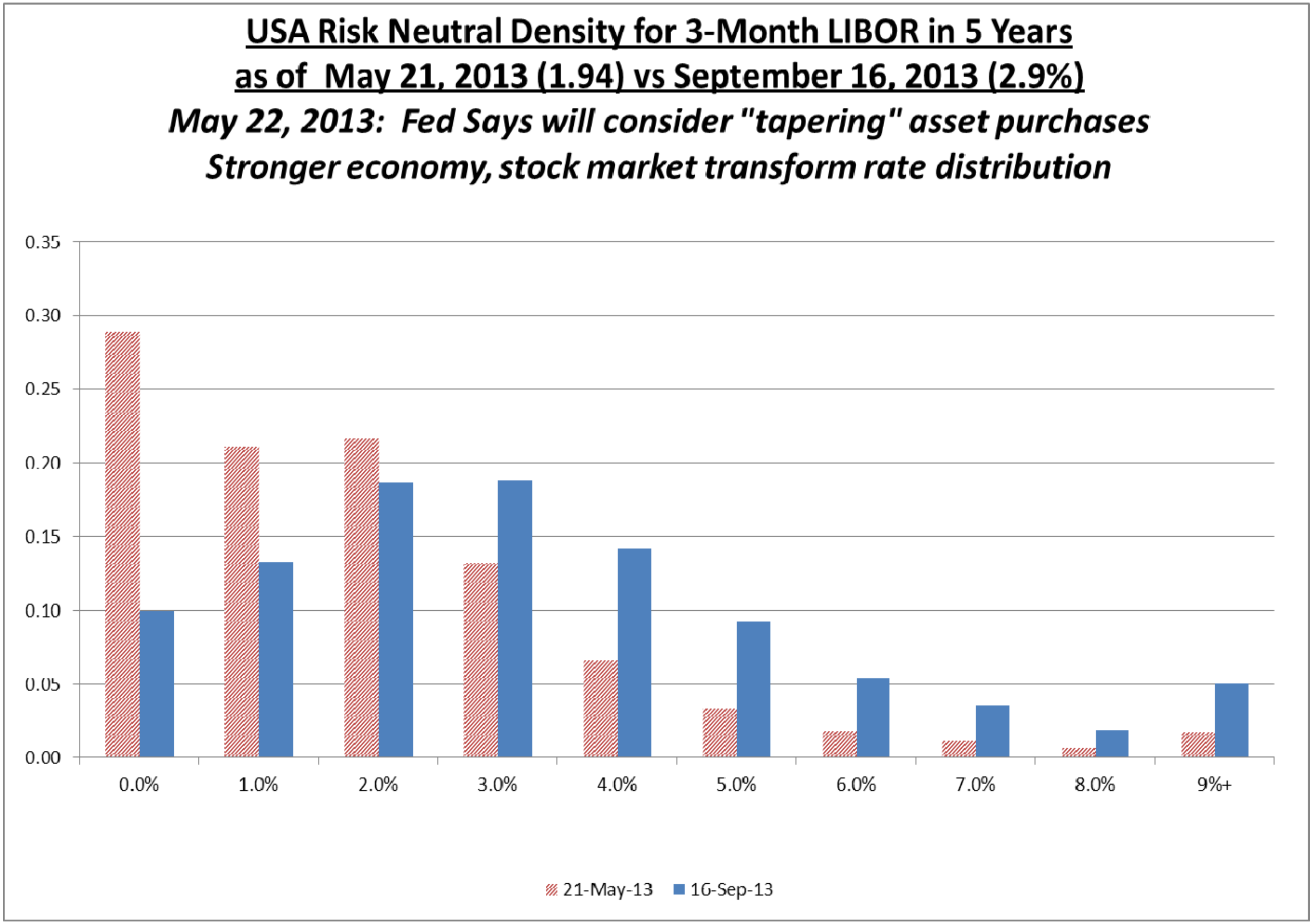
USA Risk Neutral Density for 3-Month LIBOR in 3 Years
5 Days: May 21, 2013 (1.94% 10 Yr) to May 28, 2013 (2.15%)
May 22, 2013: Fed will consider "tapering" asset purchases
Rate distribution shifts higher, especially after Memorial Day
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%
21-May-13
22-May-13
23-May-13
24-May-13
28-May-13

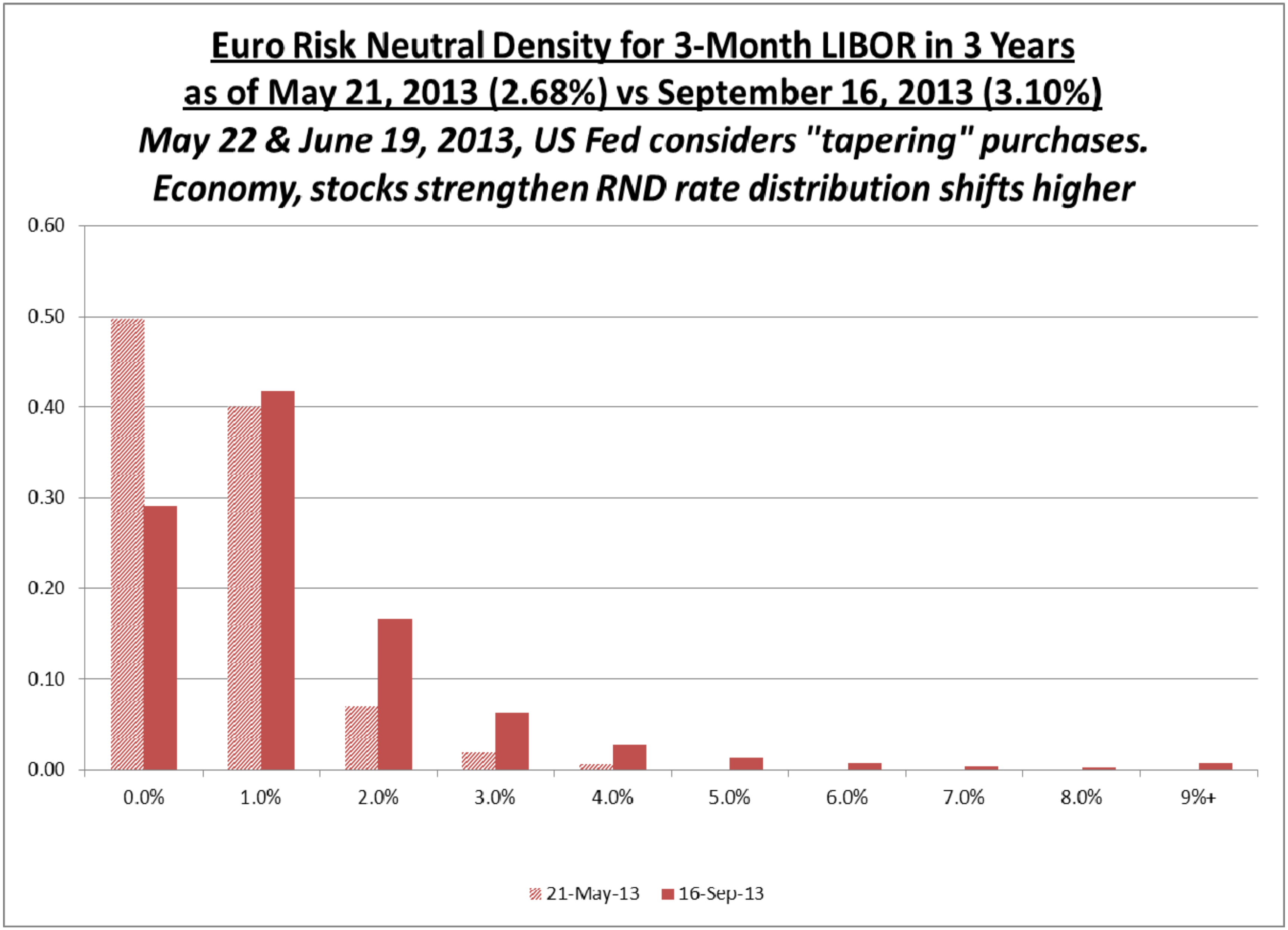

USA Risk Neutral Density for 3-Month LIBOR in 3 Years
4 Days: June 18, 2013 (2.20%) to June 21, 2013 (2.52%)
June 19, 2013: Fed consider s"tapering" asset purchases
Chairman Bernanke says housing strong and supportive
Risk neutral density moves towards highe
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
18-Jun-13
19-Jun-13
20-Jun-13
21-Jun-13


Euro Risk Neutral Density for 3-Month LIBOR in 5 Years
6 Days: May 21, 2013 (2.68% 10 Yr) to May 29, 2013 (2.81%)
May 22, 2013 US Fed considers "tapering" purchases
Little initial impact on rate distibution in Europe
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
21-May-13
22-May-13
23-May-13
24-May-13
28-May-13
29-May-13

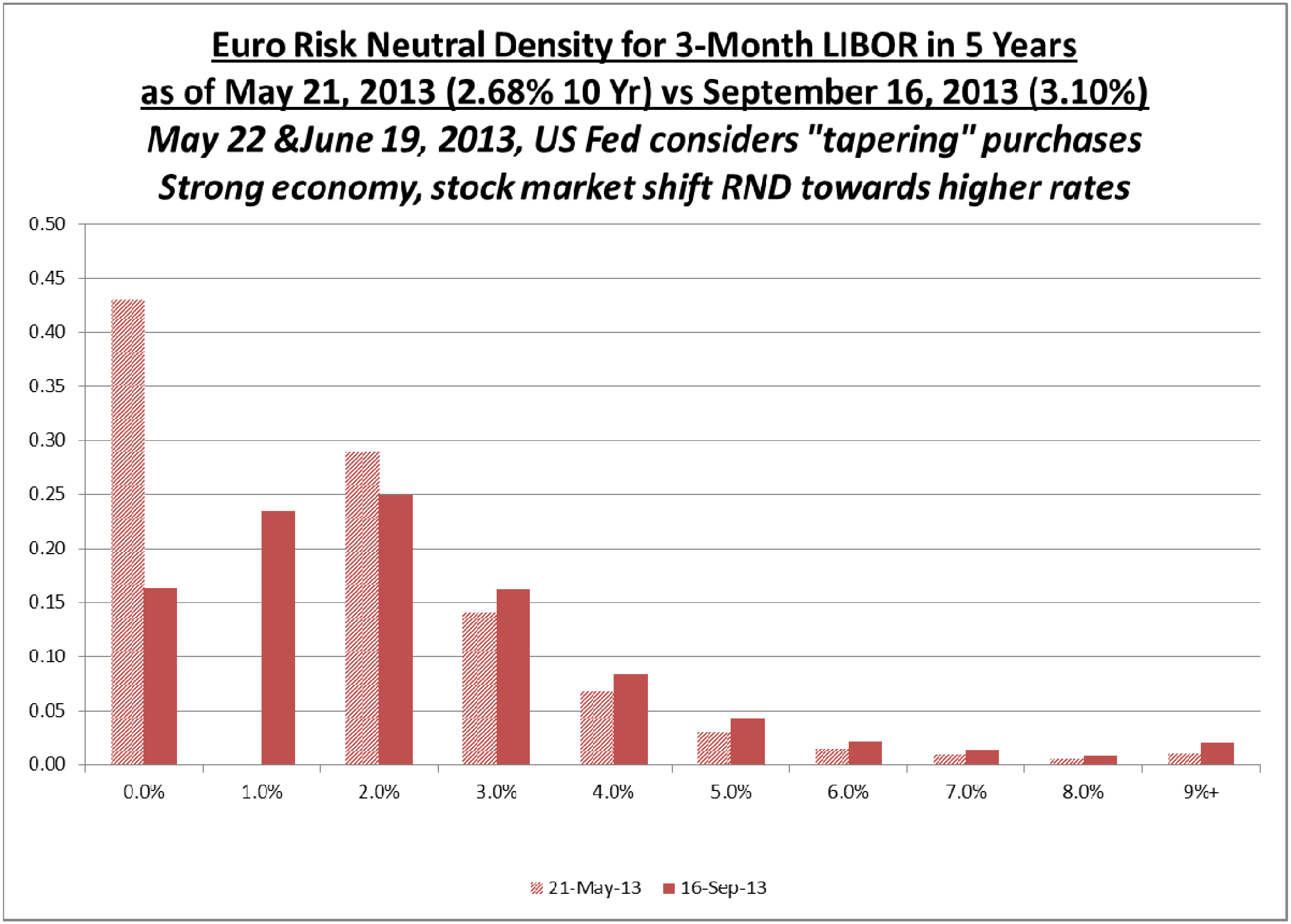

Euro Risk Neutral Density for 3-Month LIBOR in 5 Years
4 Days: Jun 18, 2013 (2.98%) to June 21, 2013 (3.11%)
June 19: US Fed considers "tapering" purchases. Housing strong.
Shift in RND to higher rates, but lower tail risk remains.
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
18-Jun-13
19-Jun-13
20-Jun-13
21-Jun-13


USA Risk Neutral Density for 3-Month LIBOR in 3 Years
as of May 21, 2013 (1.94%) vs September 16, 2013 (2.9%)
May 22, 2013: Fed will consider "tapering" asset purchases
Stronger economy, stock market transform rate distribution
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
21-May-13
16-Sep-13


USA Risk Neutral Density for 3-Month LIBOR in 5 Years
as of May 21, 2013 (1.94) vs September 16, 2013 (2.9%)
May 22, 2013: Fed Says will consider "tapering" asset purchases
Stronger economy, stock market transform rate distribution
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
21-May-13
16-Sep-13



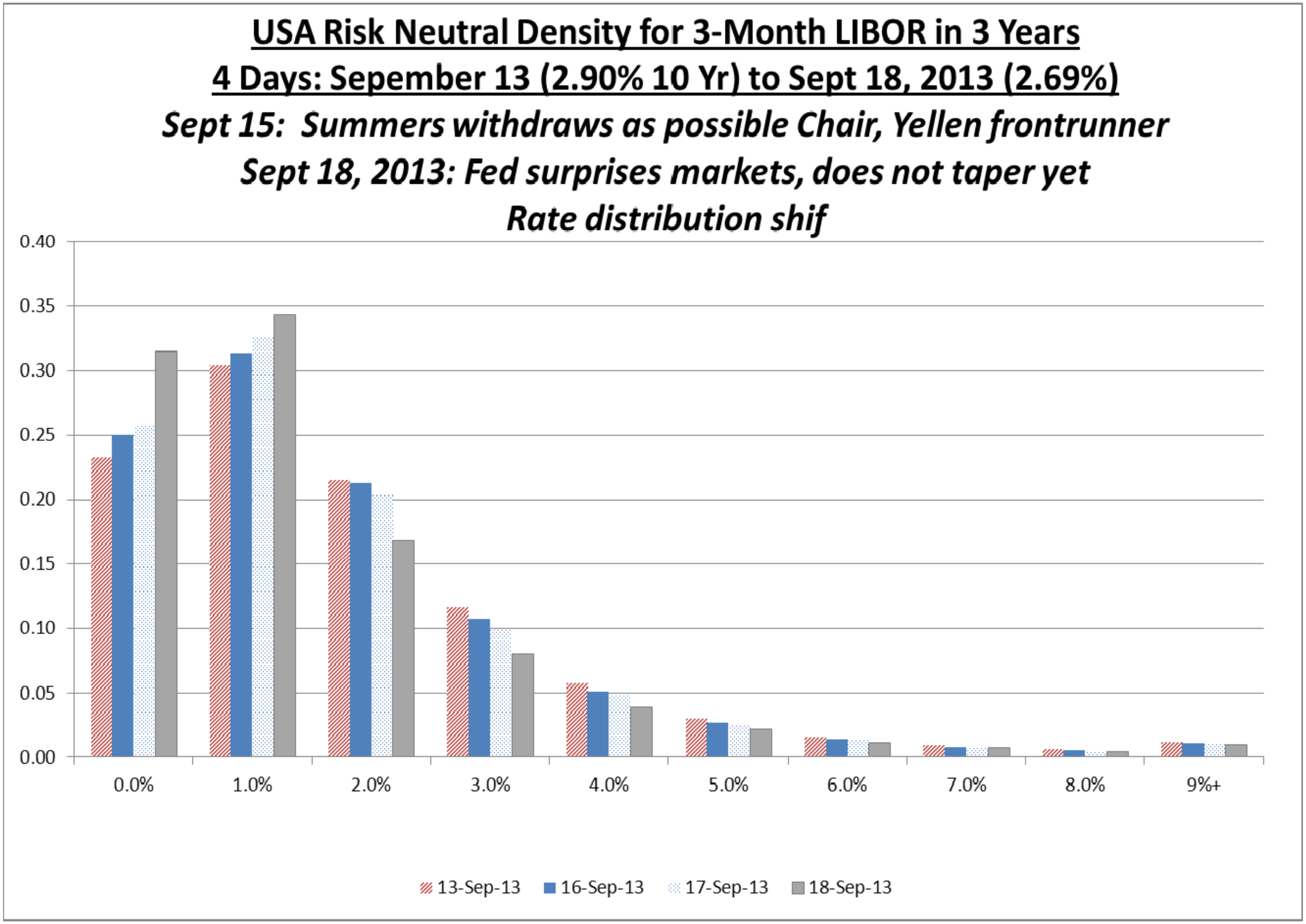
Euro Risk Neutral Density for 3-Month LIBOR in 3 Years
as of May 21, 2013 (2.68%) vs September 16, 2013 (3.10%)
May 22 & June 19, 2013, US Fed considers "tapering" purchases.
Economy, stocks strengthen RND rate distribution shifts higher
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
21-May-13
16-Sep-13


Euro Risk Neutral Density for 3-Month LIBOR in 5 Years
as of May 21, 2013 (2.68% 10 Yr) vs September 16, 2013 (3.10%)
May 22 &June 19, 2013, US Fed considers "tapering" purchases
Strong economy, stock market shift RND towards higher rates
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
21-May-13
16-Sep-13

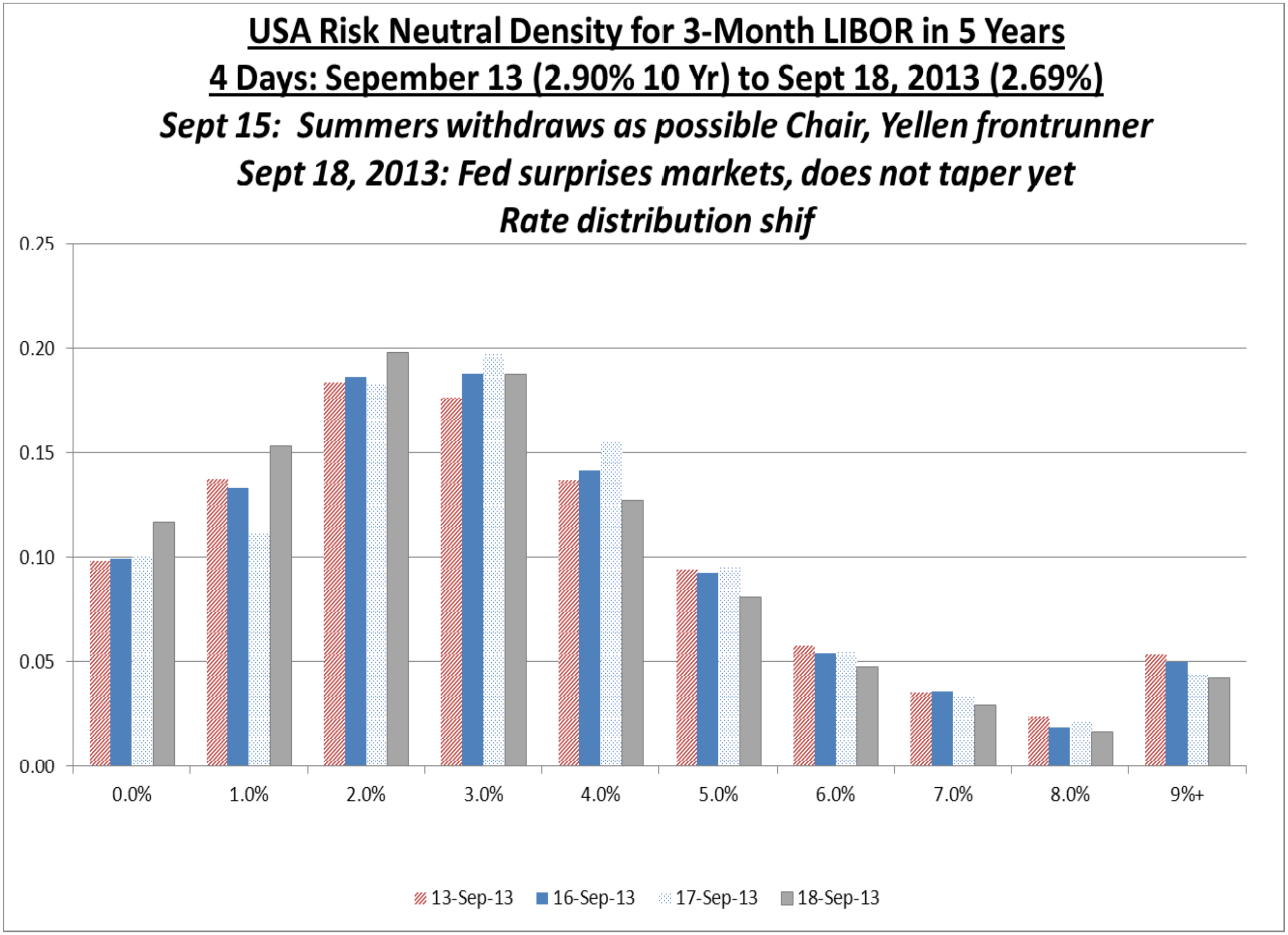
Postscript:
September 15-18, 2013 Events

September 15: Larry Summers withdraws from consideration as new Fed Chair. Janet Yellen presumed frontrunner, believed proponent of easy money longer.

September 18: Fed surprises markets and does not start "taper." Reduces growth forecasts.



USA Risk Neutral Density for 3-Month LIBOR in 3 Years
4 Days: Sepember 13 (2.90% 10 Yr) to Sept 18, 2013 (2.69%)
Sept 15: Summers withdraws as possible Chair, Yellen frontrunner
Sept 18, 2013: Fed surprises markets, does not taper yet
Rate distribution shif
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
13-Sep-13
16-Sep-13
17-Sep-13
18-Sep-13


USA Risk Neutral Density for 3-Month LIBOR in 5 Years
4 Days: Sepember 13 (2.90% 10 Yr) to Sept 18, 2013 (2.69%)
Sept 15: Summers withdraws as possible Chair, Yellen frontrunner
Sept 18, 2013: Fed surprises markets, does not taper yet
Rate distribution shif
0.25
0.20
0.15
0.10
0.05
0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
13-Sep-13
16-Sep-13
17-Sep-13
18-Sep-13

VIII. Conclusions

1. The approach of Breeden-Litzenberger 1978 is being used to estimate tail risks and risk neutral densities in practice.

2. Time spreads of interest rate caps and floors give prices for long-term call and put options (e.g., 4-5 year maturities) on 3-month LIBOR. State prices and risk neutral densities implicit in cap and floor prices are realistic and recently have been highly non-normal (very positively skewed), as near-zero interest rates occurred.

3. This approach is non-parametric, using only cap and floor prices that are generally observable. No parameter estimation needed.

3. Before and after state prices/risk neutral densities implicit in interest rate floors and caps demonstrate the efficacy (and sometimes the lack thereof) of Federal Reserve and European Central Bank policy actions on interest rate probability distributions. Distributions have changed shape quite dramatically in the past 10 years.

Appendix 1

Review of Theory and Recent Uses: Prices of State Contingent Claims Implicit in Option Prices

Stephen Ross (Yale/MIT) (1976, QJE),
Douglas Breeden-Robert Litzenberger
(Stanford/Chicago, 1978, J Business)

State Prices (Arrow Securities) Implicit in Option Prices

Breeden-Litzenberger 1978, *Journal of Business, following Ross 1976 QJE.*

In general, any derivative asset with payoffs $f(\tilde{P})$ can be priced by arbitrage from the prices of $1 "elementary claims" on $\tilde{P}$. An elementary claim on $\tilde{P}$ has a payoff of $1 contingent upon $\tilde{P}=P_1, \tilde{P}=P_2, \cdots \tilde{P}=P_N$. With these, we can price all payoffs of the form $f(\tilde{P})$.

The following construction shows that elementary claims can be created from call or put options:

				Call Option Portfolios		
	Payoffs on Call Options			Port. A	Port. B	Port.C=A-B
P	C(X=2)	C(X=3)	C(X=4)	C(2)-C(3)	C(3)-C(4)	C(2)-2C(3)+C(4)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	1	0	1
4	2	1	0	1	1	0
5	3	2	1	1	1	0
6	4	3	2	1	1	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
N	N-2	N-3	N-4	1	1	0

"Butterfly Spreads" of Options Give State Prices and Risk Neutral Densities

Generally, $e(\tilde{P}=x) = \dfrac{[c(x-\Delta)-c(x)]-[c(x)-c(x+\Delta)]}{\Delta}$

$\lim\limits_{\Delta \to 0} \dfrac{e(\tilde{P}=x)}{\Delta} = c_{xx}(x=\tilde{P}) = 2^{nd}$ partial of call price w.r.t. exercise price, evaluated at $x=\tilde{P}$.

(*Journal of Business*, 1978, vol. 51, no. 4)
© 1978 by The University of Chicago
0021-9398/78/5104-0005$02.21

Douglas T. Breeden
University of Chicago

Robert H. Litzenberger
Stanford University

Prices of State-contingent Claims Implicit in Option Prices*

TABLE 3 **Delta-Security Prices***

$\frac{Y_1}{M_0} - \frac{Y_2}{M_0}$	Time to Maturity: 3 Mos.	6 Mos.	9 Mos.	1 Yr.	2 Yrs.	3 Yrs.	4 Yrs.	5 Yrs.	10 Yrs.	20 Yrs.
0–.1										.2¢
.1–.2									.3¢	.9
.2–.3							.1¢	.3¢	1.2	1.6
.3–.4					.1¢	.3¢	.7	1.2	2.5	1.9
.4–.5					.6	1.5	2.4	3.0	3.5	2.0
.5–.6			.2¢	.5¢	2.5	4.0	4.7	4.9	4.0	1.9
.6–.7		.6¢	1.7	3.0	6.1	6.8	6.7	6.4	4.2	1.8
.7–.8	1.2¢	5.0	7.6	9.0	9.9	9.0	8.0	7.1	4.2	1.7
.8–.9	13.1	16.6	16.5	15.7	12.4	10.1	8.5	7.3	4.0	1.6
.9–1.0	34.8	26.4	21.9	18.9	12.9	10.0	8.2	6.9	3.6	1.4
1.0–1.1	32.5	24.3	20.0	17.3	11.7	9.1	7.4	6.2	3.3	1.3
1.1–1.2	13.5	14.7	13.8	12.8	9.6	7.7	6.4	5.5	3.0	1.2
1.2–1.3	2.9	6.5	7.8	8.1	7.3	6.2	5.4	4.7	2.6	1.0
1.3–1.4	.4	2.2	3.7	4.6	5.3	4.9	4.4	3.9	2.3	.9
1.4–1.5		.6	1.5	2.3	3.7	3.7	3.5	3.2	2.0	.9
1.5–1.6				1.1	2.5	2.8	2.8	2.6	1.8	.8
1.6–1.7				.5	1.6	2.1	2.2	2.1	1.5	.7
1.7–1.8				.2	1.0	1.5	1.7	1.7	1.3	.6
1.9–2.0					.4	.8	1.0	1.1	1.0	.5
2.0–2.1					.2	.5	.8	.9	.9	.5
2.1–2.2					.1	.4	.6	.7	.8	.4
2.2–2.3					.1	.3	.4	.6	.7	.4
2.3–2.4					.1	.2	.3	.5	.6	.4
2.4–2.5						.1	.3	.4	.5	.3
2.5–2.6						.1	.2	.3	.5	.3
2.6–2.7						.1	.1	.2	.4	.3
2.7–2.8							.1	.2	.4	.3
2.8–2.9							.1	.1	.3	.2
2.9–3.0							.1	.1	.3	.2
3.0–3.1							.1	.1	.2	.2
3.1–3.2								.1	.2	.2
3.2–3.3								.1	.2	.2
3.3–3.4									.2	.2
3.4–3.5									.1	.2
3.5–3.6									.1	.1
3.6–3.7									.1	.1
3.7–3.8									.1	.1
3.8–3.9									.1	.1
3.9–4.0									.1	.1
4.0–4.1									.1	.1

* Assumptions for all maturities are: $r = .06$, $\delta = .04$, $\sigma = .20$.

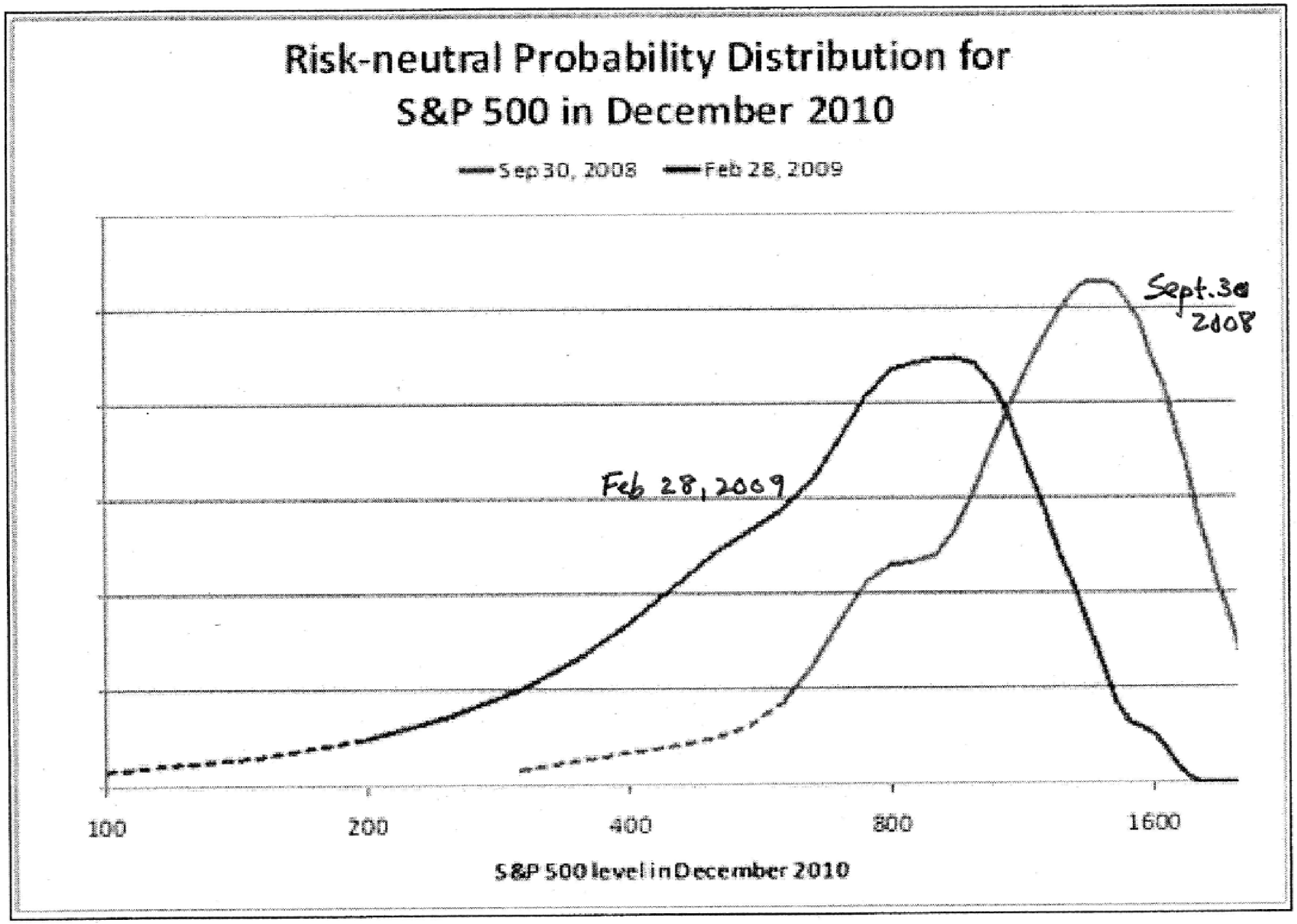
Selected Academic Articles and Extensions

Articles estimating risk neutral densities and/or applying them to price more complex securities include academic works by Banz and Miller (1978), Shimko (1993), Rubinstein (1994), Longstaff (1995), Jackwerth and Rubinstein (1996), Ait-Sahalia and Lo (1998), Ait-Sahalia, Wang and Yared (2001),

Longstaff, Santa-Clara and Schwartz (2001), Ait-Sahalia and Duarte (2003), Carr (1998, 2004, 2012), Bates (2000), Li and Zhao (2006), Figlewski (2008), Zitzewitz (2009), Birru and Figlewski (20010a,b), Kitsul and Wright (2012), Ross (2013), and Martin (2013), to name a few.

Freakonomics article: "Quantifying the Nightmare Scenarios"

Eric Zitzewitz (Dartmouth) Uses Breeden-Litzenberger 1978 Technique
In ***Freakonomics*** Blog by Justin Wolfers, March 2, 2009



9/30/2008:
S&P500= 1166
VIX = 39.4%

2/28/2009
S&P500 = 735
VIX = 46.4%

Central Bank Applications

Central banks have also estimated "option implied (risk-neutral) probability distributions" using these techniques. Central bank applications are discussed in articles of Bahra (1996, 1997), Clews, Panigirtzoglous and Proudman (2000), and Smith (2012) of the Bank of England,; Malz (1995,1997) of the Federal Reserve Board of New York; and Durham (2007) and Kim (2008) of the Federal Reserve Bank of Washington.

Kocherlakota's (2013) research group at the Federal Reserve Bank of Minneapolis use Shimko's (1993) statistical method applying the Breeden-Litzenberger formula to regularly estimate and publish RNDs and tail risks (e.g., risk neutral probabilities of moves of +/- 20% or more) for many assets, such as stocks, crude oil, wheat, real estate, and foreign exchange.

Federal Reserve Bank of Minneapolis Uses Breeden-Litzenberger Method to Estimate "Tail Risk" Every 2 Weeks For Stocks, Commodities, Currencies, Real Estate.

FEDERAL RESERVE BANK OF MINNEAPOLIS

BANKING AND POLICY STUDIES

Methodology for Estimating Risk Neutral Probability Density Functions

We estimate risk neutral probability density functions (RNPDs) for a variety of different asset classes using a variation of the technique developed by Shimko (1993). This procedure involves fitting a curve to the implied volatilities of a series of options and expressing the volatility as a function of the strike price. The implied volatilities are then translated into continuous call option prices, and the risk neutral distribution of the underlying asset is obtained through the Breeden-Litzenberger (1978) method.

References

Breeden, D. T., and Litzenberger, R. H. (1978), "Prices of state-contingent claims implicit in option prices," *Journal of Business* 51 (4), pp. 621-51.

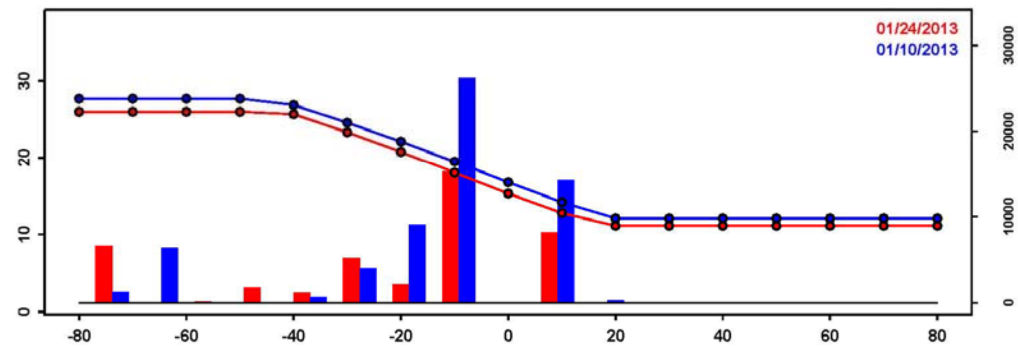
Shimko, D. C. (1993), "Bounds of Probability," *Risk*, 6 (4), pp. 33-37.

Source: Excerpts from "Methodology" tab of Federal Reserve Bank of Minneapolis website, February 1, 2013. President: Dr. Narayana Kocherlakota.

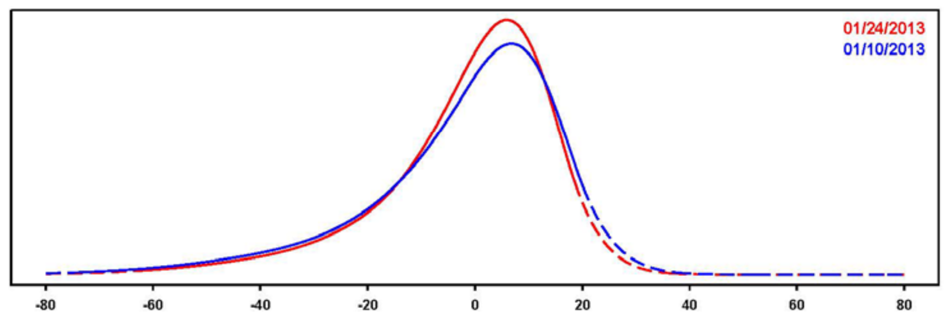
RISK NEUTRAL PROBABILITY DENSITY FUNCTIONS -- S&P 500

Log returns are based on the risk neutral density function of the underlying asset derived from options that expire in approximately 12 months.

Implied Volatilities (lines--left axis) and Volume (bars--right axis)



Risk Neutral PDF of the Log Return Distribution



Probability of a Large Change

Statistics of the Log Return Distributions

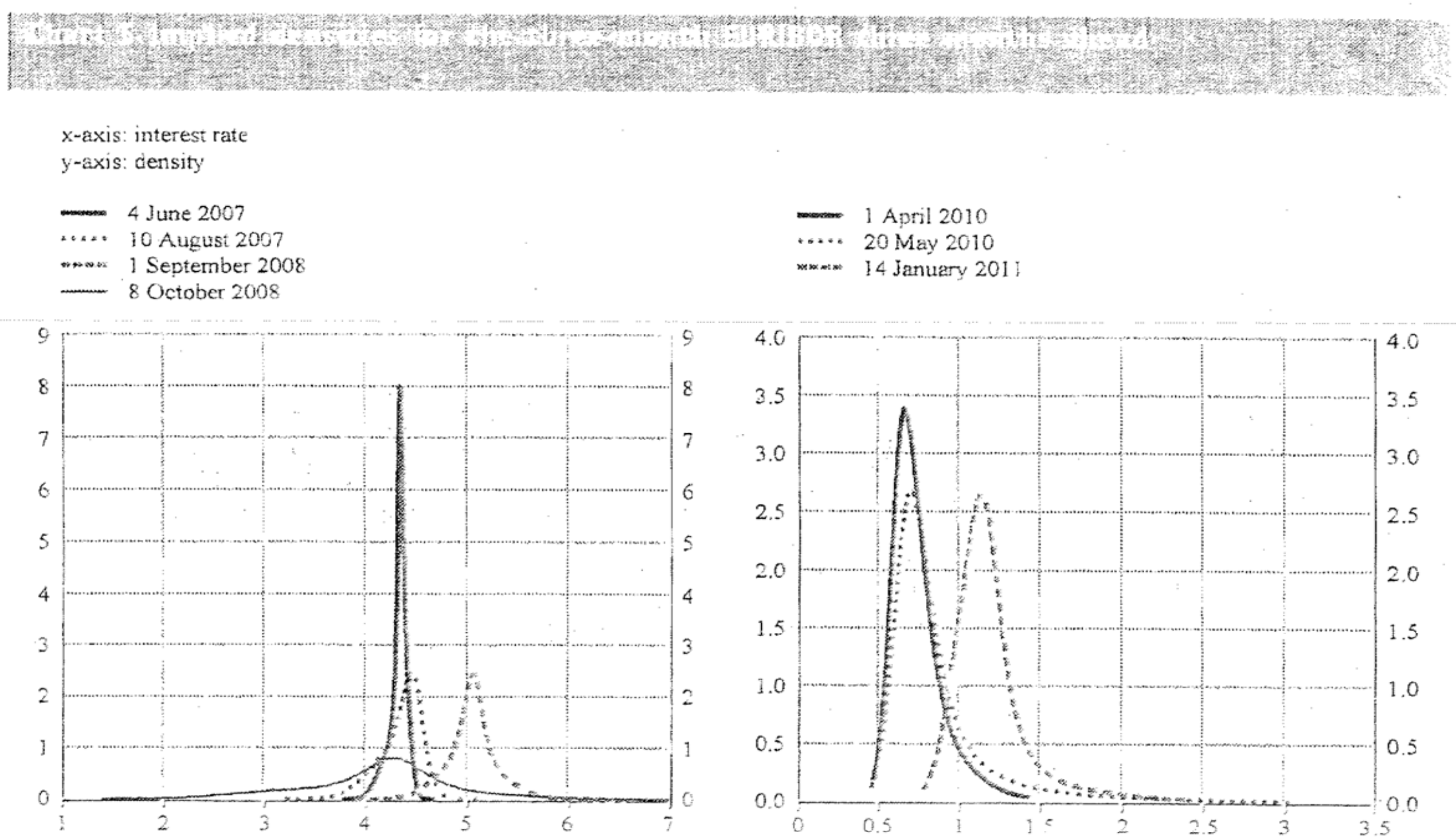
	01/10/2013	01/24/2013	Change
10th Pct	-26.01%	-23.00%	3.02%
50th Pct	1.82%	1.68%	-0.15%
90th Pct	17.19%	15.71%	-1.48%
Mean	-1.74%	-1.39%	0.35%
Std Dev	18.14%	16.40%	-1.74%
Skew	-1.17	-1.21	-0.03
Kurtosis	1.98	2.23	0.25

European Central Bank Article
ECB Monthly Bulletin, February 2011
Distributions for Euribor in 3 Months

THE INFORMATION CONTENT OF OPTION PRICES DURING THE FINANCIAL CRISIS

Financial asset prices have experienced significant volatility in reaction to the financial and economic crisis. In the context of such market volatility, investors' expectations and the level of market uncertainty as regards the future course of financial asset prices provide valuable information for analytical purposes. This article presents a technique recently adopted by ECB staff for the purposes of quantifying market participants' expectations regarding future asset prices in the form of probability distributions drawing on option prices. It shows how these techniques can be applied to money and stock markets, and the information content of measures of market expectations is discussed, with a particular focus on the behaviour of such measures during the financial crisis. These measures of market expectations allow the central bank to better understand market sentiment and behaviour. They also extend the central bank's information set and have shown themselves to be particularly relevant during periods of financial market tension.

European Central Bank Article (cont.)
ECB Monthly Bulletin, February 2011
<u>Distributions for Euribor in 3 Months</u>



Sources: NYSE Liffe and ECB calculations.

Appendix 2

True Probabilities vs. Risk Neutral Probabilities

(Normalized State Prices)

In a general state preference model:

Inserting eq. 6 for the zero coupon bond gives:

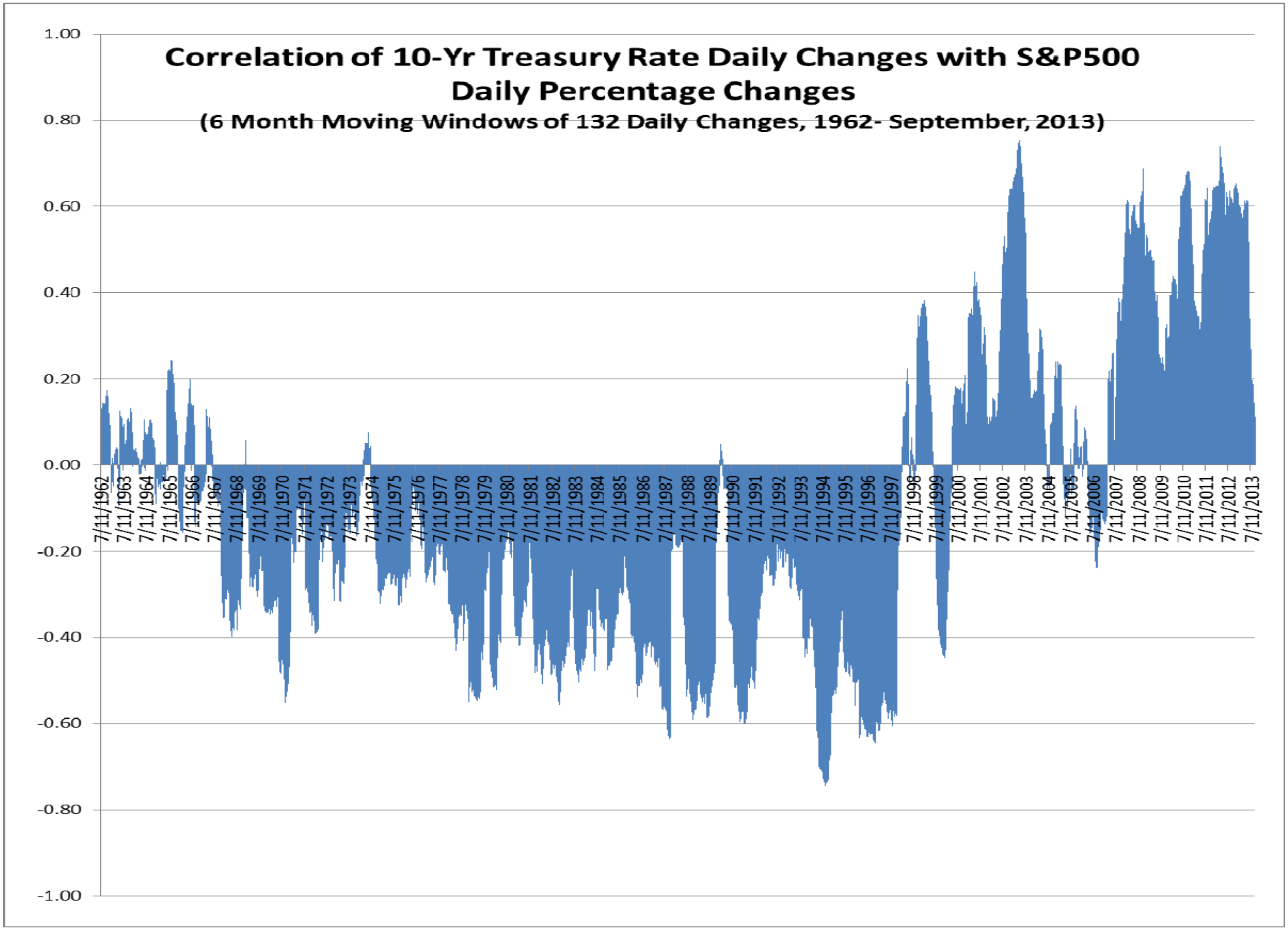
$$\frac{\phi^*_{tr_j}}{\pi_{tr_j}} = \frac{E\left[\tilde{u}'_{ts} \middle| r_j\right]}{E[\tilde{u}'_t]} \qquad (12)$$

Thus, we see that the risk-neutral probability to true probability ratio at the optimum for r_j is equal to the expected marginal utility of consumption, conditional upon the interest rate being at the specified level, divided by the unconditional expected marginal utility of consumption at time t. So if we are looking at butterfly spreads or digital options centered upon LIBOR = 2%, we need to compute the conditionally expected marginal utility of consumption, given that 2% rate.

If assume power utility (CRRA) and lognormally distributed consumption, we get a simple <u>formula for state price to probability ratios:</u>

$$\log\left(\frac{\phi^*_{ts}}{\pi_{ts}}\right) = \gamma\left[\mu_t - g_{ts} - \frac{1}{2}\gamma\sigma^2_c\right]t \qquad (19)$$

As expected, higher growth states for consumption have lower $\left(\frac{\phi^*_{ts}}{\pi_{ts}}\right)$ ratios. One could input different estimates of relative risk aversion and different states' growth rates and consumption volatility into the eq. 19 and compute the estimated log of the risk neutral probability to the true probability.


Correlation of 10-Yr Treasury Rate Daily Changes with S&P500 Daily Percentage Changes
(6 Month Moving Windows of 132 Daily Changes, 1962- September, 2013)
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
7/11/1962
7/11/1963
7/11/1964
7/11/1965
7/11/1966
7/11/1967
7/11/1968
7/11/1969
7/11/1970
7/11/1971
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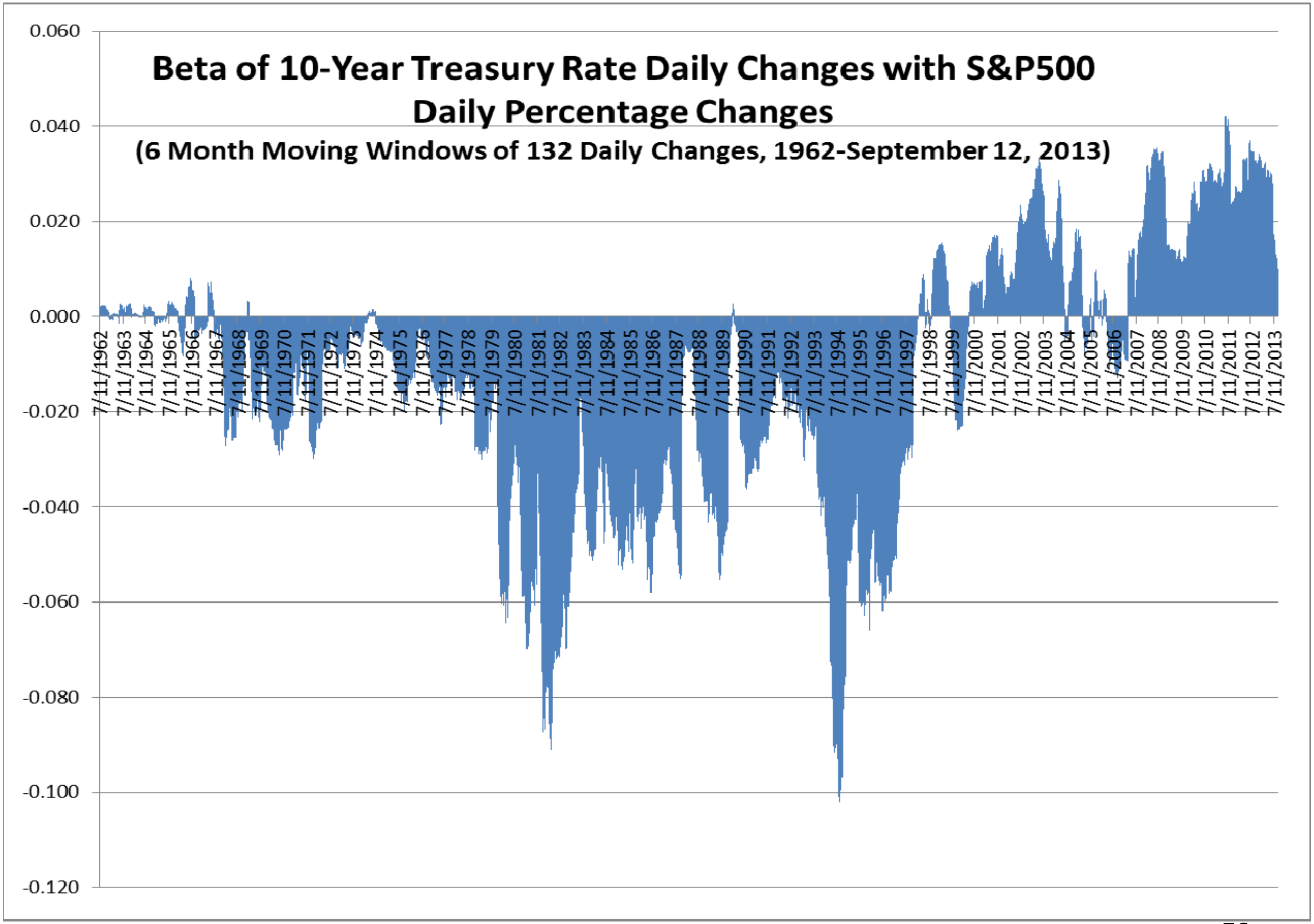
7/11/2012
7/11/2013


Beta of 10-Year Treasury Rate Daily Changes with S&P500
Daily Percentage Changes
(6 Month Moving Windows of 132 Daily Changes, 1962-September 12, 2013)
0.060
0.040
0.020
0.000
-0.020
-0.040
-0.060
-0.080
-0.100
-0.120
7/11/1962
7/11/1963
7/11/1964
7/11/1965
7/11/1966
7/11/1967
7/11/1968
7/11/1969
7/11/1970
7/11/1971
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7/11/2001
7/11/2002
7/11/2003
7/11/2004
7/11/2005
7/11/2006
7/11/2007
7/11/2008
7/11/2009
7/11/2010
7/11/2011
7/11/2012
7/11/2013

Illustration of True Probabilities Related to Risk Neutral Probabilities

True probability = K*Risk Neutral x exp(Gamma*(gts - mu))

Assumes: CRRA-Lognormal real growth model

Real Growth on Nominal Rate: 1998 to 2011 Data					Real Growth on Nominal Rate: 1977 to 1997 Data				
Intercept	-3.71	(t= -2.2)			Intercept	4.11	(t= 3.2)		
Slope	1.42	(t= 3.8)			Slope	-0.12	(t= -0.8)		
MuCgrow	3				MuCgrowt	3			
		Relative Risk Aversion (Gamma)					Relative Risk Aversion (Gamma)		
Nominal	Real	2	4	8	Nominal	Real	2	4	8
Rate	Growth	Ratio of True Probability to Risk Neutral*			Rate	Growth	Ratio of True Probability to Risk Neutral*		
1	-2.29	0.90	0.81	0.65	1	3.99	1.02	1.04	1.08
2	-0.87	0.93	0.86	0.73	2	3.87	1.02	1.04	1.07
3	0.55	0.95	0.91	0.82	3	3.75	1.02	1.03	1.06
4	1.97	0.98	0.96	0.92	4	3.63	1.01	1.03	1.05
5	3.39	1.01	1.02	1.03	5	3.51	1.01	1.02	1.04
6	4.81	1.04	1.08	1.16	6	3.39	1.01	1.02	1.03
7	6.23	1.07	1.14	1.29	7	3.27	1.01	1.01	1.02
8	7.65	1.10	1.20	1.45	8	3.15	1.00	1.01	1.01
9	9.07	1.13	1.27	1.63	9	3.03	1.00	1.00	1.00
10	10.49	1.16	1.35	1.82	10	2.91	1.00	1.00	0.99
					11	2.79	1.00	0.99	0.98
					12	2.67	0.99	0.99	0.97
					13	2.55	0.99	0.98	0.96
					14	2.43	0.99	0.98	0.96
					15	2.31	0.99	0.97	0.95
*=Up to a scalar multiple					16	2.19	0.98	0.97	0.94