

Firm Volatility in Granular Networks*

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Abstract

We propose a model of firm volatility based on customer-supplier connectedness. We assume that customers' growth rate shocks influence the growth rates of their suppliers, larger suppliers have more customers, and the strength of a customer-supplier link depends on the size of the customer firm. When the size distribution becomes more dispersed, economic activity is concentrated among a smaller number of firms, the typical supplier becomes less diversified and its volatility increases. The model is consistent with a set of new stylized facts. At the macro level, the firm volatility distribution is driven by firm size dispersion; the latter explains common movements in firm-level total and residual volatility. At the micro level, we show that the concentration of customer networks is an important determinant of firm-level volatility.

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1 Introduction

Firms experience great volatility in their growth rates and stock returns. Such fluctuations in uncertainty have important implications for investment and hiring decisions, firm value, and stock prices.¹ While much progress has been made in describing the time-series dynamics of volatility, our understanding of the underlying determinants of firm volatility is limited. Indeed, most work on volatility in economics and finance models firms that are exposed to heteroscedastic shocks without specifying the source of heteroscedasticity. In contrast, this paper explores the determinants of firm volatility both theoretically and empirically.

We propose a model in which firms are connected to other firms in a business network. Idiosyncratic and homoscedastic shocks hit each firm and propagate through the network. Each firm has a limited number of business connections. The granularity of firm networks makes firms imperfect aggregators of idiosyncratic shocks. The model links the firm size distribution to network formation, generating a rich interaction between the firm size and firm volatility distributions. As the firm size distribution becomes more dispersed, firm networks become more concentrated, and the diversification of shocks becomes impaired. Both the average firm volatility and its cross-sectional dispersion increase. The interaction between the size distribution and network structure endogenously generates heteroscedastic firm volatility. Guided by the model's predictions, we establish a new set of stylized cross-sectional and time series facts about firm volatility. A calibrated version of our model captures these facts not only qualitatively but also quantitatively.

Our network model of firm growth makes three assumptions. First, shocks travel upstream: customers' growth rate shocks influence the growth rates of their suppliers. Second, the probability of the presence of a customer-supplier link depends on the size of the supplier. Hence, large firms typically supply to a higher number of customers. Third, the importance of a customer-supplier link depends on the size of the customer. Hence, large customers have a stronger connection with their suppliers, presumably because they accounts for a large fraction of their suppliers' sales. We provide microeconomic evidence for all three assumptions

¹See Leahy and Whited (1996), Bloom, Bond, and Van Reenen (2007), Bloom (2009), Stokey (2012) among many others. The volatility of individual stock returns varies greatly over time (Lee and Engle 1993) and across different firms (Black (1976), Christie (1982), and Davis, Haltiwanger, Jarmin, and Miranda (2007)).

based on the observed customer-supplier networks among Compustat firms.

These simple assumptions have six main implications for firm volatility and its relationship to firm size. We test these predictions in the data and find support for all of them.

First, larger firms have lower volatility than smaller firms. Because they are connected to more customers, their customer network is less concentrated which improves diversification. Second, firms with more concentrated customer networks have higher volatility. Because customer size determines the strength of a link, a supplier's customer network will be more concentrated if there is high dispersion in its customers' sizes. Large customers' shocks will exert an outsized influence on the supplier, which results worse diversification and high volatility, even if the firm has a large number of customers. The data indeed shows a strong negative cross-sectional relationship between firm size and firm volatility and a strong positive correlation between a firm's out-Herfindahl, our measure of the degree of concentration of the customer network, and its volatility. The dependence of firm volatility on firm size and firm out-Herfindahl survives the inclusion of other determinants of volatility previously proposed in the empirical literature, including industry concentration and competition, R&D intensity, equity ownership composition, and firm age or cohort effects.

Third, because each supplier's network is a random draw from the entire size distribution, all firms' customer networks have equal concentration in expectation. They inherit the concentration of the entire firm size distribution. Increases in the dispersion of the size distribution hamper the diversifiability of shocks for all firms, and hence increase the volatility of all firms. Thus, the model generates a factor structure in volatilities with the firm size dispersion as the factor. In the data, lagged size dispersion explains 25% of the variation in (realized) firm volatilities, as much as is explained by lagged average volatility, a natural benchmark. Fourth, small firms have a larger exposure to this common factor, which also implies that fluctuations in the factor should account for a larger fraction of the volatility dynamics of small firms. Both are features of the data.

Fifth, as a result of the factor structure, firm size dispersion is positively correlated with mean firm volatility and with the dispersion of firm volatility. In the data, size dispersion has a 67% correlation with mean firm volatility and 61% with the dispersion of firm volatility. A persistent widening in the firm size dispersion should lead to a persistent rise in mean firm

volatility. We observe such a widening (increase in firm concentration) between the early 1960s and the late 1990s, providing a new explanation for the trend in mean firm volatility studied by Campbell et al. (2001).

Sixth, residual firm volatility, defined as the volatility of the part of firm growth that is orthogonal to aggregate growth, inherits the same factor structure. Intuitively, the factor model is misspecified because a firm's growth only depends on the growth of the customers in its network, not on a size-weighted average growth of all firms. Consistent with these predictions, Kelly, Lustig and Van Nieuwerburgh (2012) uncover a factor structure in the volatility of stock returns (1926-2010) and sales growth rates (1962-2010). Even after removing all of the common variation in returns or sales growth, as is commonly done in the literature (see, e.g., Bekaert, Hodrick, and Zhang (2010)), the puzzling finding is that the volatility of the residuals inherits the same factor structure as total volatility. Our paper provides a resolution of this puzzle.

Collectively, this evidence supports a network-based explanation of firm volatility. Next we explore whether the network model is able to quantitatively account for the correlations between firm size, firm volatility, the number of customers, and the concentration of its customer base, as well as aggregate moments of the cross-sectional distributions of size, volatility and connections. The complexities that arise from the inherent non-linearity of the network as well as its dynamics (exit and entry of firms and persistence in connections) necessitate a numerical simulation. The calibration also allows us to confront truncation and selection issues we face in the data. We find that a calibration with strong network effects and high average concentration in customer networks is able to account quantitatively for most features cross-sectional and aggregate features of the size and volatility data. Interestingly, the model displays substantial movement in aggregate moments of size and volatility distributions, despite having a large number of firms and being run for a large number of periods. The data display similar movements, but standard models of the firm size distribution imply that such aggregate moments are constant.

In addition to the empirical literatures on fundamentals-based and return-based firm volatility, our paper contributes to the literature on networks in finance.² To that literature,

²See Allen and Babus (2009) for a recent summary paper.

we contribute a model that links the size distribution to network formation and network formation to the firm volatility distribution. We use the network structure in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), but apply it at the firm level. We combine their insights with the idea that each firm only connects with a small number of customers. The granularity of the network is related to the idea that a few large firms account for a large part of aggregate output and hence of aggregate volatility (Gabaix 2011). However, our granularity operates at the firm level thereby endogenously generating firm-level volatilities rather than taking those as given. Our explicit aggregation to economy-wide volatility can be interpreted as providing micro-foundations for the granularity effect in Gabaix (2011).

Our paper brings to the fore an interesting question on the direction of shock propagation in the networks literature. The literature that tries to understand aggregate volatility of GDP by studying the connections between sectors (e.g., Long and Plosser (1987), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Carvalho and Gabaix (2013)) typically considers *downstream* transmission of shocks from intermediary goods to final goods producers. Our work and that in corporate finance (e.g., Ahern and Harford (forthcoming)) instead assumes that shocks propagate upstream. Shea (2002) lists a class of equilibrium models in which final demand shocks are transferred upstream. Thus, our specification emphasizes demand shocks over supply shocks as drivers of firm volatility. To understand the difference in predictions, the appendix studies a version of our model where shocks travel downstream from suppliers to customers instead. The implications for the size and volatility distributions are identical. Interestingly, the implications for the correlation between volatility on the one hand and customer concentration (out-Herfindahl) and supplier concentration (in-Herfindahl) on the other hand are different. On balance, the data supports the downstream transmission mechanism more strongly.

The rest of the paper is structured as follows. Section 2 sets up the network model and studies its volatility implications. Section 3 imposes three assumptions and describes the factor structure in volatility generated by the model. Section 4 presents macro evidence on the link between the firm size and the firm volatility distributions. Section 5 describes micro evidence on the link between firms' network structures and their volatility. Section 6 calibrates the model and studies the match with the macro and micro evidence. The proofs

as well as some auxiliary empirical evidence, theoretical results, and alternative calibrations are relegated to the appendix.

2 A Network Model of Firm Growth

We develop a dynamic network model of connections between firms. While the model is general at this stage, we refer to it as a model of connections between suppliers and customers. In this directed network, suppliers' growth rates depend on their own idiosyncratic shocks and the growth rates of their customers.

2.1 Firm Growth

Define $S_{i,t}$ as the size of firm i , and its growth rate as $g_{i,t+1}$, where

$$S_{i,t+1} = S_{i,t} \exp(g_{i,t+1}). \quad (1)$$

Firm i 's growth rate is defined as a linear combination of its own shock and a weighted average of the growth rates of its customers j :

$$g_{i,t+1} = \mu_g + \gamma \sum_{j=1}^N w_{i,j,t} g_{j,t+1} + \varepsilon_{i,t+1}. \quad (2)$$

The parameter $\gamma \in [0, 1)$ governs the rate of decay as a shock propagates through the network. The weight $w_{i,j,t}$ determines how strongly firm i 's growth rate is influenced by the growth rate of firm j ; it governs the strength of the connection between i and j . If i and j are not connected then $w_{i,j,t} = 0$. By convention, we set $w_{i,i,t} = 0$. The full matrix of connection weights is $\mathbf{W}_t = [w_{i,j,t}]$. We assume that all rows of \mathbf{W}_t sum to one so that its largest eigenvalue λ_{max} equals one. Connections are not symmetric: firm j can be a customer of i without i being a customer of j .³ Let \mathbf{g}_{t+1} and $\boldsymbol{\varepsilon}_{t+1} \sim N(0, \sigma_\varepsilon^2 \mathbf{I})$ be the $N \times 1$ vectors

³As an aside, these growth dynamics impute dynamics to the relative sizes, or shares, of firms that are similar to those explored by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). In contrast to their work, there is no mean reversion built into our shares. Furthermore, only the customer shares in \mathbf{W}_t are relevant for the cash flow dynamics of a firm. These shares divide by the sum of all customers not by the sum of all firms in the economy.

of growth rates and shocks, respectively, then:

$$\mathbf{g}_{t+1} = \boldsymbol{\mu}_g + \gamma \mathbf{W}_t \mathbf{g}_{t+1} + \boldsymbol{\varepsilon}_{t+1} = (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}). \quad (3)$$

We purposely imposes stark assumptions on the nature of the underlying innovations: Each firm i experiences an idiosyncratic, homoscedastic growth rate shock $\varepsilon_{i,t+1} \sim N(0, \sigma_\varepsilon)$. All dynamics in the volatility of growth rates will arise endogenously.

Acemoglu et al. (2012) derive a static version of (3) as the equilibrium outcome in a multi-sector production economy. There is also a directed network, but in their version the productivity shocks are transferred downstream from suppliers to customers.⁴ The appendix explores a version of our model where shocks are propagated downstream from suppliers to customers.

Since we focus on propagation from customer to supplier firms, a question arises as to how to think of retail firms whose customers are households. Since households are not part of the model, this would appear to be a source of leakage. Appendix A.2 discusses an extension of the model where retail firms are exposed to the shocks of other firms through the labor income and wealth shocks experienced by households.

2.2 Firm Volatility

Conditional on \mathbf{W}_t , the variance-covariance matrix of growth rates \mathbf{g}_{t+1} is given by:

$$\mathbf{V}_t(\mathbf{g}_{t+1}) = \sigma_\varepsilon^2 (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\mathbf{I} - \gamma \mathbf{W}_t')^{-1}. \quad (4)$$

The vector of firm volatilities is the square root of the diagonal of the variance-covariance matrix.

Let $\tilde{\mathbf{S}}$ denote the vector of relative sizes, with $\tilde{S}_i = S_i / \sum_k S_k$. Then we can define the

⁴There is a constant-returns to scale economy populated by a stand-in agent who has Cobb-Douglas preferences defined over all of the N different commodities produced. Productivity shocks are transferred downstream from suppliers to customers. In this economy, the weights $w_{k,j}$, are the input weights of good k for firm j in the production of commodity j while γ measures the input share in GDP. This specification emphasizes upstream supply shocks as the only drivers of firm-level volatility, while our model focuses on downstream demand shocks as the only drivers.

aggregate growth rate of the economy as:

$$g_{a,t+1} = \tilde{\mathbf{S}}_t' \mathbf{g}_{t+1} = \tilde{\mathbf{S}}_t' (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}). \quad (5)$$

The variance of the aggregate growth rate is given by:

$$V_t(g_{a,t+1}) = \sigma_\varepsilon^2 \tilde{\mathbf{S}}_t' (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\mathbf{I} - \gamma \mathbf{W}_t')^{-1} \tilde{\mathbf{S}}_t. \quad (6)$$

A standard approach to evaluating systematic versus idiosyncratic risk is to run factor model regressions. The primary factor is typically a size-weighted average of the left hand side variable. It is instructive to examine how the network model impacts this type of analysis. Let $\boldsymbol{\beta}$ denote the $N \times 1$ vector of factor exposures. Then the residual growth rate is defined as:

$$\mathbf{g}_{t+1}^{res} = \mathbf{g}_{t+1} - \boldsymbol{\beta} g_{a,t+1} = (\mathbf{I} - \boldsymbol{\beta} \tilde{\mathbf{S}}_t') (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}). \quad (7)$$

The conditional variance of the residual growth rate is given by:

$$V_t(\mathbf{g}_{t+1}^{res}) = \sigma_\varepsilon^2 (\mathbf{I} - \boldsymbol{\beta} \tilde{\mathbf{S}}_t') (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} (\mathbf{I} - \gamma \mathbf{W}_t')^{-1} (\mathbf{I} - \tilde{\mathbf{S}}_t \boldsymbol{\beta}'). \quad (8)$$

A key observation is that variation in \mathbf{W}_t and $\tilde{\mathbf{S}}_t$ induces heteroscedasticity in both total and residual growth rates, even though all underlying innovations are i.i.d. across firms and over time.

2.3 First-order Effects on Volatility

To better understand the relation between volatility and network structure, we start by studying first-order network effects. The first order approximation to the full network model allows us to develop intuition because it admits analytical expressions for firm volatility.

Under our maintained assumption that $\gamma \in [0, 1)$,

$$(\mathbf{I} - \gamma \mathbf{W}_t)^{-1} = \mathbf{I} + \gamma \mathbf{W}_t + \gamma^2 \mathbf{W}_t^2 + \gamma^3 \mathbf{W}_t^3 + \dots \approx \mathbf{I} + \gamma \mathbf{W}_t,$$

which delivers a first-order approximation of the growth rate:

$$\mathbf{g}_{t+1} \approx (\mathbf{I} + \gamma \mathbf{W}_t) (\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}). \quad (9)$$

In this approximation, firm i 's growth rate depends not on its customers' growth rates but directly on their growth rate *shocks*:

$$g_{i,t+1} \approx (1 + \gamma) \mu_g + \gamma \sum_j w_{i,j,t} \varepsilon_{j,t+1} + \varepsilon_{i,t+1}. \quad (10)$$

Under the first-order approximation, the variance of firm's i growth rate is given by:

$$V_t(g_{i,t+1}) \approx \sigma_\varepsilon^2 (1 + \gamma^2 H_{i,t}), \quad (11)$$

where

$$H_{i,t} \equiv \sum_{j=1}^N w_{i,j,t}^2 \quad (12)$$

is the Herfindahl index of supplier i 's network of customers. We refer to $H_{i,t}$ as the customer Herfindahl or the out-Herfindahl. The higher i 's out-Herfindahl, the more concentrated her network of customers is, and the higher her variance. Hence, to a first order approximation, the variance of a firm's growth rate is determined by its customer Herfindahl $H_{i,t}$, the volatility of the underlying innovations σ_ε^2 , and the importance of (rate of shock decay in) the network captured by the parameter γ .

The first-order approximation of firm variance delivers a lower bound on the true variance in the full network model.

Proposition 1. *If $\gamma \in (0, 1)$, average firm variance is bounded below by a quantity that depends on the average out-Herfindahl:*

$$\frac{1}{N} \sum_i V_t(g_{i,t+1}) \geq \sigma_\varepsilon^2 \left(1 + \gamma^2 \frac{1}{N} \sum_i H_{i,t} \right) \quad (13)$$

The proof of this and all ensuing propositions are in Appendix [A.1](#). The higher-order

feedback effects that the first-order approximation abstracts from can only increase firm-level volatility.

2.4 Higher-order Network Effects on Volatility

The strength of higher-order network effects depends on γ and the properties of the network matrix \mathbf{W}_t . We can derive bounds on firm volatility in two polar cases. The first case features maximally strong feedback effects and no diversification benefits from linkages. Each firm only has one connection, it's neighbor to the right. The corresponding \mathbf{W}_t matrix has ones above the main diagonal and zeros elsewhere, except for a one in the $(N, 1)$ position. The second polar case considers a network with maximal diversification. Each firm is connected to all other firms in the network with equal strength. The corresponding \mathbf{W}_t matrix has zeros on the main diagonal and $\frac{1}{N-1}$ elsewhere.

Proposition 2. *In a symmetric network, the variance of each firm i at each time t is bounded by:*

$$\sigma_\varepsilon^2 \left[\frac{\left(\frac{N-2}{N-1}\gamma - 1\right)^2 + \frac{1}{N-1}\gamma^2}{\left(\frac{1}{N-1}\gamma^2 + \frac{N-2}{N-1}\gamma - 1\right)^2} \right] \leq V_t(g_{i,t+1}) \leq \frac{\sigma_\varepsilon^2(1 - \gamma^{2N})}{(1 - \gamma^2)(1 - \gamma^N)^2}. \quad (14)$$

As the number of firms in the economy goes to infinity these bounds become:

$$\sigma_\varepsilon^2 \leq V_t(g_{i,t+1}) \leq \frac{\sigma_\varepsilon^2}{1 - \gamma^2}$$

The lowest firm variance can be in a large economy is the original shock variance σ_ε^2 . The upper bound for volatility obtained under maximal concentration grows without bound as γ approaches one. In the limit, shocks do not decay as they propagate through the network. In our calibration below, which does not impose symmetry, firm variances lie between these two bounds. The data require a high value for γ , producing a wide range of possible volatilities.

3 Size and Volatility

We now introduce two key assumptions on the probability and the strength of connections between customers and suppliers. These assumptions connect the firms size distribution to the network structure of the economy. We provide micro-economic evidence for these assumptions in section 5.

3.1 Size Effects in Network Structure

The linkage structure at time t is determined by the firm size distribution coming into period t . The existence of a link between i and j is captured by

$$b_{i,j,t} = \begin{cases} 1 & \text{if } i \text{ connected to } j \text{ at time } t \\ 0 & \text{otherwise,} \end{cases}$$

Each element of the connections matrix, $\mathbf{B}_t = [b_{i,j,t}]$, is drawn from a Bernoulli distribution with $P(b_{i,j,t} = 1)$. This probability of connection is assumed to be linear in supplier size:

$$P(b_{i,j,t} = 1) \equiv p_{i,t} = S_{i,t} / \left(Z \sum_j (S_{j,t}) \right), \quad (15)$$

While the functional form only matters quantitatively, the crucial assumption is that the probability of a connection depends on the size of the supplier, not on the size of the customer. It follows immediately that larger suppliers have more connections on average. This is the model's first size effect.

Conditional on a link existing between supplier i and customer j ($b_{i,j,t} = 1$), the strength of that link is linear in customer size:

$$w_{i,j,t} = \frac{b_{i,j,t} S_{j,t}}{\sum_{k=1}^N b_{i,k,t} S_{k,t}}, \quad \forall i, j, t. \quad (16)$$

This weighting scheme assumes that larger customers have a larger impact on a supplier's growth rate. This assumption is the second size effect in our network.

3.2 Factor Structure in Volatility

Because all firms, large and small, draw their connections from the same economy-wide size distribution, their customer networks have equal concentration in expectation. The only difference in the network structure across suppliers of different size is the *number* of customers they have. This commonality in network concentration naturally leads to a factor structure in firm volatilities.

To derive closed-form approximations, we consider what happens to the network structure when the number of firms N becomes large. This allows us to appeal to the law of large numbers. We characterize the expected network structure conditional on the size distribution, and the latter's relation to firm volatility.

3.2.1 First-Order Network Model

Under the first-order approximation to the full network model, firm variance depends on the firm's out-Herfindahl $H_{i,t}$ as in equation (11). In a large economy, a firm's out-Herfindahl converges to the economy-wide Herfindahl H_t divided by the probability p_i :

Proposition 3. *In the first-order approximation, firm variance displays a factor structure:*

$$V_i(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \frac{1}{p_{i,t}} \gamma^2 H_t \right). \quad (17)$$

as the number of firms N grows large.

Equation (17) shows that all firms' volatilities are exposed to a common factor: the Herfindahl index of the *entire* firm size distribution. It also shows that cross-sectional differences in volatility are captured by one over the linkage probability, $p_{i,t}$. Larger firms have a higher p_i and therefore lower exposure to the common factor than smaller firms. Intuitively, larger firms typically connect to more customers, achieve better shock diversification, and display lower volatility. The lower exposure also makes large firms less sensitive to fluctuations in H_t (the firm size distribution): they display lower volatility of volatility.

If the firm size distribution is log-normal with mean $\mu_{s,t}$ and variance $\sigma_{s,t}^2$, the natural candidate for the volatility factor is the firm size dispersion:

Corollary 1. *If the firm size distribution is log-normal, firm variance displays a factor structure with the firm size dispersion as factor*

$$V(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \frac{1}{p_{i,t}} \gamma^2 \frac{\exp(\sigma_{s,t}^2)}{N} \right), \quad (18)$$

in the first-order version of the network for large N .

Proposition 4. *If the size distribution is log-normal, the volatility distribution is also log-normal with mean $E[\log(V(g_{i,t+1}) - \sigma_\varepsilon^2)] \approx \log(\sigma_\varepsilon^2 \gamma^2 Z) + \frac{3}{2} \sigma_{s,t}^2$ and volatility $V(\log(V(g_{i,t+1}) - \sigma_\varepsilon^2)) \approx \sigma_{s,t}^2$. In the first-order version with large N .*

Corollary 1 and Proposition 4 imply that the cross-sectional firm size dispersion at time t drives both the cross-sectional mean and the cross-sectional dispersion of the firm variance distribution, where the variance is measured between t and $t + 1$. Below, we show that the log size and log variance distributions are accurately described by normal distributions in the data and that the dispersion in log firm size distribution Granger-causes both the mean and the dispersion of the log variance distribution.

3.2.2 Higher-Order Network Model

We can extend our large- N approximation to include higher-order network effects. The key result is that the economy-wide Herfindahl survives as the common factor driving all firm volatilities.

Proposition 5. *If the number of firms N grows large, firm variance is given by:*

$$V_t(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \kappa_0 \frac{S_{i,t}}{NE[S_t]} + \kappa_{i,t} H_t \right)$$

where $\kappa_0 = \frac{2\gamma^2}{1-\gamma}$ and $\kappa_{i,t} = \frac{\gamma^2}{p_{i,t}} + \frac{2\gamma^3}{1-\gamma} + \frac{\gamma^4}{(1-\gamma)^2}$.

Once we include higher-order effects, there are two countervailing size effects. The first effect is the standard diversification effect: larger firms have higher p_i , larger networks, and hence a smaller volatility. The second effect is a higher-order network effect captured by κ_0 :

larger firms have more links and hence are more exposed to “echo” effects from their own shocks. In our calibrated model, the first effect clearly prevails.

3.2.3 Residual Volatility

In the literature, idiosyncratic volatility is typically constructed by first removing the correlated component of growth rates (or returns) with a statistical model such as principal component analysis, then calculating the volatilities of the residuals. In a granular network model like ours, a standard factor regression is misspecified. There is no dimension-reducing factor that can capture commonalities in growth rates since, by virtue of the network, *every* firm’s shock may be systematic. A sign of the misspecification of the factor model is that the residuals exhibit a volatility factor structure that looks very similar to the factor structure for total firm volatility.

Proposition 6. *In a first-order version, the volatility of residuals also inherits an approximate factor structure:*

$$V_t(g_{i,t+1}^{res}) \approx \sigma_\varepsilon^2 \left(1 + \frac{1}{p_{i,t}} \gamma^2 H_t - \left[\frac{S_{i,t}}{NE[S_{i,t}]} + \gamma H_t \right]^2 \right). \quad (19)$$

3.2.4 Aggregate Volatility

Finally, we consider the impact on aggregate volatility. In the first-order version of our economy, aggregate volatility also is governed by the same factor H_t .

Proposition 7. *In the first-order version, aggregate volatility inherits an approximate factor structure:*

$$V(g_{a,t+1}) \approx \sigma_\varepsilon^2 H_t (1 + \gamma)^2. \quad (20)$$

4 Macro Evidence on Size Dispersion and Volatility

This section documents new stylized facts about the joint evolution of the firm size and firm volatility distributions.

4.1 Data

We conduct our analysis using stock market data from CRSP over the period 1926–2010 and cash flow data from CRSP/Compustat over 1952–2010. We consider market and fundamental measures of firm size and firm volatility calculated at the annual frequency. For size, we use equity market value at the end of the calendar, or total sales within the calendar year. Market volatility is defined as the standard deviation of daily stock returns during the calendar year. Fundamental volatility in year t is defined as the standard deviation of quarterly sales growth (over the same quarter the previous year) within calendar years t to $t + 4$.⁵

Appendix B.1 establishes that log size and log volatility are approximately normally distributed in the data. The near-log normality of size and volatility distributions is convenient in that the dynamics of each distribution may be summarized with two time series: the cross section mean and standard deviation of the log quantities. We next examine these time series in detail.

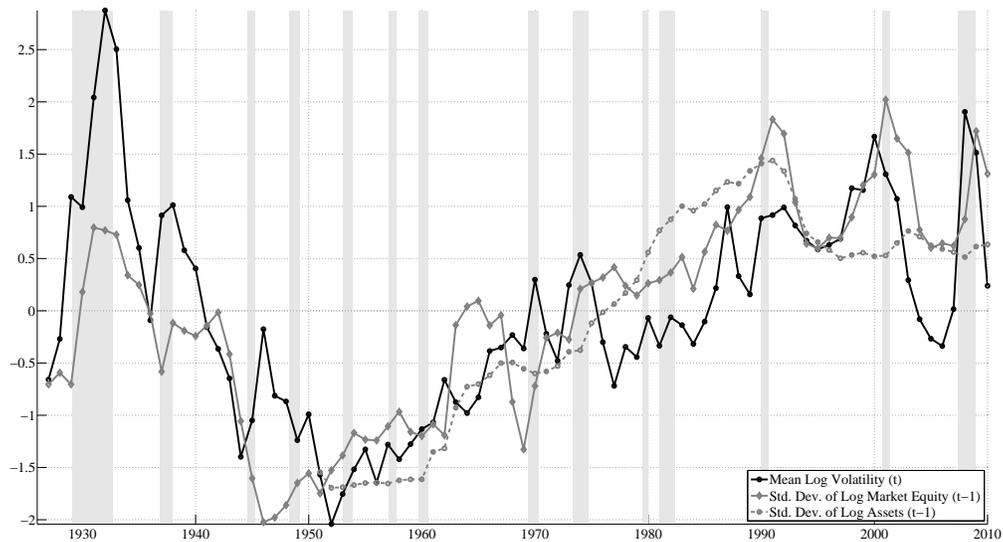
4.2 Comovement of Size and Volatility Distributions

Figure 1 plots the cross-sectional *average* of log firm variance against two different measures of lagged firm size dispersion: the dispersion of log sales and of log market equity across firms. The correlation between average volatility and the market-based measure of size dispersion is 67% while the correlation with the sales-based measure is 61%. Figure 2 plots the cross-sectional *dispersion* of log firm variance against the same two measures of size dispersion. The correlation between volatility dispersion and the market-based measure of lagged size dispersion is 60% while the correlation with sales-based dispersion is 41%. Hence, as predicted by the model, we observe a strong positive association relation between firm

⁵We also consider fundamental volatility measured by the standard deviation of quarterly sales growth within a single calendar year. The one and five year fundamental volatility estimates are qualitatively identical, though the one year measure is noisier because it uses only four observations.

size dispersion on the one hand and the average firm volatility and the dispersion of firm volatility on the other hand.

Figure 1: AVERAGE VOLATILITY AND DISPERSION IN FIRM SIZE

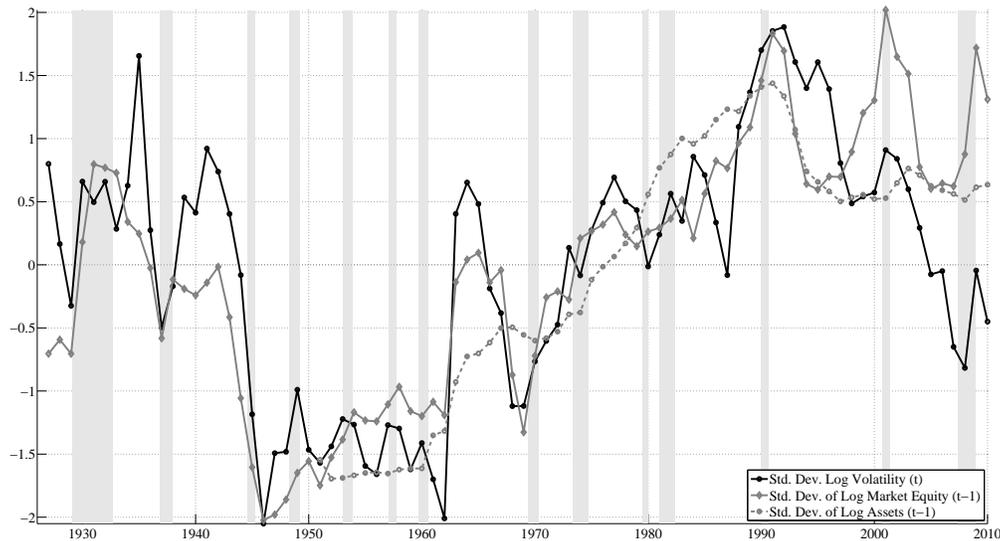


Notes: The figure plots the cross-sectional dispersion of log firm size, once measured based on market equity values and once based on sales, and the cross-sectional mean of the log variance distribution, where the variance is measured based on daily stock returns. All series are rescaled for the figure to have mean zero and variance one.

Figure 3 plots the mean (top panel) and dispersion (bottom panel) of the log variance distribution where variances are calculated either from stock return (solid line) or from sales growth data (dashed line). Average return volatility and average sales growth volatility have an annual time series correlation of 64% and their dispersions have a correlation of 49%. This demonstrates a high degree of similarity between market volatilities and its (more coarsely measured) fundamental counterpart. Any explanation of these volatility facts, including “financial explanations,” must confront this similarity.

The positive correlations between firm size dispersion and both the mean and dispersion of firm variance are very robust. Appendix B.2 shows that these correlations hold across broad industries, across size terciles, for NYSE firms and non-NYSE firms, for firms that have been publicly listed at least 50 years and for random samples of 500 firms. Hence, these correlations unlikely to be driven by changes in the nature of publicly listed firms. Appendix B.3 shows that the positive correlation between size dispersion and average volatility holds

Figure 2: DISPERSION IN VOLATILITY AND DISPERSION IN FIRM SIZE



Notes: The figure plots the cross-sectional dispersion of log firm size, once measured based on market equity values and once based on sales, and the cross-sectional dispersion of the log variance distribution, where the variance is measured based on daily stock returns. All series are rescaled for the figure to have mean zero and variance one.

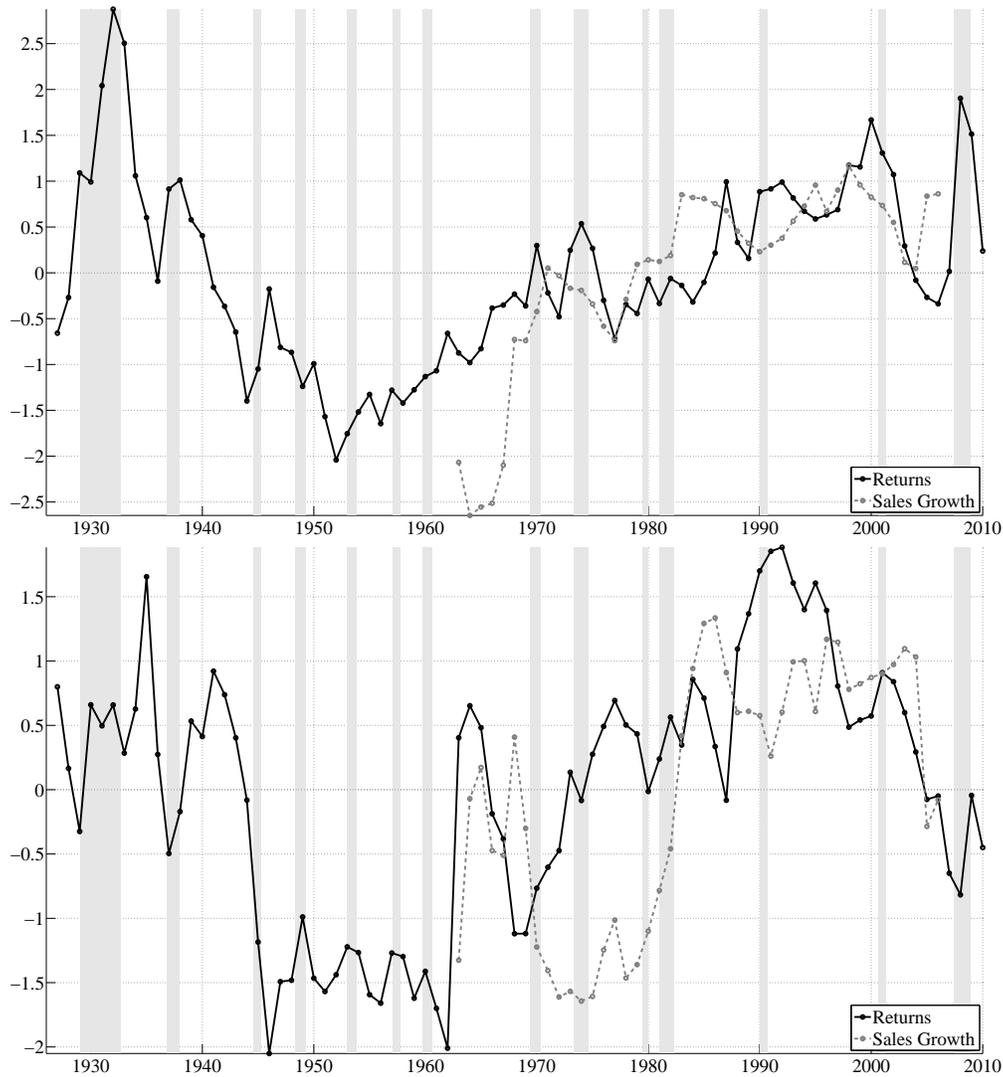
both at low and business cycle frequencies.

Finally, the changes in the firm size distribution that drive these volatility dynamics are not specific to publicly traded firms. Appendix B.4 documents that the dispersion in the log size distribution, where size is measured by number of employees, displays similar dynamics for publicly listed and privately-held firms. In particular, the sample of private companies also displays a secular increase in firm size dispersion between the mid-1970s and mid-2000s. While we do not aim to explain why the firm size dispersion grew -other than to provide a model where firm size and volatility dynamics feed into each other- we conjecture that the secular decline in firm exit and entry rates is related to the increase in size concentration over this period (Appendix B.5).

4.3 Granger Causality Tests

Our network model predicts that movements in the size distribution precede changes in the volatility distribution. When the size distribution spreads out (contracts) at time t , the network structure for the subsequent period adjusts, and diversification of growth rate shocks is

Figure 3: FUNDAMENTAL AND MARKET VOLATILITY



Notes: The figure plots the cross-sectional mean (top panel) and standard deviation (bottom panel) of the log variance distributions based on returns and based on sales growth. All series standardized to have mean zero and variance one.

hindered (enhanced). We use Granger causality tests to evaluate whether dispersion in firm sizes predicts the mean and standard deviation of the volatility distribution, after controlling for own lags of the dependent variable. Table 1 presents the results from these tests. We find that firm size dispersion (based on log market equity) has statistically significant predictive power for mean firm volatility (based on returns), and dispersion in volatility. The reverse is not true. After controlling for own lags of size moments, moments of the volatility

Table 1: GRANGER CAUSALITY TESTS

Dependent Variable		Independent Variables		
		Intercept	$\mu_{\sigma,t-1}$	$\sigma_{s,t-1}$
$\mu_{\sigma,t}$	Coeff	-1.18	0.74	0.07
	<i>t</i> -stat.	-2.82	8.31	2.21
$\sigma_{S,t}$	Coeff	-0.28	-0.11	0.97
	<i>t</i> -stat.	-0.47	-0.88	16.01
		Intercept	$\sigma_{\sigma,t-1}$	$\sigma_{s,t-1}$
$\sigma_{\sigma,t}$	Coeff	-0.02	0.61	0.04
	<i>t</i> -stat.	-1.48	5.86	3.12
$\sigma_{s,t}$	Coeff	0.20	-1.19	1.04
	<i>t</i> -stat.	1.47	-2.25	15.88

Notes: Annual data 1926–2010. The table reports results of Granger causality tests for the ability of log firm size dispersion ($\sigma_{S,t-1}$) to predict the mean ($\mu_{\sigma,t}$) and standard deviation ($\sigma_{\sigma,t}$) of the log volatility distribution. We use the market based volatility measure constructed from stock returns and the market equity measure of size.

distribution do not predict the size distribution.⁶ This evidence suggests that size dispersion leads the volatility distribution, consistent with the model.

4.4 Volatility Factor Structure

Recent research has documented a puzzling degree of common variation in the panel of firm-level volatilities. Kelly et al. (2012) show that firm-level stock return volatilities share a single common factor that explains roughly 35% of the variation in log volatilities for the entire panel of CRSP stocks.⁷ This R^2 is nearly twice as high for the 100 Fama-French

⁶Lagged dispersion in log volatility appears to Granger-cause size dispersion, but the coefficient has the wrong sign. The hypothesis has the one-sided alternative that volatility dispersion *positively* predicts size dispersion; thus this negative result leads us to fail to reject the null.

⁷Similarly, Engle and Figlewski (2012) document a common factor in option-implied volatilities since 1996, and Barigozzi, Brownlees, Gallo, and Veredas (2010) and Veredas and Luciani (2012) examine the factor structure in realized volatilities of intra-daily returns since 2001. The contribution of this paper is to provide an economic explanation for a common factor in firm-level volatility, and argue that the cross-sectional dispersion of firm size is natural candidate for this volatility factor.

portfolios. They also show that this strong factor structure is not only a feature of return volatilities, but also of sales growth volatilities. The puzzling aspect of this result is that the factor structure remains nearly completely intact after removing all common variation in returns (or sales growth rates) by extracting principal components. Hence, common volatility dynamics are unlikely to be driven by an omitted common return (or sales growth) factors.

The results of Section 3 suggest that our granular network model may be able to explain the puzzling comovement in firm volatility. Propositions 3 and 5 show an approximate factor structure among the volatilities of all firms, and suggests that concentration of the lagged economy-wide size distribution is the appropriate factor. Furthermore, equation (24) predicts that factor model residuals will possess a similar degree of volatility comovement, despite residual growth rates themselves being nearly uncorrelated.

Panel A of Table 2 shows results of panel volatility regressions for three different factor models. The left three columns uses the volatility of total returns and total sales growth rates, while the next three columns use residual volatilities. Residual volatilities are calculated in a one factor model regression of stock returns (sales growth rates) on the value-weighted market return (sales-weighted average growth rates). In both cases, residuals have average pairwise correlations that are below 2% in absolute value, despite the original series having average correlations over 25% on average. Columns (1) and (4) considers the dispersion of lagged log market-based firm size as a factor. Columns (2) and (5) consider the lagged weighted-average volatility. This lagged cross-sectional average volatility is a natural benchmark for factor model comparison because it is essentially a first principal component of volatilities and because the lag maintains a comparable timing with the conditioning factor implied by our model. The third column instead uses the contemporaneous average volatility as a factor. Because it uses finer conditioning information, it can be considered an upper bound on the explanatory power of a single factor. We report the panel R^2 values based on each factor.

The table shows that the lagged size dispersion has the same degree of explanatory power as the lagged average volatility. Both factors capture about 25% of the panel variation in return volatility. The contemporaneous mean volatility explains closer to 40% of the variation. The results for sales-based volatility are similar, where our size dispersion factor

explains about 22% of the panel variation, close to the 23% and 24% for the lagged and contemporaneous average volatility factor. Columns (4) to (6) repeat these regressions for the panel of residual volatilities, and find similar results.

Table 2: R^2 OF VOLATILITY FACTOR MODELS

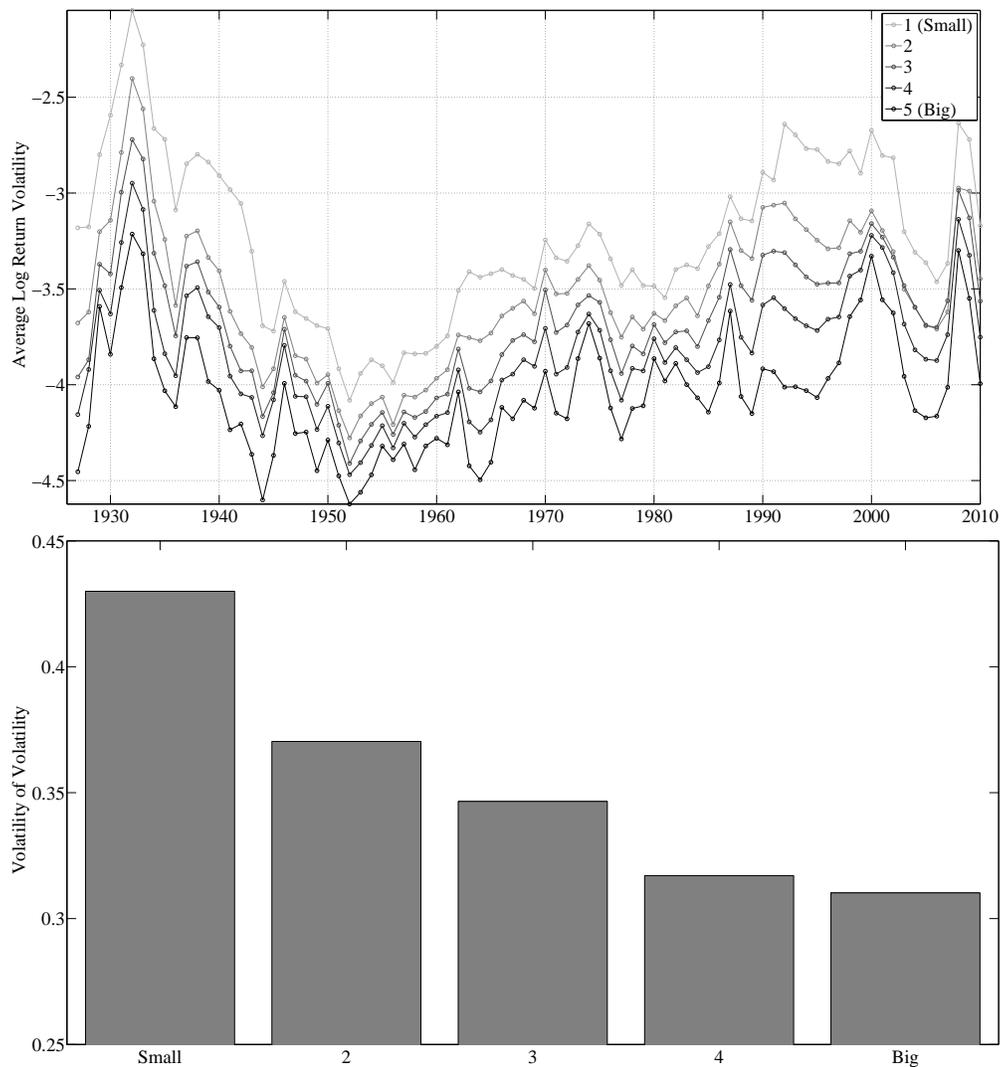
	Total Volatility			Residual Volatility		
	(10)	(2)	(3)	(4)	(5)	(6)
	Factors			Factors		
	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$
Panel A: Factor Model R^2, All Firms						
Return Vol.	24.4	25.9	39.3	24.7	26.5	37.5
Sales Gr. Vol.	21.8	23.4	24.3	21.4	25.8	27.8
Panel B: Return Volatility Loadings by Size Quintile						
Q1	1.25	1.09	1.18	1.32	1.10	1.19
Q5	0.80	0.65	0.84	0.81	0.55	0.71
Panel C: Return Volatility R^2 by Size Quintile						
Q1	47.9	76.5	90.9	52.6	77.2	90.6
Q5	37.7	53.7	87.2	48.9	49.2	79.1

Notes: The table reports factor model estimates for the panel of firm-year volatility observations. In Panel A, total volatility is measured as standard deviation of daily returns within the calendar year, and residual volatility is estimated from daily regressions of firm returns on the value-weighted market portfolio within the calendar year. In Panel B, total volatility in year t is measured as standard deviation of quarterly observations of year-on-year sales growth for each stock in calendar years t to $t + 4$. Residual volatility is measured from regressions of firms sales growth on the sales-weighted average growth rate for all firms. All volatility factor regressions take the form $\log \sigma_{i,t} = a_i + b_i \text{factor}_t + e_{i,t}$. We consider three different volatility factors. The first, motivated by our network model, is the lagged cross section standard deviation of log market equity, $\sigma_{S,t-1}$. The second and third factors we consider are the lagged and contemporaneous cross section average log volatility, $\mu_{\sigma,t-1}$ and $\mu_{\sigma,t}$. We report the pooled factor model R^2 in percent.

The model also predicts that larger firms have lower loadings on the size dispersion factor, which lowers both the level of their volatilities relative to small firms and their volatility of volatility. Panel B of Table 2 shows the loading of firm volatility on each factor, averaged within each quintile of the firm size distribution. Small firms have loadings that are 50%

larger on the factor than large firms. Panel C shows that the common factor explains a larger share of the volatility dynamics of small firms than that of large firms. Finally, Figure 4 plots the average log firm volatility each year (top panel) and the time-series volatility of average volatility (bottom panel) for each size quintile. Large firms have lower levels of volatility and also less variation in volatility. Table 2 and Figure 4 are consistent with the model's prediction that dispersion in firm sizes predicts the entire panel of firm-level volatilities.

Figure 4: AVERAGE FIRM VOLATILITY BY SIZE QUINTILE



Notes: The top panel plots average log return volatility within CRSP market equity quintiles. The bottom panel reports time series standard deviation of average volatility within each quintile.

4.5 Aggregate Volatility Accounting

The volatility of firm-level stock returns has increased from an average of 26% per year in the 1950s to 63% per year since 1990 (see, for example, Campbell, Lettau, Malkiel, and Xu (2001)). This increase is also present in residual volatilities from a factor model for returns. The increase in total and residual firm volatility puzzles financial economists. Candidate explanations can be divided into real and financial explanations. Our stylized facts about the joint distribution of volatility and size are present in both fundamental- and market-based measures, which would seem to point in the direction of a real explanation rather than a financial one. We contribute to this literature a new explanatory variable for the dynamics of firm volatility: firm size dispersion.

We find that accounting for changes in the size distribution nullifies the volatility trends of the 1950s-1990s sample. Figure 5 plots average market-based log firm variance as well as the residual from that same average variance on our market-based measure of size dispersion (the cross-sectional dispersion in log market equity). This regression has an R^2 of 51.3%. The dotted lines show estimated time trends in the original volatility data (black) and in the residual (gray). The figure shows that, after controlling for fluctuations in the firm size distribution over time, there is no anomalous trend in average firm volatility in the post-war era.

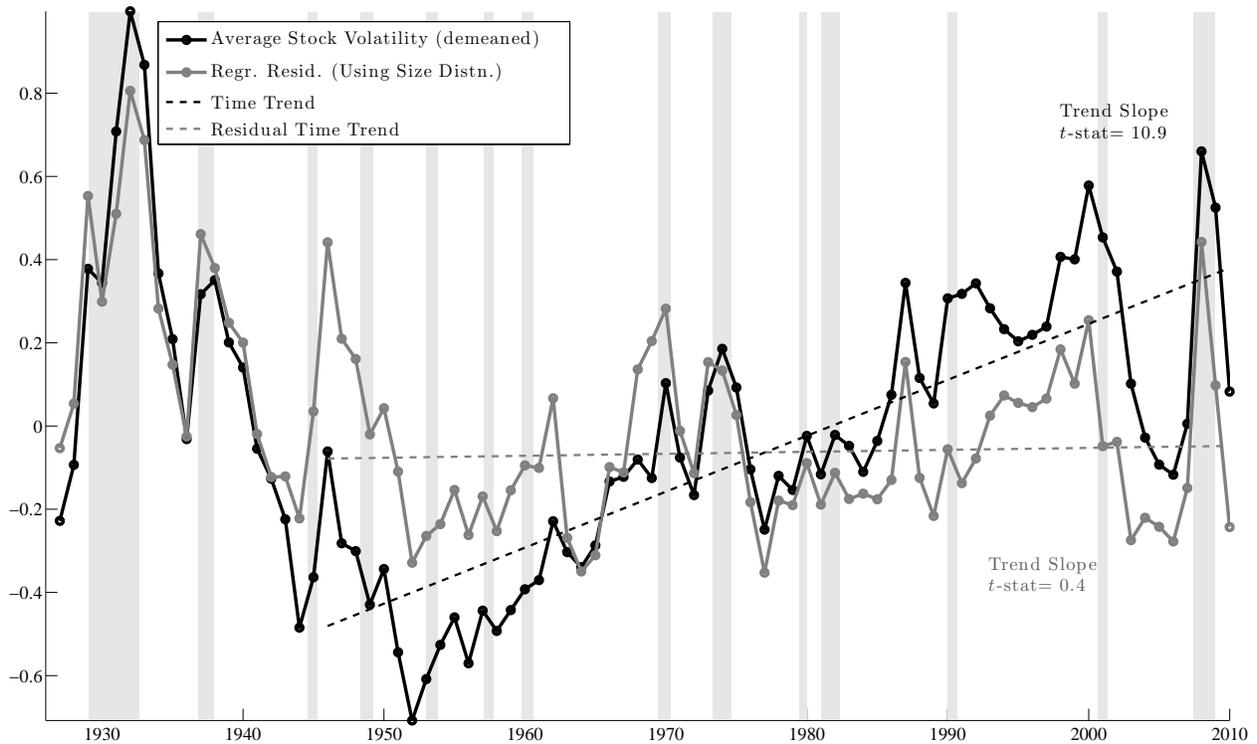
5 Micro Evidence of Granular Networks

This section discusses the micro evidence on business networks.

5.1 Firm-level Network Data

Our data for annual firm-level linkages comes from Compustat. It includes the fraction of a firm's dollar sales to each of its major customers. Firms are required to supply customer information in accordance with Financial Accounting Standards Rule No. 131, in which a major customer is defined as any firm that is responsible for more than 10% of the reporting seller's revenue. Firms have discretion in reporting relationships with customers that account

Figure 5: TRENDS IN AVERAGE FIRM VOLATILITY



Notes: The figure plots the average of log stock return variance (black line) as well as the residual from a regression of mean variance on our market-based measure of log firm size dispersion (gray line). It also shows estimated time trends in the original volatility data (black dotted line) and in the residual (gray dotted line), and reports the t -statistic for the time trend coefficient estimate.

for less than 10% of their sales, and this is occasionally observed (23% of firms). The Compustat data has been carefully linked to CRSP market equity data by Cohen and Frazzini (2008), which allows us to associate information on firms' network connectivity with their market equity size and their return volatility.⁸ The data set covers the period 1980-2009, and includes 48,839 customer-supplier-year observations. Appendix B.6 provides more details on the network data.

⁸Cohen and Frazzini (2008) used the same data to show that news about business partners does not immediately get reflected into stock prices. We are grateful for sharing their data with us. Atalay, Hortaçsu, Roberts, and Syverson (2011) also use these data to develop a realistic model of the buyer-supplier networks in the U.S. economy.

Table 3: OVERVIEW OF SIZE, VOLATILITY, AND CUSTOMER-SUPPLIER NETWORK STRUCTURE

	Supplier					Customer				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$w_{i,j,t}$	$\log S_{i,t}$	$\log S_{i,t}$	$H_{i,t}^{out}$	$H_{i,t}^{out}$	$w_{i,j,t}$	$\log S_{i,t}$	$\log S_{i,t}$	$H_{i,t}^{in}$	$H_{i,t}^{in}$
	$S_{j,t}$	$K_{i,t}^{out}$	$H_{i,t}^{out}$	$\log \sigma_{i,t}(r)$	$\log \sigma_{i,t}(s)$	$S_{j,t}$	$K_{i,t}^{in}$	$H_{i,t}^{in}$	$\log \sigma_{i,t}(r)$	$\log \sigma_{i,t}(s)$
Panel A: Raw data										
Avg.	0.20	0.03	-0.31	0.15	0.27	0.78	0.37	-0.25	0.12	0.08
$t - stat$	3.61	0.86	-8.69	2.79	3.53	23.44	21.63	-4.34	0.93	0.80
Panel B: Truncated data										
Avg.	0.19	0.01	-0.31	0.15	0.25	0.89	0.37	-0.26	0.13	0.08
$t - stat$	2.03	0.21	-7.08	3.49	5.53	35.92	22.79	-4.00	0.98	0.85

Notes: The table reports annual cross section correlations between various features of firm’s customer-supplier network (based on Compustat data) with firm’s size and volatility. Annual data for 1980-2009. The table reports the time-series average as well as the t-statistic, measured as the time-series average divided by the time-series standard deviation estimated over the annual 1980-2009 sample. In Panel A, we use all linkage data. In panel B, we impose the 10% truncations, which implies that we delete all customer-supplier pairs that represent less than 10% of supplier sales.

5.2 Justifying Modeling Assumptions

The first use the customer-supplier linkage data is in providing direct evidence on the main modeling assumptions we made in Section 2. Table 3 reports the relevant correlation statistics. We calculate cross-sectional correlations for each yearly network realization in the Compustat linkage data and report the time series average of annual correlations. We also report a t-statistic, measured as the ratio of the time series mean of the correlation to its time-series standard deviation. In these calculations, size is defined as annual firm sales. Columns (1)-(5) take the perspective of the suppliers and the connections with their customers while Columns (6)-(10) focus on costumers’ connections to their suppliers. In Panel A we consider all customer-supplier linkages while in Panel B we only keep those pairs where the customer represents at least 10% of the supplier’s sales. Thus, Panel B imposes the truncation also on the firms that voluntarily report more customer data than required.

The model assumes that suppliers have stronger connections with their larger customers, and Column (1) shows that this is indeed a strong feature of the data. Specifically, for each supplier i , we calculate the correlation between its customers’ total sales and the fraction

of i 's revenue that is due to each customer. We then average across all suppliers in a given year. In an average year, that correlation is 20% and supports our assumption that linkage weight w_{ij} is closely related to the size of the customer S_j .

We also assume that larger suppliers are connected to more customers on average (higher K^{out}). The Compustat data cannot speak directly to this assumption due to the truncation of all linkages whose weights fall below 10%. For example, a small firm that has a single customer accounting for 100% of its revenue can show up as having more links than a firm with 11 equally important customers with weights of 9.1% due to truncation. Column (2) shows that any correlation that might exist between size and number of customers is largely destroyed by truncation, with an average annual correlation of 3%. Our model simulations below will confirm the large bias caused by truncation.

Column (3) reports the correlation between a supplier's log size and the Herfindahl of its customer network. As in the model, a supplier i 's out-Herfindahl $H_{i,t}^{out} = H_{i,t}$ is calculated based on the supplier's weighted out-degrees with its customers. Our model predicts that larger suppliers are more diversified and thus have lower out-Herfindahls. The average of -31% is consistent with this prediction. Column (4) compares suppliers' out-Herfindahls to their log return volatilities calculated from stock return data. The model suggests that well-diversified suppliers (those with lower out-Herfindahls) should have lower volatility. This relationship is confirmed in the linkage data with a 15% average correlation between Herfindahl and log firm variance. Column (5) repeats this calculation using quarterly sales growth volatility in place of return volatility. It uncovers a similar association with average correlations of 27%. All correlations in Columns (1)-(3) are statistically significant at the 1% level.

It is instructive to also study the network from the customer perspective. Columns (6)-(10) report the corresponding correlations with the customer rather than the supplier as the unit of observation. Column (7) reports a 37% correlation between customer log size and the number of connections that customer has (K^{in}). In our model, the probability that a connection exists between supplier i and customer j depends only on i 's size and is independent of firm j 's size. Thus, the model predicts a zero correlation between size and in-degree in the *absence* of link truncation. But because links with weights below 10%

are not observed, and because larger customers are associated with higher linkage weights, then we would expect to see a strong association between customer size and in-degree in the truncated data. For instance, firms like Apple or Walmart dominate their suppliers' sales due to their massive sizes. While we assume smaller firms are equally likely to be customers as large firms, truncation will eliminate the links with small customers, leaving large firms counted as customers far more often. Column (8) shows that there is a significant -25% association between a customer's log size and its in-degree Herfindahl. That is, large customers have a less concentrated network of suppliers. However, Columns (9) and (10) show that the correlation between volatility and in-Herfindahl is not statistically different from zero.

A comparison of results in Columns (1)-(5), which summarize features of suppliers networks, to results on customer networks in Columns (6)-(9) suggests an interesting interpretation of shock propagation through customer-supplier networks. First, the strength of links depend strongly on customer size as in our model. Second, the association between a supplier's size and its number of customers in the data is also consistent with our model assumption once the data truncation is taken into account. Third, customer network concentration associates strongly with suppliers' volatility, while this is not the case for customer volatility and supplier network concentration. This is consistent with the notion that shocks are propagated upstream, from customers to suppliers, and that the effects of these shocks on firms' volatility depends on how well-diversified a firm is in its customer base. We provide additional evidence in the next subsection.

Appendix B.7 studies the association between size, out-degree, out-Herfindahl, in-degree, and in-Herfindahl at the industry rather than at the firm level. It uses annual data from the BEA on 65 industries between 1998 and 2011. This data has the advantage that it does not suffer from the truncation issue that plagues the Compustat firm-level data. The results are consistent with those at the firm-level, with one exception. Now we find a strong positive correlation of 61% between log size and the number of customers (out-degree), which is strongly significant. This finding lends credence to the positive association between size and number of customers assumption we make in the model. It also confirms the large effect truncation has in reducing this correlation in the Compustat data.

5.3 Determinants of Firm-level Volatility

A large literature has examined the determinants of firm level volatility on the basis of firm characteristics, including Black (1976) who proposed that differences in leverage drive heterogeneity in firm volatility, Comin and Philippon (2006) who argue for industry competition and R&D intensity, Davis et al. (2007) who favor age effects, and Brandt et al. (2010) who argue that institutional ownership is a key driver of volatility. Our model predicts a negative correlation between volatility and firm size and a positive correlation between volatility and customer network concentration (out-Herfindahl).

Table 4 shows results of panel regressions of firm-level log annual return volatility on size and the out-Herfindahl, while controlling for a range of firm characteristics including log age, leverage, industry concentration, institutional holdings, as well as industry and cohort fixed effects. Consistent with our model, we find that the two most important determinants of volatility are size and out-Herfindahl. A 100% increase in the size of the firm decreases volatility by between 12% and 16%. An increase of customer Herfindahl from zero to one increases volatility by 88% without controlling for size; the effect is 17% when we control for size. Note that, in our model, size and network concentration are redundant since size determines network structure. Within our network model, a supplier's size and its customer Herfindahl are strongly negatively correlated in the cross-section. Given that concentration in the sales network is measured with substantial noise, it is likely that size captures an important part of the true network concentration effect.

When we replace the out-Herfindahl with the in-Herfindahl in this multivariate regression, network concentration is no longer a significant determinant of firm-level volatility after we include the control variables. The results of this section further support the upstream transmission of shocks we assume in our network model.

Table 4: DETERMINANTS OF FIRM-LEVEL VOLATILITY

	Dependent Variable: Log Firm Volatility														
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Log Sales	-0.16		-0.14	-0.15		-0.12	-0.14		-0.12	-0.13		-0.12	-0.13		-0.12
	<i>-16.71</i>		<i>-15.57</i>	<i>-25.56</i>		<i>-23.93</i>	<i>-19.92</i>		<i>-12.10</i>	<i>-33.10</i>		<i>-19.93</i>	<i>-20.61</i>		<i>-17.78</i>
H^{out}		0.88	0.17		0.63	0.17		0.53	0.15		0.55	0.13		0.74	0.17
		<i>7.14</i>	<i>3.09</i>		<i>8.74</i>	<i>3.43</i>		<i>7.01</i>	<i>2.65</i>		<i>8.69</i>	<i>3.27</i>		<i>28.54</i>	<i>8.81</i>
Log Age				-0.05	-0.26	-0.11	-0.08	-0.22	-0.11						
				<i>-3.07</i>	<i>-11.32</i>	<i>-6.60</i>	<i>-3.52</i>	<i>-6.58</i>	<i>-6.10</i>						
Leverage				0.29	0.10	0.24	0.30	0.04	0.23	0.32	0.13	0.26			
				<i>3.32</i>	<i>1.08</i>	<i>3.10</i>	<i>4.53</i>	<i>0.48</i>	<i>3.45</i>	<i>4.71</i>	<i>1.87</i>	<i>3.95</i>			
Ind. Conc.				0.32	0.36	0.35	0.44	0.41	0.37	0.32	0.30	0.29			
				<i>1.51</i>	<i>1.32</i>	<i>1.62</i>	<i>2.13</i>	<i>1.35</i>	<i>1.70</i>	<i>2.06</i>	<i>1.45</i>	<i>1.81</i>			
Inst. Hldg.							-0.03	-0.50	-0.06						
							<i>-0.39</i>	<i>-5.09</i>	<i>-0.61</i>						
Constant	-3.91	-3.52	-3.78	-3.92	-3.03	-3.64	-3.80	-2.93	-3.60	-4.01	-3.64	-3.88	-3.86	-3.51	-3.75
	<i>-88.45</i>	<i>-54.43</i>	<i>-76.93</i>	<i>-49.72</i>	<i>-39.79</i>	<i>-42.73</i>	<i>-32.42</i>	<i>-30.61</i>	<i>-30.98</i>	<i>-72.31</i>	<i>-50.66</i>	<i>-59.97</i>	<i>-111.89</i>	<i>-94.74</i>	<i>-82.52</i>
FE	None	None	None	None	None	None	None	None	None	Cohort	Cohort	Cohort	Ind.	Ind.	Ind.
Obs.	171,034	38,030	37,202	145,070	32,887	32,887	113,425	31,318	31,318	145,247	32,901	32,901	171,034	38,030	37,202
Adj. R^2	0.329	0.048	0.361	0.347	0.228	0.402	0.354	0.281	0.413	0.405	0.307	0.448	0.386	0.218	0.425

Notes: The table reports panel regressions of firm's return volatility on a range of characteristics including size (log sales), customer network Herfindahl, age, book leverage, competition measured by industry size concentration, ownership composition measures as fraction of shares held by 13f-reporting institutions, as well as cohort and industry fixed effects in certain specifications.

6 Joint Dynamics of the Size and Volatility Distribution

To develop a quantitative understanding of the interaction between the size and volatility distribution, we introduce entry/exit and network persistence into the granular network model of Section 2. The calibrated model aims to match dynamics of the aggregate firm size and firm volatility distributions, as well as features of the customer-supplier network found in Compustat data.

6.1 Feedback from Volatility to Size Distribution

In our multi-period network model there is feedback from volatility to the firm size distribution. To understand this feedback effect, consider a dynamic model with no exit/entry. The persistence parameter γ governs the strength of the network effect. When γ is zero, there are no network effects, and the model satisfies Gibrat's law: The mean growth rate and the variance of the growth rate are constant across firms, regardless of size, because the growth innovations are i.i.d. over time. In the absence of exit/entry, the size distribution diverges because the standard deviation of the log size distribution $\sigma_{S,t} = \sigma_\epsilon \sqrt{t}$ grows at rate \sqrt{t} . This is true even if the size drift, μ_g , is zero (see de Wit (2005)).

The network's feedback effects contribute a term to the variance of the log size distribution that grows exponentially in t . Hence, $\sigma_{S,t}$ grows at a faster rate than \sqrt{t} , especially when γ is large.

Proposition 8. *If the higher-order cumulants are positive, then the variance of the log size distribution at t is bounded below by:*

$$E[\sigma_{S,0}^2] + t\sigma_\epsilon^2 + \sigma_\epsilon^2 \gamma^2 Z \sum_{\tau=1}^t \exp [2 (\tau\sigma_\epsilon^2 + \sigma_{S,0}^2)], t \geq 0$$

The inequality follows directly from the recursive expression for the variance of the log size distribution in Proposition 8. The network effects add an exponential growth term to

the cross-sectional variance of size. This term increases in γ . To guarantee the existence of a stationary firm size distribution, we introduce firm entry and exit.

6.2 Network Dynamics

The model begins at time 0 with an initial $N \times 1$ firm size distribution \mathbf{S}_0 , where each $S_{i,0}$ is drawn from a log normal distribution. The linkage structure at time 0 is determined by the initial firm size distribution. Each element of the connections matrix \mathbf{B}_0 is drawn from a Bernoulli distribution with $P(b_{i,j,0} = 1) = p_{i,0}$ as in (15). To induce persistence in links, we modify equation (15) for $t > 1$, giving suppliers a relatively high probability of reconnecting to its customers from the previous period. The probability that a firm i that was connected to firm j in period $t - 1$ is again connected to that same firm j at time t is given by $\min\{p_{i,j,t} + \kappa, 1\}$, $t > 0$, where $\kappa > 0$. In each period, a fraction δ of firms die randomly. These rules for the connection dynamics and firms' birth and death completely specify the network evolution and the growth rate process in (2).

6.3 Calibration

We simulate our model for $N = 2000$ firms and 1300 periods (years). We discard the first 300 observations to let the network settle down to its long run distribution, and compute model statistics by averaging over the last $T = 1000$ years. In each period, we report moments based on the sample of the largest 1,000 firms. Focusing on the largest firms is the model's way of recognizing that there are firms in the real world that are not measured in our Compustat sample (e.g., private firms), and that the firms that are measured are connected to these unmeasured firms affecting the former firms' network, size growth, and volatility. Our choice of $N = 2000$ is dictated by computational considerations. To further improve comparability between model and data, we compare the model results for a constant sample of 1000 firms to an appropriately corresponding Compustat subsample. We focus on the top-33%, a sample which contains 1,000 firms *on average* in the data. For completeness, we also report the empirical moments for the entire distribution of publicly traded firms (about 3,000 firms on average).

Because network data are truncated, as explained above, we implement the same truncation inside the model. That is, we set the weight $w_{ij,t}$ equal to zero whenever $w_{ij,t} < 10\%$. By calibrating a censored simulation against censored data, we are able to draw inferences about network relationships among firms in the full, uncensored model economy.⁹

In order to compare variance moments in model and data, we want to take into account that empirical variances are estimated with noise. Therefore, we report “estimated” variance moments in the model, which we compute as:

$$\log(\widehat{Var}_t[g_{t+1}]) = \log(Var_t[g_{t+1}]) + e_{t+1}$$

where $e \sim \mathcal{N}(0, \sigma_e^2)$ and σ_e is the time series average of the cross-sectional standard deviation of $\log(Var_t[g_{t+1}])$.

We choose the following parameters for our benchmark model. We set the mean exogenous firm growth rate μ_g equal to zero, which is the growth rate we observe for real market capitalization in the full cross-section.¹⁰ The initial firm size distribution is log normal with mean $\mu_{S_0} = 10.20$ and standard deviation $\sigma_{S_0} = 1.06$. These numbers are chosen to match the time-series average of the cross-sectional mean and variance of the top-33% firm-size distribution. The exogenous firm destruction rate δ is set to 5%, close to the time-series average firm exit rate in our sample of 4.2%. The probability of forming a supplier-customer connection features parameter Z , which governs the baseline likelihood of a connection for a firm i . We set $Z = 0.35$ which implies an average of 10 customer connections (out-degrees) and an average of 2 truncated out-degrees. The parameter κ gives the additional probability of a connection when firm i and j were already connected in the previous period. This additive term is not there for new firms (with no pre-existing connections). Thus, κ governs the persistence of connections. We set κ equal to 0.5 to match the observed 54% time-series average death rate of truncated links. In our benchmark model the average death rate of

⁹The second step we take to address censoring issues is to show that the size and volatility dynamics uncovered among public firms continues to hold when we pool public firm data with coarser data on private firms available from the Census Bureau; see Appendix B.4.

¹⁰The observed growth annual rate is 8% per year in the top of the firm size distribution, but this number is upwardly biased due to selection. Firms that leave the top of the size distribution are excluded from the growth calculations while firms that enter the top group are newly included.

truncated links is 57% whereas the average death rate of untruncated links is 46%. The two key parameters are γ , which governs the persistence of the network, and σ_ε , which is the fundamental shock volatility. The parameter γ governs how important shocks to customers' growth are to a supplier. The closer γ is to one, the more important are higher-degree (indirect) effects, i.e., effects beyond the direct supplier-customer relationship. These two parameters are set to match as best as possible the mean and dispersion of firm volatility.

6.4 Calibration Targets

This section documents the features of the size, volatility and in/out-degree distribution of U.S. firms we would like our model to explain.

6.4.1 Size Distribution Target Moments

Table 5 reports moments of the cross-sectional log size distribution. All reported moments are time-series averages unless explicitly mentioned otherwise. The first column reports moments for the full cross-section of firms observed in Compustat. The second column reports results for the top-33% largest Compustat firms in each year. Column 2 is the main column of interest in the data. For example, 11.63 is the time-series average of the average log size of the large-firm sample, which is naturally higher than the 9.61 for the full Compustat sample. The dispersion of the log size distribution is 1.06 for the top-33% group and 1.79 for the full sample. Panel B reports *aggregate* moments of the size distribution. The first two rows show that size dispersion (the cross-sectional standard deviation of log size) has high variability over time and is highly persistent. The time series standard deviation is 14% for the top-33% firms and 24% for all firms. The third row shows that the cross-sectional standard deviation of size *growth* (log size changes) is also volatile over time. The time series standard deviation is 15% for all firms and 11% for the top-33% firms. These results confirm that the size distribution moves around considerably over time. Finally, the economy-wide Herfindahl index is 0.012 on average over time, and similar across both samples of firms.

Table 5: FIRM SIZE DISTRIBUTION TARGET MOMENTS

	All	Top-33%	Model
Panel A: Cross-sectional Moments of Log Size			
Avg	9.61	11.63	11.63
SD	1.79	1.06	1.07
5%	6.86	10.38	10.32
10%	7.39	10.47	10.44
25%	8.33	10.78	10.79
Med	9.48	11.39	11.42
75%	10.77	12.25	12.25
90%	12.05	13.10	13.10
95%	12.76	13.64	13.68
Panel B: Time Series Properties of Size Distribution			
SD of $\sigma_{S,t}$	0.24	0.14	0.19
AR(1) $\sigma_{S,t}$	0.947	0.939	0.995
SD of $\sigma_{g,t}$	0.15	0.11	0.05
H_t	0.010	0.012	0.020

Notes: All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first column reports the full cross-section of firms. The second column reports results for the top-33% of firms in each year. Column 3 reports the corresponding moments for the model. Panel A reports moments of the log size distribution, where size is defined in the data as market equity. Panel B reports the time-series standard deviation and time-series persistence of size dispersion $\sigma_{S,t}$, defined as the cross-sectional standard deviation of log size, the time-series standard deviation of the cross section standard deviation of log growth rates $\sigma_{g,t}$ and the time-series average of the economy-wide Herfindahl index H_t .

6.4.2 Volatility Distribution

Table 6 reports moments of the cross-sectional log variance distribution. In columns (1) and (2), we report market-based log variance. In columns (3) and (4), we compute sales-based log variance. Because volatilities (standard deviations in levels) are more intuitive than log variances, we exponentiate the moments of log variance and then take their square root, which is what is reported in Table 6. All reported moments are time-series averages unless explicitly mentioned otherwise.

Panel A shows that average market-based volatility is 30% per year for the large-firm sample and 40% per year for the full cross-section. Average sales-based volatility is 24% for the large-firm sample and 29% for the full sample. The range of return-based (sales-based) volatilities goes from 17% (8%) at the fifth percentile to 54% (77%) at the 95th percentile for the latter group. The cross-sectional dispersion in return volatility is 73% in Column (2) and 97% in the full cross-section reported in Column (1). Sales-based volatility has even larger dispersion of 142% and 151%, in Columns (4) and (3) respectively, possibly because sales-based volatility is measured with more noise.

Panel B shows moments of the joint cross-sectional distribution of firm size and firm volatility. The first row computes the cross-sectional correlation between log size at time t and log variance at time $t + 1$, for each t and then reports the time-series average. The second row reports the slope coefficient (beta) of a cross-sectional regression of log variance at time $t + 1$ on a constant and log size at time t ; it reports the time-series average of that slope. Both are strongly negative in the data showing that large firms have lower volatility over the next period.

Panel C reports aggregate moments of the the volatility distribution and the joint size-volatility distribution. The first row shows the time-series standard deviation of average volatility: 64% in the top-33% sample and 69% in the full sample. Average volatility is highly variable over time. The second row shows that the cross-sectional standard deviation of log variance moves substantially over time in both samples and for both ways of measuring volatility. The time series standard deviation is 18% in the full sample and 12% in the top-33% sample; both are lower than the time-series volatility of mean firm volatility. The

third row reports two key moments in our paper (see Figures 1 and 2): the time-series correlation between size dispersion at t (the cross-sectional standard deviation of log size) and mean volatility (the cross-sectional mean of log variance) at time $t + 1$. The two are strongly positively correlated in the data: 0.71 in the full sample and 0.55 in the top sample. Similarly, size dispersion is strongly positively correlated with volatility dispersion (the cross-sectional standard deviation of log variance) over time: 0.76 in both samples. The last row reports the volatility of aggregate growth which is 20%.

6.4.3 Network Moments

Table 7 reports the median and 99th percentile of the cross-sectional distribution of the number of customers that a supplier is connected to, K^{out} , and the number of suppliers a customer is connected to, K^{in} . We also report the median and 99th percentile of the cross-sectional distribution of the out- and in-degree Herfindahl indices H^{out} and H^{in} . All calculations impose the 10% truncation uniformly. We find a median (truncated) out-degree of 1 and (truncated) 99th percentile of 3.3 connections. The median (truncated) in-degree is 1 with a 99th percentile of 17 or 32 connections depending on the sample. The Herfindahl indices, which are also based on truncated degree information, are less biased because large customers receive a large weight and are more likely to be in the database.

We repeat the cross-sectional correlations of in- and out-degrees and in- and out-Herfindahls at time t with log size at t and log variance at $t + 1$, previously reported in Panel B of Table 3. As discussed above, we find a nearly zero correlation between truncated out-degree K^{out} and log size, which we argue is due to truncation. We find a strong negative -31% correlation between supplier log size and out-Herfindahl. Large firms have a better-diversified portfolio of customers, and hence small out-Herfindahls. We also find a positive relationship between supplier log variance and out-Herfindahl: the correlation is 15% based on returns and 29% based on sales. Firms with a more diversified portfolio of customers and smaller Herfindahls have lower volatility because they more effectively diversify the shocks that hit their customers. This also helps explain the negative correlation between firm size and firm volatility in Panel B of Table 6. In Panel B, we find a positive 37% correlation between in-degree and size, but we argue based on the model that truncation induces an upward bias in this

Table 6: FIRM VOLATILITY DISTRIBUTION TARGET MOMENTS

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	Model
Panel A: Cross-Sectional Moments of Firm Volatility					
Avg	0.40	0.30	0.29	0.24	0.37
SD	0.96	0.73	1.51	1.42	0.45
5%	0.19	0.17	0.09	0.08	0.26
10%	0.22	0.19	0.11	0.10	0.28
25%	0.29	0.24	0.17	0.14	0.31
Med	0.40	0.30	0.29	0.23	0.36
75%	0.56	0.38	0.48	0.38	0.42
90%	0.75	0.48	0.77	0.60	0.49
95%	0.90	0.54	1.05	0.77	0.54
Panel B: Cross-Sectional Moments of Size-Volatility Distribution					
$Corr(S_t, V_{t+1})$	-0.57	-0.33	-0.33	-0.18	-0.42
$\beta(S_t, V_{t+1})$	-0.32	-0.23	-0.27	-0.22	-0.18
Panel C: Time Series Properties of Volatility Distribution					
SD of $\mu_{\sigma^2,t}$	0.69	0.64	0.46	0.42	0.20
SD of $\sigma_{\sigma^2,t}$	0.18	0.12	0.13	0.13	0.05
$Corr(\sigma_{S,t}, \mu_{\sigma^2,t})$	0.71	0.55	0.51	0.51	0.95
$Corr(\sigma_{S,t}, \sigma_{\sigma^2,t})$	0.76	0.76	0.57	0.38	0.77
SD of $g_{agg,t}$	0.21	0.20	0.21	0.20	0.20

*Notes:*All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first and third columns report return volatility and sales growth volatility, respectively, for the full cross-section of firms. The second and fourth columns report results for the largest 33% of firms by market capitalization in each year. Annual variances in Columns (1) and (2) are calculated from daily stock return data, while variances in columns (3) and (4) are based on 20 quarters of sales growth rates (in the current and the next four years). Moments are computed based on the log variance distribution, but are expressed as volatilities in levels for exposition. That is, we exponentiate each moment of the log distribution and take the square root. Column (5) reports the corresponding moments for the model. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

Table 7: NETWORK TARGET MOMENTS

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	Model
Panel A: Out-degree Moments					
median K^{out}	1.00	1.00	–	–	1.71
99 th % K^{out}	3.38	3.32	–	–	4.66
median H^{out}	0.05	0.03	–	–	0.60
99 th % H^{out}	0.95	0.54	–	–	1.00
$Corr(K_t^{out}, S_t)$	0.01	–0.07	–	–	0.34
$Corr(H_t^{out}, S_t)$	–0.31	–0.08	–	–	–0.71
$Corr(H_t^{out}, V_{t+1})$	0.15	0.08	0.29	0.10	0.54
Panel B: In-degree Moments					
median K^{in}	1.00	1.12	–	–	1.90
99 th % K^{in}	31.61	16.80	–	–	7.39
median H^{in}	0.00	0.00	–	–	0.11
99 th % H^{in}	0.28	0.24	–	–	0.85
$Corr(K_t^{in}, S_t)$	0.37	0.49	–	–	0.49
$Corr(H_t^{in}, S_t)$	–0.26	–0.20	–	–	0.03
$Corr(H_t^{in}, V_{t+1})$	0.13	0.08	0.08	0.08	0.01

Notes: All reported moments are time-series averages unless explicitly mentioned otherwise. The first and second columns report data for the cross-section of firms for which we have customer information from Compustat. The sample is 1980-2009. The variance in Column (1)-(2) are calculated based on daily stock return data, while the variance in Column (3)-(4) are based on 20 quarters worth of annual sales growth (in the current and the next four years). Column (5) reports the corresponding moments for the model. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

correlation. This provides evidence that the likelihood of a connection between a supplier and a customer does not strongly depend on the customer’s size. We find a modestly positive (and statistically insignificant) correlation between in-Herfindahl and volatility.

6.5 Simulation Results

This section reports the simulation results. We focus on the benchmark calibration and leave a detailed discussion of the alternative calibrations for Appendix C.1.

6.5.1 Benchmark Calibration

As can be seen from column (3) in Table 5, the benchmark model matches the main moments of the firm size distribution. It matches the average and the dispersion in firm size exactly, by virtue of the calibration. The other moments of the size distribution constitute over-identifying restrictions. The inter-quartile range for log firm size is [10.79,12.25], matching the [10.78,12.25] range in the top-33% sample closely. At both extremes of the distribution, the model also compares favorably to the data. Panel B shows that size dispersion moves around dramatically over time in the model and is very persistent. The model produces the same high time-series variability in the size dispersion as observed (0.19 compared to 0.14 in column 2 and 0.24 in column 1), but not that of the dispersion in size growth rates (0.05 versus 0.11 in column 2 and 0.15 in column 1). The benchmark model overstates firm concentration somewhat: the economy-wide Herfindahl index is 0.020 compared to 0.012 in the data.

Next, we turn to the volatility distribution. Column (5) of Table 6 shows that the model generates high average firm volatility of 37%, in between the return volatility in the full sample of 40% and that in the top-33% sample of 30%. The model is capable of generating a wide range of volatility outcomes. The cross-sectional dispersion in volatility is 0.45, compared to 0.73 for the return-based measure (column 2), and 1.42 for the sales-based measure (column 4). The model generates the right volatility at the 90 and 95th percentiles. The least volatile firms have a volatility of 26% in the model, but only 17% in the return data. Panel B shows that the model generates a -42% correlation between size and volatility at the firm level, similar to the -33% in the data. The slope of the relationship between log variance and log size is similar to the one in the data (slope coefficient of -0.18 versus -0.23). Panel C shows that the model generates substantial variability over time in both the cross-sectional mean of firm variance and in its dispersion, but still under-predicts these moments compared to the data. The model generates a high volatility of aggregate growth rates of 20%, matching the data. In the time-series, size dispersion is strongly positively correlated with both the mean of the log firm variance distribution (0.95 correlation) and its dispersion (0.77 correlation). These correlations are high in the data as well (0.55 and 0.76).

These correlations are some of the key moments we focus on in this paper.

Column (5) of Table 7 shows the network-related properties of the model. Like the data, the model features a small median number of supplier-customer relationships after truncation: 1.71 on average in the model. The 99th percentile of the truncated out-degree distribution is 4.66 in the model and 3.32 in the data. Truncation severely affects the number of out-degrees. The median of the *untruncated* K^{out} distribution in the model is 3.71 and the 99th percentile is 112. In the model, larger firms have more connections.

The cross-sectional correlation between the truncated out-degree and log size is 0.34 in the model, which is 0.26 lower than the 0.60 correlation between the *untruncated* out-degree and log size in the model. This large downward bias means that the observed correlation between truncated out-degree and log size in the data (0.01 for the full sample and -0.07 for the top-33% sample) is severely downward biased. Applying the model-generated bias, the data would show a positive correlation between *untruncated* out-degree and log size (albeit a smaller one as in the model). That is, larger suppliers have more connections. A reverse bias occurs for the correlation between in-degree and log size. The observed value of 0.49 for the top-33% exactly matches the 0.49 value in the benchmark model, where both numbers are based on the *truncated* in-degree distribution. However, the correlation between *untruncated* in-degree and log size is 0.04 in the model, which implies that this correlation is severely upward biased due to truncation. The magnitude of the bias is such that the (unobserved) correlation between untruncated in-degree and log size could easily be zero. That zero correlation between customer size and number of connections is an important assumption of the model.

The model matches some properties of Herfindahls, but not others. The model generates a strong negative correlation between out-degree Herfindahl and size (-71%) and a positive correlation between out-degree Herfindahl and volatility (54%). Both are key features of the data, as we discussed above. The model also matches the weak correlation pattern between in-Herfindahls and volatility. In the model, these correlations are similar whether we use truncated or untruncated Herfindahls, suggesting no bias in their empirical counterparts. The main shortcoming of the benchmark calibration is that its truncated out-degree Herfindahl is too high. The median is 0.60, higher than the 0.05 value we observe in the data.

The reason is that the number of connections a typical supplier has is small (the median out-degree is 3.7), and that large customers receive a large weight (w_{ij} is strongly increasing in size of the supplier S_j). To match the dispersion in firm volatilities in a setting with only uncorrelated shocks, the model requires too much concentration in customer networks.

Finally, we re-estimate the volatility factor regressions of Table 2, but now based on simulation-generated data from the model. The corresponding R^2 statistics for panel regressions of total volatility are 36%, 37%, and 40% for the three factors considered in that table (from left to right). These numbers quite close to their empirical counterparts of 24%, 26%, and 39%. Thus, the model quantitatively replicates the factor structure in volatilities.

6.5.2 Alternative Calibrations

We compute several other models that help illustrate the various ingredients of the model. These are discussed at length in Appendix C.1. These alternative model features show that strong network persistence is important (high γ , and a model with higher-order network effects). They also show that we can match the low observed out-Herfindahls by changing the parameters that govern the number and the importance of connections, without jeopardizing the main predictions of the model. That calibration, however, cannot quite generate enough dispersion in firm volatility.

6.5.3 Downstream Transmission

Finally, we study a version of our model with downstream transmission of shocks instead of upstream transmission in Appendix C.2. That model delivers the same size and volatility moments, except for the network moments, where in-degree and out-degree moments are reversed. Truncation affects the network moments; see appendix for a detailed discussion. This downstream version of our model matches the correlations between in-Herfindahl and size and volatility better, but completely misses the correlation between out-Herfindahl and size and volatility. In the data, the correlation of volatility with out-Herfindahl was statistically much stronger than that with in-Herfindahl. We conclude that the data are most supportive of an upward shock transmission.

7 Conclusion

We document new features of the joint evolution of the firm size and firm volatility distribution and propose a new model to account for these features. In the model, shocks are transmitted upstream from customers to suppliers. Firms sell products to an imperfectly diversified portfolio of customers. The larger the supplier, the more customer connections the supplier has, the better diversified it is and the lower its volatility. Large customers have a relatively strong influence on their suppliers, so shocks to large firms have an important effect throughout the economy. When the size dispersion of the economy increases, such large firms become more important and many firms' network of customers becomes less diversified. In those times, average firm volatility is higher as is the cross-sectional dispersion of volatility. We provide direct evidence of such linkages, and use the supplier-customer relationship data to calibrate our model, and we show that the calibrated model replicates the most salient features of the firm size and the volatility distribution.

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A Theoretical Appendix

A.1 Proofs of Propositions

- Proof of Proposition 1:

Proof. We will derive a lower bound on the average variance. We use the following notation to denote the matrix inverse:

$$\mathbf{v} = (\mathbf{I} - \gamma \mathbf{W}_t)^{-1} = \mathbf{I} + \gamma \mathbf{W}_t + \gamma^2 \mathbf{W}_t^2 + \gamma^3 \mathbf{W}_t^3 + \dots$$

The average variance is given by $\frac{1}{N} \sigma_\epsilon^2 \sum_i \mathbf{e}'_i \mathbf{v} \mathbf{v}' \mathbf{e}'_i$. First, note that:

$$\mathbf{e}'_i \mathbf{v} \geq \mathbf{e}'_i (\mathbf{I} + \gamma \mathbf{W}_t),$$

because the \mathbf{W} matrix only has positive entries. Hence, in turn, it follows that:

$$\mathbf{e}'_i \mathbf{v} \mathbf{v}' \mathbf{e}'_i \geq \mathbf{e}'_i (\mathbf{I} + \gamma \mathbf{W}_t) (\mathbf{I} + \gamma \mathbf{W}'_t) \mathbf{e}'_i$$

We have derived the following lower bound on the average variance:

$$\frac{1}{N} \sum_i V_t(g_{i,t+1}) \geq \sigma_\epsilon^2 \left(1 + \frac{2}{N} \gamma \sum_{i=1}^N w_{ii} + \frac{1}{N} \gamma^2 \sum_{i=1}^N \sum_{j=1}^N w_{ij,t}^2 \right)$$

We can restate this expression using the Herfindahl:

$$\frac{1}{N} \sum_i V_t(g_{i,t+1}) \geq \sigma_\epsilon^2 \left(1 + \gamma^2 \frac{1}{N} \sum_i H_{i,t} \right)$$

□

- Proof of Proposition 2, upper bound

Proof. In a network where each firm i only has one connection to its neighbor to the right, firm growth is given by:

$$g_i = \mu_g + \gamma g_{i+1} + \epsilon_i, \tag{21}$$

where firm $N+1$ is firm 1 by convention. Without loss of generality, this \mathbf{W}_t matrix can be represented with ones above the main diagonal and zeros elsewhere, except in the $(N, 1)$ position. By backward substitution, we obtain the following expression for the growth rate of firm 1:

$$g_1 = \frac{1}{1 - \gamma^N} \left[\mu \sum_{i=1}^{N-1} \gamma^i + \sum_{i=1}^{N-1} \gamma^i \epsilon_{i+1} \right]$$

Hence, the variance of a typical firm is given by

$$V(g) = \frac{\sigma_\epsilon^2 (1 - \gamma^{2N})}{(1 - \gamma^2)(1 - \gamma^N)^2}$$

□

- Proof of Proposition 2, lowerbound

Proof. Next, we consider a network structure in which each firm is connected to all other firms and all links have the same weight on the other firms' growth rate. That weight is given by $\frac{1}{N-1}$. The

weighting matrix \mathbf{W} has zeros on the diagonal and $\frac{1}{N-1}$ everywhere else. The growth equation is given by $(\mathbf{I} - \gamma\mathbf{W})^{-1}(\boldsymbol{\mu} + \boldsymbol{\varepsilon})$. $(\mathbf{I} - \gamma\mathbf{W})$ has ones on the main diagonal and $\frac{-\gamma}{N-1}$ everywhere else. $(\mathbf{I} - \gamma\mathbf{W})^{-1}$ has $\frac{(N-2)\gamma - (N-1)}{\gamma^2 + (N-2)\gamma - (N-1)}$ on the diagonal and $\frac{-\gamma}{\gamma^2 + (N-2)\gamma - (N-1)}$ on the off-diagonal. We need to compute the diagonal elements of the variance-covariance matrix. This yields the following expression:

$$\begin{aligned}
V(g_i) &= \sigma_\varepsilon^2 \text{diag} \left([\mathbf{I} - \gamma\mathbf{W}]^{-1} [\mathbf{I} - \gamma\mathbf{W}]^{-1} \right) \\
&= \sigma_\varepsilon^2 \left[\frac{((N-2)\gamma - (N-1))^2 + (N-1)\gamma^2}{(\gamma^2 + (N-2)\gamma - (N-1))^2} \right] \\
&= \sigma_\varepsilon^2 \left[\frac{\left(\frac{N-2}{N-1}\gamma - 1 \right)^2 + \frac{1}{N-1}\gamma^2}{\left(\frac{1}{N-1}\gamma^2 + \frac{N-2}{N-1}\gamma - 1 \right)^2} \right]
\end{aligned} \tag{22}$$

□

- Proof of Proposition 3:

Proof. Note $E[b_{i,j,t}] = p_{i,t}$, and $b_{i,j,t}$ is independent of customer j 's size. By the law of large numbers, this implies that

$$\frac{1}{N} \sum_i b_{i,j,t} S_{j,t}^x \rightarrow E[b_{i,j,t} S_{j,t}^x] = p_{i,t} E[S_{j,t}^x], \quad x = 1, 2.$$

Therefore, for sufficiently large N , a supplier's customer concentration Herfindahl is approximately

$$H_{i,t} = \frac{\frac{1}{N} \sum_j b_{i,j,t} S_{j,t}^2}{N \left(\frac{1}{N} \sum_j b_{i,j,t} S_{j,t} \right)^2} \approx \frac{E[S_{j,t}^2]}{p_{i,t} N E[S_{j,t}]^2}.$$

This bears close resemblance to the Herfindahl of the economy-wide size distribution, H_t . By the same law of large numbers rationale,

$$H_t = \frac{1}{N} \sum_i \frac{S_{i,t-1}^2}{(\sum_i S_{i,t-1})^2} \approx \frac{E[S_{j,t-1}^2]}{N E[S_{j,t-1}]^2}.$$

Together, these results imply the following common factor structure for growth rate volatilities of firms in this network economy:

$$V_i(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \frac{1}{p_{i,t}} \gamma^2 H_t \right). \tag{23}$$

□

- Proof of Corollary 1

Proof. If the firm size distribution is log normal at t , then

$$E[S_{i,t}^x] = \exp \left(x\mu_{s,t} + \frac{x^2}{2} \sigma_{s,t}^2 \right)$$

where $\mu_{s,t} \equiv E[\log S_{i,t}]$ and $\sigma_{s,t}^2 \equiv V(\log S_{i,t})$. As a result, the firm size Herfindahl is

$$H_t \approx \frac{E[S_{j,t}^2]}{N E[S_{j,t}]^2} = \frac{1}{N} \exp(\sigma_{s,t}^2).$$

Hence, in the first-order approximation, the common variance dynamics for all firms are determined by cross section standard deviation of the size distribution, $\sigma_{s,t}^2$:

$$V(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \gamma^2 \frac{\exp(\sigma_{s,t}^2)}{N p_{i,t}} \right).$$

□

- Proof of Proposition 4:

Proof. If $p_{i,t} = S_{i,t} / \left(Z \sum_j (S_{j,t}) \right)$, then approximate log normality of $S_{i,t}$ implies $V(g_{i,t+1})$ is also approximately log normal. Furthermore, the parameters of the $V(g_{i,t+1})$ distribution depend only on the dispersion in firm sizes. This is evident from Equation 17, which implies

$$\begin{aligned} \log(V(g_{i,t+1}) - \sigma_\varepsilon^2) &= \log(\sigma_\varepsilon^2 \gamma^2 Z) - \log(S_{i,t}) + \log\left(\sum_j S_{j,t}\right) + \log(H_t) \\ &= \log(\sigma_\varepsilon^2 \gamma^2 Z) - \log(S_{i,t}) + \log(NE[S_{j,t}]) + \log(H_t). \end{aligned}$$

Assuming that $S_{i,t}$ is log normal and noting that $\log\left(\sum_j S_{j,t}\right) \approx \log(NE[S_{j,t}])$, we see that $V(g_{i,t+1})$ is a shifted log normal where

$$E[\log(V(g_{i,t+1}) - \sigma_\varepsilon^2)] \approx \log(\sigma_\varepsilon^2 \gamma^2 Z) + \frac{3}{2} \sigma_{S,t}^2$$

and

$$V(\log(V(g_{i,t+1}) - \sigma_\varepsilon^2)) \approx \sigma_{S,t}^2.$$

□

- Proof of Proposition 5:

Proof. Let weight matrix W be given as in our paper. Consider $(I - \gamma W)^{-1} = I + \gamma W + \gamma^2 W^2 + \dots$. We assume

$$w_{ij} = \frac{B_{ij} S_j}{\sum_j B_{ij} S_j}$$

with $B_{ij} \perp S_j$. I want to derive a large N approximation to the higher order matrix products. Begin with W^2 ,

$$[W^2]_{ij} = \frac{\sum_k B_{ik} S_k \left(\frac{B_{kj} S_j}{\sum_l B_{kl} S_l} \right)}{\sum_k B_{ik} S_k}.$$

Recall

$$\sum_k B_{ik} S_k \approx p_i NE[S]$$

and

$$p_i = \frac{S_i}{Z \sum_i S_i} = \frac{S_i}{Z NE[S]}$$

which implies

$$\begin{aligned}
[W^2]_{ij} &= S_j \sum_k \frac{B_{ik} S_k B_{kj}}{(\sum_l B_{kl} S_l) (\sum_k B_{ik} S_k)} \\
&\approx S_j \sum_k \frac{B_{ik} S_k B_{kj}}{p_i p_k N^2 E[S]^2} \\
&= \frac{S_j}{p_i N E[S]} \sum_k \frac{B_{ik} S_k B_{kj}}{S_k N E[S] / (Z N E[S])}, \quad (\text{using } p_k = \frac{S_k}{Z \sum_i S_i}) \\
&\approx \frac{S_j}{p_i N E[S]} \sum_k Z B_{ik} B_{kj} \\
&\approx \frac{S_j}{p_i N E[S]} Z p_i \sum_k p_k, \quad (\text{using LLN and } E[B_{ik} B_{kj}] = p_i p_k) \\
&\approx \frac{S_j}{p_i N E[S]} Z p_i \sum_k \frac{S_k}{Z N E[S]} \\
&= \frac{S_j}{N E[S]}
\end{aligned}$$

that is, for large N , the rows of W^2 are identical, and are just the (scaled) vector of firm sizes. Denote this matrix as \bar{W} . Next, consider multiplying W^2 by W . We have

$$\begin{aligned}
[WW^2]_{ij} &\approx \sum_k \frac{B_{ik} S_k S_j}{(\sum_l B_{kl} S_l) N E[S]} \\
&= \frac{S_j}{N E[S]}.
\end{aligned}$$

It follows by induction, then, that $W^z = \bar{W}$, $z \geq 2$. This further implies that the economy-wide firm-size Herfindahl is the only quantity we need to describe firm i 's diversification (other than the firm's connection probability p_i). For calculating growth variance, it's also necessary to know the following:

$$[WW']_{ij} = \sum_k \frac{B_{ik} B_{jk} S_k^2}{(\sum_l B_{il} S_l) (\sum_k B_{jk} S_k)} \approx \begin{cases} H & i \neq j \\ H/p_i & i = j \end{cases}$$

$$[W\bar{W}]_{ij} \approx \sum_k \frac{B_{ik} S_k^2}{(\sum_l B_{il} S_l) N E[S]} \approx \frac{E[S^2]}{N E[S]^2} = H$$

$$[W\bar{W}']_{ij} \approx \sum_k \frac{S_k^2}{N^2 E[S]^2} \approx H$$

Large N growth rates can be written

$$\begin{aligned}
g &= (I - \gamma W)^{-1} (\mu + \epsilon) \\
&\approx \left(I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W} \right) (\mu + \epsilon) \\
&\approx \left(I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W} \right) (\mu + \epsilon)
\end{aligned}$$

and growth variance of firm i is the i^{th} diagonal element of

$$\begin{aligned}
V(g) &\approx \sigma^2 \left(I + \gamma W + \frac{\gamma^2}{1-\gamma} \bar{W} \right) \left(I + \gamma W + \frac{\gamma^2}{1-\gamma} \bar{W} \right)' \\
&= I + \gamma W + \frac{\gamma^2}{1-\gamma} \bar{W} + \gamma W' + \gamma^2 W W' + \frac{\gamma^3}{1-\gamma} \bar{W} W' \\
&\quad + \frac{\gamma^2}{1-\gamma} \bar{W}' + \frac{\gamma^3}{1-\gamma} W \bar{W}' + \frac{\gamma^4}{(1-\gamma)^2} \bar{W} \bar{W}' \\
V(g_i) &\approx \sigma^2 \left(1 + \kappa_0 \frac{S_i}{NE[S]} + \kappa_i H \right)
\end{aligned}$$

where $\kappa_0 = \frac{2\gamma^2}{1-\gamma}$ and $\kappa_i = \frac{\gamma^2}{p_i} + \frac{2\gamma^3}{1-\gamma} + \frac{\gamma^4}{(1-\gamma)^2}$. □

- Proof of Proposition 6

Proof. The covariance between an individual growth rate and aggregate growth is approximated by

$$\begin{aligned}
Cov_t(g_{i,t+1}, g_{a,t+1}) &\approx Cov_t \left(\varepsilon_{i,t+1} + \gamma \sum_j w_{i,j,t} \varepsilon_{j,t+1}, \sum_j \varepsilon_{j,t+1} \left[\tilde{S}_{j,t} + \gamma \sum_i \tilde{S}_{i,t} w_{i,j,t} \right] \right) \\
&\approx \sigma_\varepsilon^2 (1 + \gamma) \left(\frac{S_{i,t}}{NE[S_{i,t}]} + \gamma H_t \right)
\end{aligned}$$

Therefore the regression coefficient is approximated by

$$\beta_{i,t} = \frac{Cov_t(g_{i,t+1}, g_{a,t+1})}{V(g_{a,t+1})} \approx \frac{1}{1 + \gamma} \left(\gamma + S_{i,t} \frac{E[S_{i,t}]}{E[S_{i,t}^2]} \right).$$

From here, the close similarity in volatility factor structure for raw growth rate volatility and residual volatility becomes apparent. Residual growth from a factor model, defined as $g_{i,t+1}^{res} = g_{i,t+1} - \beta_i g_{a,t+1}$ has variance

$$\begin{aligned}
V_t(g_{i,t+1}^{res}) &= V_t(g_{i,t+1}) - 2\beta_{i,t} Cov_t(g_{i,t+1}, g_{a,t+1}) + \beta_{i,t}^2 V_t(g_{a,t+1}) \\
&= V_t(g_{i,t+1}) - Cov_t(g_{i,t+1}, g_{a,t+1})^2 / V_t(g_{a,t+1}) \\
&\approx \sigma_\varepsilon^2 \left(1 + \frac{1}{p_{i,t}} \gamma^2 H_t - \left[\frac{S_{i,t}}{NE[S_{i,t}]} + \gamma H_t \right]^2 \right).
\end{aligned}$$

□

- Proof of Proposition 7

Proof. The first order approximation for $g_{a,t+1}$ is

$$g_{a,t+1} \approx \sum_j \varepsilon_{j,t+1} \left(\tilde{S}_{j,t} + \gamma \sum_i \tilde{S}_{i,t} w_{i,j,t} \right).$$

We can rewrite this expression (suppressing t) as

$$\sum_i \tilde{S}_i w_{i,j} = \frac{\sum_i S_i b_{i,j} S_j}{\sum_i S_i \sum_j b_{i,j} S_j} \approx \frac{S_j \sum_i S_i \frac{b_{i,j}}{p_i}}{N^2 E[S_i]^2}.$$

Under weak regularity, $\frac{1}{N} \sum_i S_i \frac{b_{i,j}}{p_i} \xrightarrow{N \rightarrow \infty} E \left[S_i \frac{b_{i,j}}{p_i} \right] = E \left[S_i E \left[\frac{b_{i,j}}{p_i} | S_i \right] \right] = E[S_i]$, where the last equality follows from $b_{i,j}$ being Bernoulli(p_i). As a result, $\sum_k \tilde{S}_k w_{k,j} \approx S_j / (NE[S_j]) = \tilde{S}_j$. Noting that $\sum_k \tilde{S}_{k,t}^2 = H_t$ and $\sum_k \tilde{S}_{k,t} w_{k,j,t} \approx S_{j,t} / (NE[S_{j,t}])$, the aggregate growth rate variance can be approximated as

$$\begin{aligned} V(g_{a,t+1}) &\approx \sigma_\varepsilon^2 \sum_j \left(\tilde{S}_{j,t} + \gamma \sum_i \tilde{S}_{i,t} w_{i,j,t} \right)^2 \\ &\approx \sigma_\varepsilon^2 H_t (1 + \gamma)^2. \end{aligned} \quad (24)$$

□

- Proof of Proposition 8

Proof. If the firm size distribution is not log normal at t , then we can still use the cumulant-generating function to characterize volatility. Note that the expected value of any moment of firm size is given by:

$$E[S_t^x] = \exp \left(\sum_{j=1}^{\infty} \frac{x^j}{j!} \kappa_{s,j,t} \right)$$

where $\mu_{s,j,t} \equiv E[(\log S_t)^j]$ denotes the j^{th} central conditional moment and $\kappa_{1,s,t} = \mu_{1,s,t}$, $\kappa_{2,s,t} = \mu_{2,s,t}$, $\kappa_{3,s,t} = \mu_{3,s,t}$ and $\kappa_{4,s,t} = \mu_{4,s,t} - 3\mu_{2,s,t}^2$. As a result, the economy-wide Herfindahl is given by:

$$H_t \approx \frac{E[S_{j,t}^2]}{NE[S_{j,t}]^2} = \frac{1}{N} \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 2}{j!} \kappa_{s,j,t} \right).$$

Hence, in the first-order approximation, the common variance dynamics for all firms are determined by cross section moments of the size distribution:

$$V(g_{i,t+1}) \approx \sigma_\varepsilon^2 \left(1 + \frac{\gamma^2}{N p_{i,t}} \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 2}{j!} \kappa_{s,j,t} \right) \right).$$

For large N , we can approximate the first-order growth dynamics of firm i as:

$$g_{i,t+1} \approx \mu + \eta_{t+1}, \eta_{t+1} \sim N \left(0, \sigma_\varepsilon^2 \left(1 + \frac{\gamma^2}{\tilde{S}_{i,t} N} Z \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 2}{j!} \kappa_{s,j,t-1} \right) \right) \right)$$

This follows directly from the variance approximation and the specification for p_i . For large N , this can be further simplified to yield:

$$g_{i,t+1} \approx \mu + \eta_{t+1}, \eta_{t+1} \sim N \left(0, \sigma_\varepsilon^2 \left(1 + \frac{\gamma^2}{\tilde{S}_{i,t}} Z \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 1}{j!} \kappa_{s,j,t-1} \right) \right) \right)$$

The log size of firm i at time t is given by $\log S_{i,t} = \log S_{i,0} + \sum_{\tau=1}^t g_{i,\tau}$. We use $\sigma_{s_{i,t}}^2$ to denote the variance of $\log S_{i,t}$ conditional on $S_{i,0}$. Hence, the variance of the log size distribution at t in the first-order version of this economy is given by:

$$\sigma_{s_{i,t}}^2 \approx \sigma_{s_{i,t-1}}^2 + \sigma_\varepsilon^2 \left(1 + \frac{\gamma^2}{\tilde{S}_{i,t-1} N} Z \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 2}{j!} \kappa_{s,j,t-1} \right) \right), t \geq 1,$$

which is the variance of the cumulative growth rate of firm i . This implies that the average variance of the size of firms in the first-order version of this economy is given by:

$$E[\sigma_{s_i,t}^2] \approx E[\sigma_{s_i,t-1}^2] + \sigma_\epsilon^2 \left(1 + \gamma^2 Z \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 1 + (-1)^j}{j!} \kappa_{s,j,\tau-1} \right) \right), t \geq 1$$

Finally, note that:

$$\text{Var}(\log S_t) \geq E[\sigma_{s_i,t}^2] = E[\text{Var}(\log S_t | S_0)].$$

This follows directly from the Cauchy-Schwarz inequality. Hence, in the first-order version of our economy, for large N , there is a lower bound on the cross-sectional variance of the log size distribution given by:

$$E[\sigma_{s_i,0}^2] + \sum_{\tau=1}^t \sigma_\epsilon^2 \left(1 + \gamma^2 Z \exp \left(\sum_{j=2}^{\infty} \frac{2^j - 1 + (-1)^j}{j!} \kappa_{s,j,\tau-1} \right) \right), t \geq 1.$$

In the log-normal case, the last sum reduces to $\exp(2\sigma_{s,\tau-1}^2)$:

$$E[\sigma_{s_i,0}^2] + \sum_{\tau=1}^t \sigma_\epsilon^2 (1 + \gamma^2 Z \exp(2\sigma_{s,\tau-1}^2)), t \geq 1$$

The result stated in the proposition follows directly if the higher-order cumulants are positive. \square

A.2 Incorporating Retailers

In the data, we also consider retailers who do not have any customer connections with other firms, at least not in the standard sense. Our simple model can be extended to include retailers. Consider a group of retailers $l = N + 1, \dots, N + k$. In the strictest interpretation of the model, retailers have $w_{lm} = 0, m = 1, \dots, N$. They contribute k zero rows to the adjacency matrix \mathbf{W} . However, if markets are incomplete, then some of the labor income risk that is specific to firms would show up in the consumption decisions of workers at these firms. That in turn would expose retail firms to some of the upstream risk.

Let $v_{l,m,t}$ denote the link strength of retailer l to customers working at supplier m .

$$g_{i,t+1} = \mu_g + \gamma \sum_{j=1}^{N+k} w_{i,j,t} g_{j,t+1} + \varepsilon_{i,t+1}, i = 1, \dots, N. \quad (25)$$

$$g_{l,t+1} = \mu_g + \psi \sum_{m=1}^{N+k} v_{l,m,t} g_{m,t+1} + \varepsilon_{l,t+1}, l = N + 1, \dots, N + k. \quad (26)$$

ψ governs how much firm-specific idiosyncratic risk is transferred to consumption; when markets are complete, $\psi = 0$. We adopt specification (16) for these retail-customer weights and impose that they sum to one. Larger retailers are connected to a larger number of consumers, and larger consumers are more important. Retail firms create network leakage because of the zero rows in the \mathbf{W} matrix.

B Empirical Appendix

This appendix discusses several additional empirical results.

B.1 Approximate Log Normality of Size and Volatility

We first we present evidence that the cross section distribution of size and volatility are approximately log normal. There is an extensive literature studying the firm size distribution that we do not cover here.

We merely note that our sample of firm sizes is approximately log normal. Hall (1987) characterized the literature on US firm sizes saying “The size distribution of firms conforms fairly well to the log normal, with possibly some skewness to the right.” Axtell (2001) provides evidence that firm sizes follow a power law in a large sample including all private US firms. Cabral and Mata (2003) argue that firm sizes evolve toward a log normal distribution over time. Rossi-Hansberg (2007) and Wright show that establishment and enterprise sizes for public and private US non-farm businesses have tails that are thinner than power law. We discuss the association between public and private firm size distributions in Appendix B.4.

Figure 6 plots histograms of the empirical cross section distribution of firms’ market equity value (Panel A) and sales (Panel B), taking the log of both quantities. The left figure in each panel shows the distribution of firm sizes pooling all firm-years from (1926–2010 for market equity, 1970–2010 for sales) and the right figure shows a one-year snapshot for 2010. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure also reports the skewness and kurtosis of the data in the histogram. The pooled empirical distribution, and the distributions for all individual years, appear nearly normally distributed. They demonstrate only slight skewness (always less than 0.5 in absolute value) and do not possess substantive leptokurtosis (always less than 3.3).

Figure 7 shows cross section distributions of yearly return volatility (Panel A) and sales growth volatility (Panel B) in logs for all CRSP/Compustat firms. Volatility also appears to closely fit a log normal distribution, with skewness no larger than 0.4 and kurtosis never exceeding 3.7. Figure 7 demonstrates near log normality of total return and growth rate volatility. Kelly, Lustig and Van Nieuwerburgh (2012) show that the same feature holds for idiosyncratic volatility.

B.2 Correlation Size Dispersion with Mean and Dispersion of Volatility

Table 8 shows that the strong correlation between moments of the volatility distribution and lagged size dispersion across markets, size quantiles and industries. We find that the the size distribution’s predictive correlation with both mean volatility and dispersion in volatility is robust in each of these sample decompositions. Size dispersion predicts volatility moments among either NYSE or NASDAQ stocks, among large and small stocks, and within all industries.

B.3 Frequency Decomposition

To study the trend and cycle in the size and volatility moments, we apply the Hodrick-Prescott filter with a smoothing parameter of 50. The top panel of Figure 8 shows the trend component. The bottom panel shows the cycle component. The top panel plots the trend in the cross-sectional average of log firm-level volatility against the cross-sectional dispersion of log market equity and log market volatility. The correlation between the trend components in average volatility and the market-based measure of lagged size dispersion is 90%. The correlation between lagged size dispersion and the trend component in market volatility is 55%. The correlation between the cyclical component in average log volatility and lagged market-based size dispersion is 26%. These results suggest that the correlation between the dispersion in the firm size distribution and moments of the volatility distribution occurs at lower frequencies. However, there is also significant evidence of correlation between cyclical volatility and size dispersion.

B.4 Size Dispersion with Private Firms

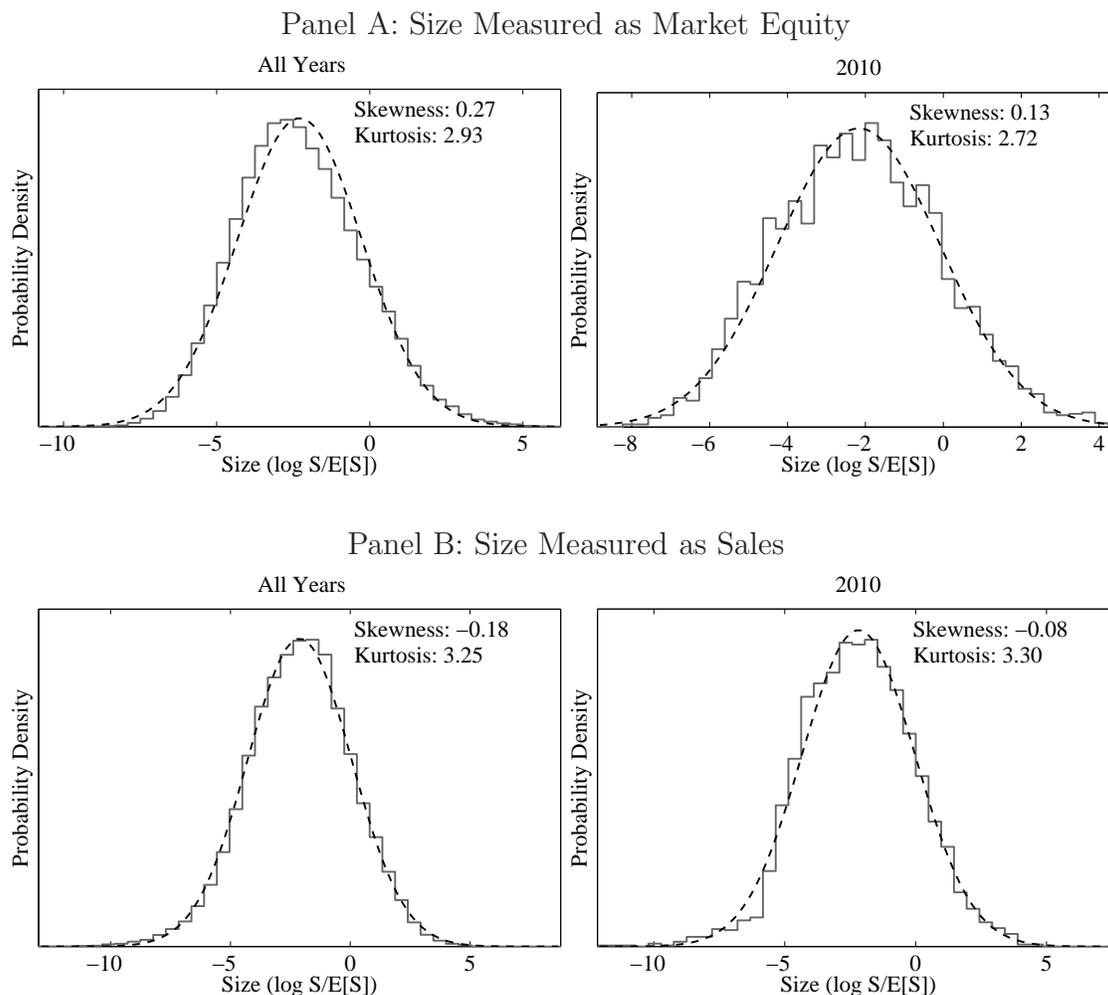
We investigate whether the dynamics in the firm size distribution, and in particular the dynamics of size dispersion, are similar for the same of publicly traded firms and the sample of all (public and private firms). For this, we resort to employment data as our measure of firm size. We use Census data on private sector firms sorted by employment, provided by the Business Dynamic Statistics. The sample is annual and covers 1977-2009. The Census reports employment data in 12 employment bins ranging from 1 – 4 at the low end to 10,000+ at the high end. We construct the same bins using the Compustat employment data. We also create a spliced series that divides the 10,000+ binds into 25 sub-bins using an imputation from the Compustat size data. At the start of the sample, Census reports 728 firms with more than 10,000 employees, while

Table 8: COMPOSITION

	# Firms	$\rho(\sigma_{\text{subset},s,t}, \sigma_{s,t})$	$\rho(\mu_{\sigma,t}, \sigma_{s,t-1})$	$\rho(\sigma_{\sigma,t}, \sigma_{s,t-1})$
All stocks	3004	-	71.7%	79.3%
By sample period / exchange				
NYSE only	1158	64.2%	62.1%	77.6%
Non-NYSE	3018	89.9%	58.1%	40.7%
At least 50 yrs	347	78.1%	44.5%	62.7%
Random 500	500	90.5%	64.9%	80.7%
By size				
Smallest third	1000	67.7%	71.7%	51.9%
Middle third	1000	87.7%	61.6%	69.8%
Largest third	1003	86.8%	55.9%	73.4%
By industry				
Consumer Non-Dur.	248	91.4%	64.4%	72.3%
Consumer Durables	107	87.6%	33.4%	74.9%
Manufacturing	528	91.9%	53.1%	79.8%
Energy	140	72.9%	68.4%	67.9%
Technology	413	88.1%	85.2%	58.2%
Telecom	55	23.6%	14.3%	10.6%
Retail	310	86.5%	69.0%	69.8%
Healthcare	188	69.5%	69.4%	50.6%
Utilities	112	67.8%	19.7%	64.8%
Other	904	82.8%	63.3%	65.7%

Notes: Annual data 1926-2010. We use the market based volatility measure constructed from stock returns and the market-based measure of size (market equity). The first column reports the time series average of the cross section number of firms. The second column reports the time-series correlation between the size dispersion of the sub-cross section and the dispersion of the full cross section. Column three reports the correlation between average log volatility ($\mu_{\sigma,t}$) and lagged log size dispersion ($\sigma_{S,t-1}$) and column four reports the correlation between dispersion in log volatility ($\sigma_{\sigma,t}$) and lagged log size dispersion.

Figure 6: LOG FIRM SIZE: EMPIRICAL DENSITY VERSUS NORMAL DENSITY

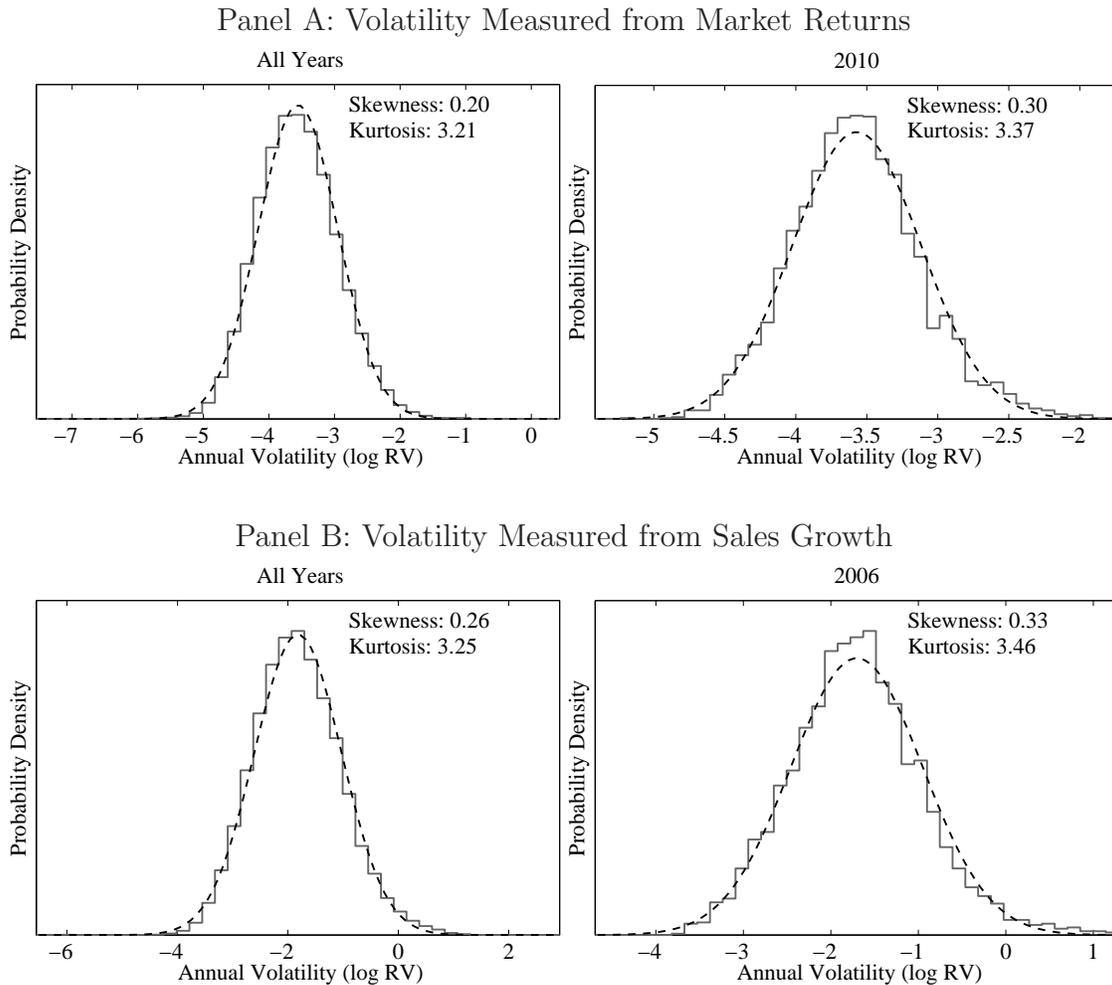


Notes: The figure plots histograms of the empirical cross section distribution of log annual firm market equity (Panel A) and log annual firm sales (Panel B). The left-hand histogram pools all years (1926-2010) and the right-hand histogram is the one-year snapshot for 2010. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.

Compustat reports 677. So, we have fairly comprehensive coverage at the start of the sample. However, at the end of the sample, there are 1,975 with 10,000+ firms, only 1,016 of which show up in Compustat. There are more large, private firms at the end of the sample.

Figure 9 plots the cross-sectional variance of log employment in Census and Compustat data. The evolution of the size distribution when considering the entire universe of firms seems similar to the one in Compustat. The correlation between the cross-sectional variance of log size in Compustat and the spliced series (shown in Panel A) is 62%. The correlation between the non-spliced Census measure (shown in Panel B) and the Compustat measure is 65%. Hence, the changes in the firm size distribution that we have documented do not seem specific to the universe of publicly-traded firms.

Figure 7: LOG VOLATILITY: EMPIRICAL DENSITY VERSUS NORMAL DENSITY

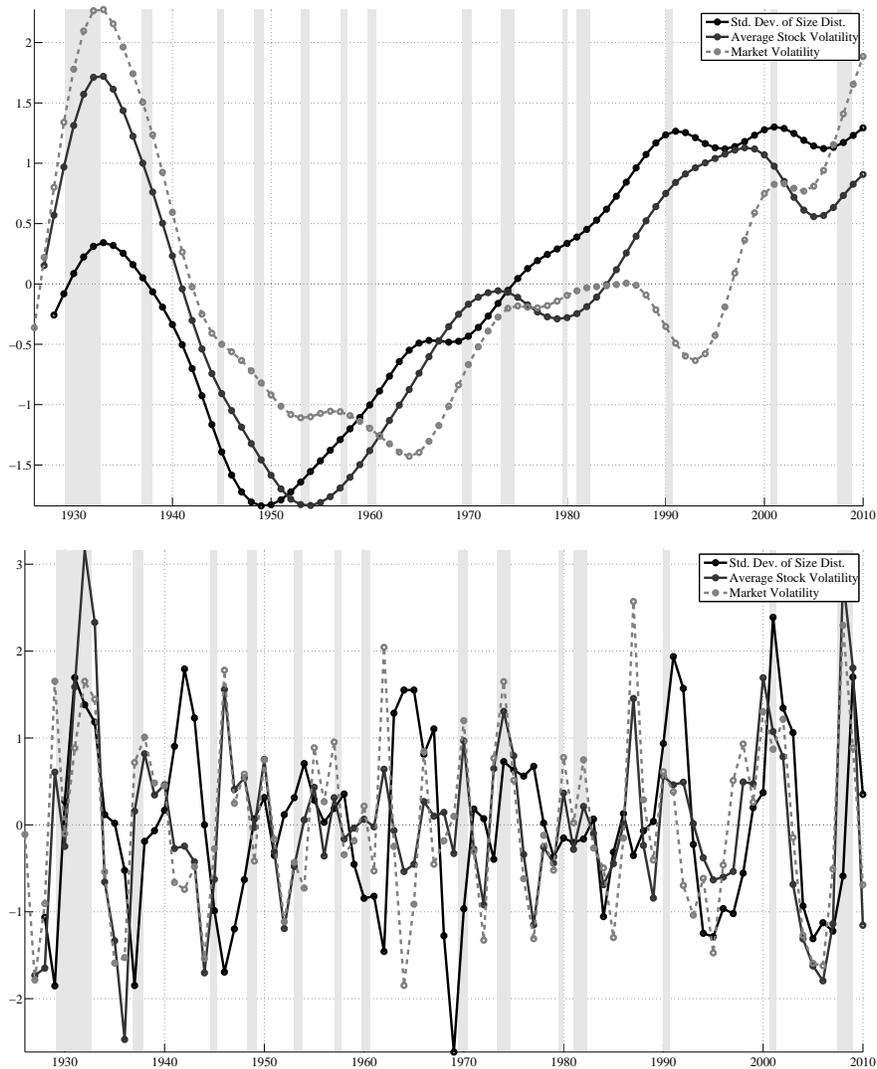


Notes: The figure plots histograms of the empirical cross section distribution of annual firm-level volatility (in logs). Panel A reports return volatility where, within each calendar year, we calculate the standard deviation of daily returns for each stock. Panel B reports sales growth volatility where, for each year t , we calculate volatility as the standard deviation of quarterly observations of year-on-year sales growth for each stock in calendar years t to $t + 4$. The left-hand histogram pools all years (1926-2010) and the right-hand histogram is the one-year snapshot for 2010 for returns and 2006 for sales growth (since sales growth volatility is calculated over a five-year window).

B.5 Firm Exit and Entry Rates

What drives the evolution of the firm size dispersion in the U.S.? We do not aim to explain these dynamics, other than to point out (and model) the two-way feedback effects between firm size distribution and firm volatility distribution. However, a relevant piece of information would seem to be the secular decline in entry and exit rates that we observe in the firm universe. Figure 10 plots the entry and exit rates for all U.S. firms based on Census data for the period 1977 to 2009. These secular changes may have contributed to the changes in the size distribution that we have documented.

Figure 8: TREND AND CYCLE IN SIZE AND VOLATILITY DISTRIBUTIONS



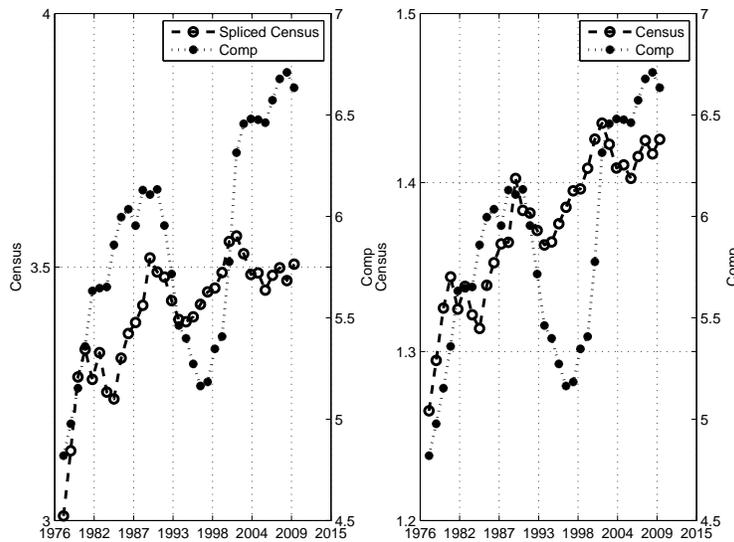
Notes: The figure plots HP-filtered trend and cycle components of time series moments of size and volatility distributions using smoothing parameter of 50.

B.6 Summary Statistics Network Data

The number of customers a firm has, the “out-degrees,” ranges between 1 and 24 while the number of suppliers a firm has, the “in-degrees,” ranges between 1 and 130. We expect that both of these distributions are highly distorted due to the truncated nature of the data. Out-degrees can reach 24 since some suppliers voluntarily report customers that fall below the 10% sales threshold. The maximum out-degrees falls to 5 when we strictly impose the 10% sales truncation.

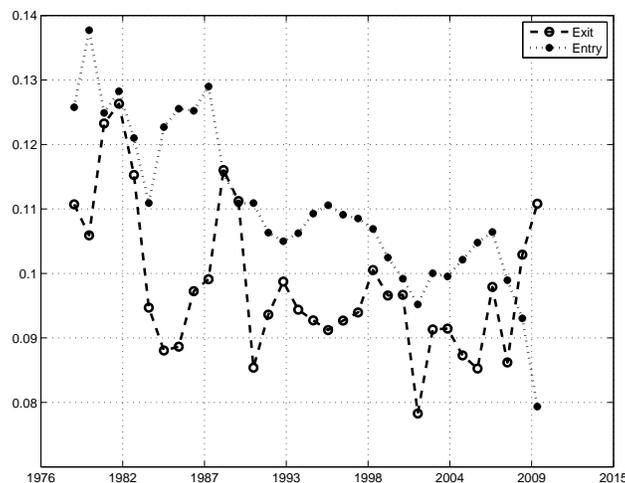
Figure 11 provides a summary of network connections for customers and suppliers in the Compustat linkage data. The left panel shows the distribution of number of links by supplier (out-degree) on a log-log scale, while the right panel shows the distribution of links by customer (in-degree). Out-degrees range between one and 24, while in-degrees range from one to 130. We expect that both of these distributions are highly distorted due to the truncated nature of the data. Out-degrees can reach 24 since some suppliers

Figure 9: VARIANCE OF LOG EMPLOYMENT: COMPUSTAT VS. CENSUS DATA



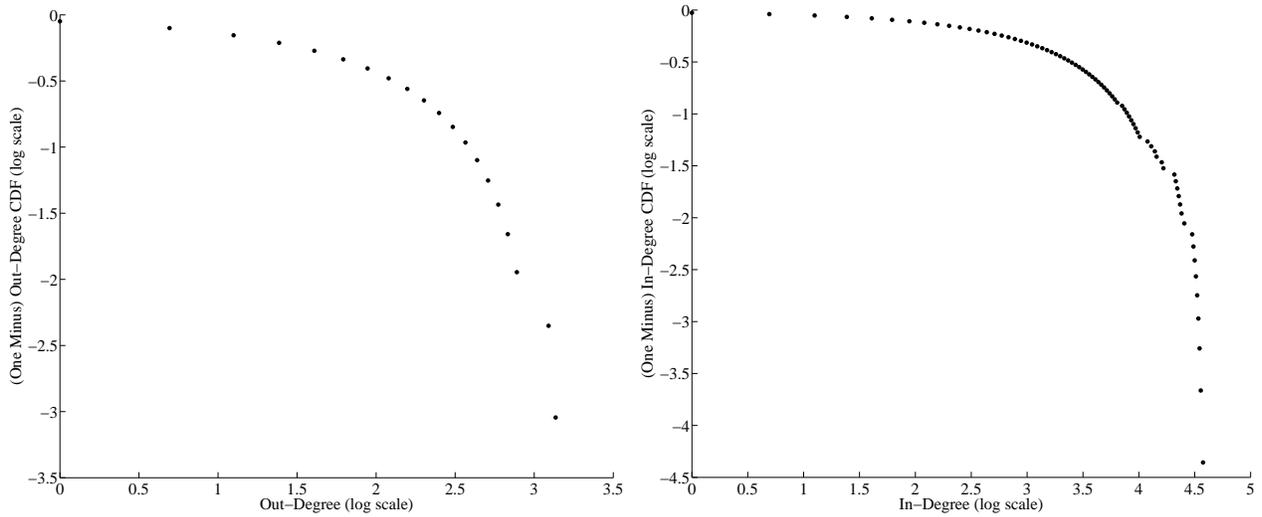
Notes: The figure plots the cross-sectional variance of log employment in the Compustat and Census data (left-hand-side axis). The data is annual and the sample covers 1977-2009. Source: U.S. Small Business Administration, Office of Advocacy, from data provided by the U.S. Census Bureau, Business Dynamics Statistics.

Figure 10: FIRM DYNAMICS



Notes: The figure plots the entry and exit rates for all U.S. firms based on Census data. The data is annual and the sample covers 1977-2009. Source: U.S. Small Business Administration, Office of Advocacy, from data provided by the U.S. Census Bureau, Business Dynamics Statistics.

Figure 11: CUSTOMER-SUPPLIER NETWORK DEGREE DISTRIBUTIONS



Notes: The figure plots log-log survivor plots of the out-degree and in-degree distributions of the Compustat customer-supplier network data pooling all firm-years for 1980–2009.

(23%) voluntarily report customers that fall below the 10% sales threshold. The maximum out-degrees falls to 5 when we strictly impose the 10% sales truncation.

Figure 12 reports histograms of weights of customer-supplier sales linkages pooling all supplier-year observations. The distribution for the raw data, in which some suppliers voluntarily report customers below the 10% sales threshold, is on the left. The right panel shows the weight distribution when we strictly impose the 10% sales truncation in our calibration below.

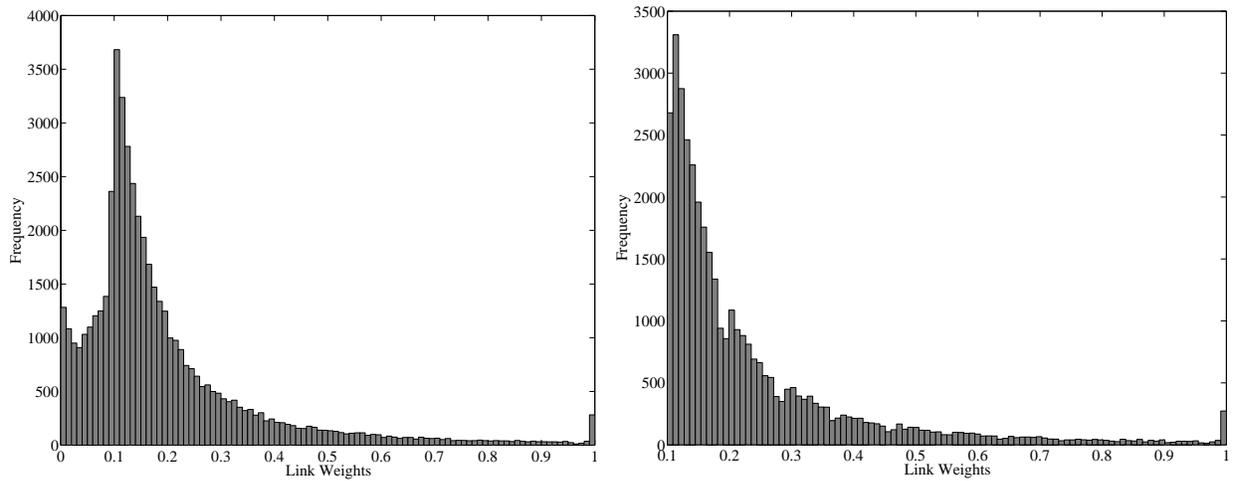
These correlations are visualized in Figure 13, which presents a snapshot of the Compustat customer-supplier network in 1980, zooming in on a region of the network dominated by retail firms (Sears, KMart and JC Penney), automotive manufacturers (Ford and General Motors), technology/telecom firms (IBM and AT&T) and industrials (General Electric and United Technologies). The size of each nodes is proportional to a firm’s log sales, and edges between nodes are proportional to $w_{i,j,t}$. Arrows on edges point from customer to supplier.

B.7 Industry-level Network Data

In further support of the main assumptions in the model, we present evidence the industry-level input-output networks appear largely consistent with the network structure that we have proposed. Industry input-output data are from the Bureau of Economic Analysis (BEA). Because industry definitions vary quite dramatically over time, we focus on a set of 65 industries we can track consistently over time between 1998 and 2011. This data is informative for evaluating cross-sectional correlations between industry size and network structure since it does not suffer the truncation issue that plagues the Compustat firm-level data. In related work, Ahern and Harford (forthcoming) use the network topography implied by the BEA industry data to show that the properties of these networks have a bearing on the incidence of cross-industry mergers.

Table 9 reports industry network correlations analogous to the firm network correlation in Table 3. We find the log total output of a supplier industry and the number of industries that it supplies to has an average correlation of 61%. The correlation is strongly positive and lends further credence to the positive association between size and number of customers assumption we make in the model. It is also suggestive of the large effect truncation may have in reducing this correlation in the firm-level data. Similarly to the firm-level

Figure 12: CUSTOMER-SUPPLIER NETWORK LINKAGE WEIGHTS



Notes: The figure plots the histogram of Compustat customer-supplier sales weights for the raw data (left panel), and when we strictly imposed the 10% sales threshold in our calibration. Plots pool all firm-years for 1980–2009.

evidence, we find a strong -39% correlation between an industry’s log size and its customer concentration measure (out-Herfindahl). Both of these facts, that larger supplier industries have more connections and are better diversified, are consistent with our model. On the customer side, we find an average correlation of industry size with in-degree of 46%, and with supplier Herfindahl of 10%. Our model would not predict that an industry’s number of suppliers would increase with its size. But neither do we find evidence to suggest that purchaser industries are experiencing any diversification benefits from their supplier network, since larger industries have a *higher* in-degree concentration, not lower. On balance, we find the BEA industry network data to be consistent with the network structure we propose.

Table 9: BEA INDUSTRY SIZE, VOLATILITY AND NETWORK STRUCTURE

Correlation between ... and ...	$\log S_{i,t}$ $K_{i,t}^{out}$	$\log S_{i,t}$ $H_{i,t}$	$\log S_{i,t}$ $K_{i,t}^{in}$	$\log S_{i,t}$ $H_{i,t}$
Avg.	0.61	-0.39	0.46	0.10
$t - stat$	28.00	-12.63	19.33	5.99

Notes: Cross section correlations of industry size, out-degree, out-Herfindahl, in-degree, and in-Herfindahl from BEA industry benchmark input-output tables. The number of industries is 65 and the data are annual from 1998 until 2011.

C Calibration Appendix

C.1 Alternative Calibrations

To better understand the various parts of the model, we explore a number of alternative models to our benchmark model (M1). Table 10 lists the parameters. The results for size, volatility, and network moments are reported in Tables 11, 12, and 13, respectively. In all simulations, we use the exact same random draws (and seed for the random number generator) to enhance comparability across models. All variation across models is thus produced by endogenously changing size and volatility distributions, not by random differences across simulations.

Table 10: CALIBRATION

Parameter	Description	M1	M2	M3	M4	M5
1. N	# firms reported each period	1000	1000	1000	1000	1000
2. T	# years in simulation (after burn-in)	1000	1000	1000	1000	1000
3. μ_g	Exogenous firm growth rate	0	0	0	0	0
4. μ_{So}	Mean initial log size distribution	10.20	10.20	10.20	10.20	10.20
5. σ_{So}	Standard deviation initial log size distribution	1.06	1.06	1.06	1.06	1.06
6. δ	Exogenous firm destruction rate	0.05	0.05	0.05	0.05	0.05
7. κ	Governs the survival rate of connections	0.5	0.5	0.5	0.5	0.5
8. Z	Governs likelihood of new connections	0.35	0.35	0.35	0.35	0.10
9. ψ	Governs the importance of connections	1	1	1	1	0.1
10. γ	Importance of the network	0.95	0.95	0.95	0.75	0.95
11. σ	Fundamental shock volatility (i.i.d.)	0.22	0.22	0.22	0.30	0.25
12. network	Full-degree or first-degree approximation	F	A	F	F	F
13. $p(\cdot)$	Governs probability of a connection	B	B	A	B	B

Notes: In row 12, F indicates full-degree network and A indicates first degree-approximation. In row 13, B indicates benchmark and A indicates alternative assumption.

Model 2 (M2) is the first-degree approximation to the benchmark model, discussed in Section 3, with all parameters left unchanged. It produces dramatically different size and volatility results, underscoring the importance of network persistence. Out-degree Herfindahls in M2 are lower than in M1. Because of less concentrated networks, firms are better able to diversify the shocks that hit their customer network. Mean volatility (27%) and the dispersion in volatility across firms (26%) are substantially lower in the first-degree approximation as in the full-degree network. The median firm is 15% smaller while the largest firms are 34% smaller than in M1. In terms of macro moments, the size dispersion fluctuations almost completely disappear (4% time-series standard deviation versus 19%). The same is true for average firm volatility (3% time-series standard deviation versus 20%), aggregate growth volatility (4% versus 20%), and the volatility of the cross-sectional dispersion of volatility. The dramatic reduction in aggregate volatilities shows that higher-order connectivity amplifies shocks and is crucial in producing the patterns in aggregate moments we observe in the data.

Model 4 (M4) tells a similar story. Here, we revert to the full-degree network, but lower the persistence parameter of the network to $\gamma = 0.75$. Because lowering γ lowers average volatility, we adjust the volatility

Table 11: MODEL COMPARISON: FIRM SIZE DISTRIBUTION

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	All	Top-33%	M1	M2	M3	M4	M5
Panel A: Cross-sectional Moments of Log Size							
Avg	9.61	11.63	11.63	11.47	11.66	11.78	11.57
SD	1.79	1.06	1.07	0.97	1.19	1.30	1.00
5%	6.86	10.38	10.32	10.34	10.21	10.33	10.41
10%	7.39	10.47	10.44	10.43	10.34	10.44	10.50
25%	8.33	10.78	10.79	10.72	10.73	10.80	10.80
Med	9.48	11.39	11.42	11.26	11.43	11.47	11.34
75%	10.77	12.25	12.25	11.99	12.35	12.43	12.08
90%	12.05	13.10	13.10	12.78	13.30	13.54	12.91
95%	12.76	13.64	13.68	13.34	13.93	14.32	13.51
Panel B: Time Series Properties of Size Distribution							
SD of $\sigma_{S,t}$	0.24	0.14	0.19	0.04	0.43	0.11	0.15
AR(1) $\sigma_{S,t}$	0.947	0.939	0.995	0.959	0.995	0.981	0.997
SD of $\sigma_{g,t}$	0.15	0.11	0.05	0.01	0.01	0.02	0.03
H_t	0.010	0.012	0.020	0.018	0.036	0.088	0.034

Notes: All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first column reports the full cross-section of firms. The second column reports results for the top-33% of firms in each year. Column 3 reports the corresponding moments for the benchmark model (M1). Column 4 shows the first-degree approximation of the benchmark model (M2). Columns 5–7 show three variations of the full-degree network model for different parameter values, described in the main text, and labeled M2–M5. Panel A reports moments of the log size distribution, where size is defined in the data as market equity. Panel B reports the time-series standard deviation and time-series persistence of size dispersion $\sigma_{S,t}$, defined as the cross-sectional standard deviation of log size, the time-series standard deviation of the cross section standard deviation of log growth rates $\sigma_{g,t}$ and the time-series average of the economy-wide Herfindahl index H_t .

of the shocks σ upwards to 0.30. This model behaves similarly to M2 in that it has a more compact firm volatility distribution and much lower variability in size dispersion, mean variance, and variance dispersion than M1. Unlike M2, M4 has a much higher economy-wide Herfindahl index (0.088) indicating strong firm size concentration. This arises from the higher shock volatility which helps to produce large firms. The largest firms are too large compared to the data. Matching mean volatility with weaker network effects thus has some counterfactual effects for the firm size distribution, as well as for volatility dispersion.

In Model 3 (M3), we modify the probability that a supplier is connected to a customer and make it less steep in size. Specifically, we change the probability of a connection from the benchmark equation (??) to the alternative

$$p_{i,j,t} = \frac{1}{Z + \log\left(\frac{\max_k\{S_{k,t}\}}{S_{i,t}}\right)}; \quad \text{prob of conn} = \max\{p_{i,j,t} + \kappa, 1\}$$

where we use the same scalar values for Z and κ than in the benchmark specification. This radically changes the average number of *untruncated* out-degrees from 10 in the benchmark to 441, while the *truncated* average out-degree actually falls. The median (truncated or untruncated) out-Herfindahl drops from 0.60 to 0.06, bringing it in line with the data. Because smaller firms now have bigger networks, they achieve better diversification. The dispersion of volatility is dramatically lower (0.08 in M3 versus 0.45 in M1) and this dispersion fluctuates much less over time. The cross-sectional correlation between size and volatility becomes positive, but there is much less dispersion in volatility in the first place.

In Model 5 (M5) we consider a calibration that delivers a substantially lower out-degree Herfindahl than the benchmark model, bringing it closer to the data. We change the function $w_{i,j,t}$, which governs the importance of customer j in supplier i 's network:

$$w_{i,j,t} = \frac{b_{i,j,t}S_{j,t}^{\psi}}{\sum_{k \neq i} b_{i,k,t}S_{k,t}^{\psi}} \quad \forall j \neq i$$

We make the importance of a given customer less steep in customer size by setting the curvature parameter $\psi = 0.1$. All other calibrations set $\psi = 1$. To further lower the out-degree Herfindahl, we lower Z from its benchmark value of 0.35 to 0.10, making the probability of a connection steeper in supplier size. This generates more connections on average. Finally, we increase the volatility of the shocks to 25% so that the model generates the same mean variance as in the data. By giving large customers a smaller weight in the network, M5 reduces the median out-degree Herfindahl from 0.60 in M1 to 0.11; the truncated counterpart is 0.05 matching the data. The average number of (untruncated) connections is 32 and the median is 12. The larger number of connections helps to further lower the Herfindahls. M5 continues to generate substantial firm size dispersion and the right amount of average firm volatility, in part because of the higher shock volatility ($\sigma_{\varepsilon} = 0.25$ versus 0.22). However, the volatility dispersion (0.20) is severely curtailed, like in M3 (0.08), compared to the 0.45 in M1 and the even higher numbers in the data. M5 cannot generate enough highly volatile firms, compared to the data. Aggregate moments such as size dispersion, mean variance, variance dispersion, and aggregate growth still display meaningful time variation, but less than in the benchmark model. Despite this drawback, this model has arguably a more realistic network structure, and continues to generate interesting variation in the key moments of interest.

C.2 Downstream Results

We study a version of our model where the direction of shock transmission is reversed from upstream to downstream. A firm's growth rate now depends on its own shock and on the growth rate of its suppliers. We use the exact same calibration as in the benchmark model. If we abstract from truncation, then reversing the direction only changes the network moments reported in Table 7; the moments in the size and volatility tables 5 and 6 are identical. In terms of network moments, the untruncated in-degree and out-degree moments are reversed. However, after truncation (which operates at the level of suppliers), the results look different. Table 14 reports results for the downstream transmission case. This version of the model has more success matching the in-degree moments, but then it does considerably worse matching the out-degree moments in the top panel. Most importantly, this version of the model implies counter-factually that there is no correlation between out-Herfindahls and volatility. In the data, we found that this relation is statistically

Table 12: MODEL COMPARISON: FIRM VOLATILITY DISTRIBUTION

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	All	Top-33%	All	Top-33%	M1	M2	M3	M4	M5
	Returns	Returns	Sales	Sales					
Panel A: Cross-Sectional Moments of Firm Volatility									
Avg	0.40	0.30	0.29	0.24	0.37	0.27	0.51	0.40	0.31
SD	0.96	0.73	1.51	1.42	0.45	0.26	0.08	0.29	0.20
5%	0.19	0.17	0.09	0.08	0.26	0.22	0.48	0.31	0.27
10%	0.22	0.19	0.11	0.10	0.28	0.23	0.48	0.33	0.27
25%	0.29	0.24	0.17	0.14	0.31	0.24	0.49	0.36	0.29
Med	0.40	0.30	0.29	0.23	0.36	0.27	0.50	0.40	0.30
75%	0.56	0.38	0.48	0.38	0.42	0.29	0.52	0.44	0.32
90%	0.75	0.48	0.77	0.60	0.49	0.32	0.53	0.48	0.35
95%	0.90	0.54	1.05	0.77	0.54	0.33	0.54	0.50	0.36
Panel B: Cross-Sectional Moments of Size-Volatility Distribution									
$Corr(S_t, V_{t+1})$	-0.57	-0.33	-0.33	-0.18	-0.42	-0.49	0.20	-0.50	-0.37
$\beta(S_t, V_{t+1})$	-0.32	-0.23	-0.27	-0.22	-0.18	-0.13	0.01	-0.11	-0.07
Panel C: Time Series Properties of Volatility Distribution									
SD of $\mu_{\sigma^2,t}$	0.69	0.64	0.46	0.42	0.20	0.03	0.82	0.09	0.15
SD of $\sigma_{\sigma^2,t}$	0.18	0.12	0.13	0.13	0.05	0.01	0.02	0.01	0.07
$Corr(\sigma_{S,t}, \mu_{\sigma^2,t})$	0.71	0.55	0.51	0.51	0.95	0.86	0.76	0.90	0.93
$Corr(\sigma_{S,t}, \sigma_{\sigma^2,t})$	0.76	0.76	0.57	0.38	0.77	0.09	0.34	-0.54	0.94
SD of $g_{agg,t}$	0.21	0.20	0.21	0.20	0.20	0.04	0.62	0.15	0.15

*Notes:*All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first and third columns report return volatility and sales growth volatility, respectively, for the full cross-section of firms. The second and fourth columns report results for the largest 33% of firms by market capitalization in each year. Annual variances in Columns 1 and 2 are calculated from daily stock return data, while variances in columns 3 and 4 are based on 20 quarters of sales growth rates (in the current and the next four years). Moments are computed based on the log variance distribution, but are expressed as volatilities in levels for exposition. That is, we exponentiate each moment of the log distribution and take the square root. Column 5 reports the corresponding moments for the benchmark model (M1). Column 6 shows the first-degree approximation of the benchmark model (M2). Columns 7-9 show three variations of the full-degree network model for different parameter values, described in the main text. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

Table 13: MODEL COMPARISON: NETWORK MOMENTS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	All	Top-33%	All	Top-33%	M1	M2	M3	M4	M5
	Returns	Returns	Sales	Sales					
Panel A: Out-degree Moments									
median K^{out}	1.00	1.00	–	–	1.71	1.99	0.75	1.12	1.03
99 th % K^{out}	3.38	3.32	–	–	4.66	4.94	2.24	4.05	7.93
median H^{out}	0.05	0.04	–	–	0.60	0.51	0.04	0.88	0.05
99 th % H^{out}	0.95	0.54	–	–	1.00	1.00	0.23	1.00	0.72
$Corr(K_t^{out}, S_t)$	0.01	–0.07	–	–	0.34	0.25	–0.08	0.45	–0.55
$Corr(H_t^{out}, S_t)$	–0.31	–0.08	–	–	–0.71	–0.67	–0.12	–0.72	–0.53
$Corr(H_t^{out}, V_{t+1})$	0.15	0.08	0.29	0.10	0.54	0.70	0.45	0.63	0.58
Panel B: In-degree Moments									
median K^{in}	1.00	1.12	–	–	1.90	2.00	0.00	1.35	2.06
99 th % K^{in}	31.61	16.80	–	–	7.39	7.42	45.41	5.90	6.93
median H^{in}	0.00	0.00	–	–	0.11	0.11	0.00	0.07	0.01
99 th % H^{in}	0.28	0.24	–	–	0.85	0.83	0.00	0.76	0.05
$Corr(K_t^{in}, S_t)$	0.37	0.49	–	–	0.49	0.46	0.31	0.38	0.16
$Corr(H_t^{in}, S_t)$	–0.26	–0.20	–	–	0.03	0.03	0.21	–0.07	0.01
$Corr(H_t^{in}, V_{t+1})$	0.13	0.08	0.08	0.08	0.01	–0.00	0.17	0.05	0.03

Notes: All reported moments are time-series averages unless explicitly mentioned otherwise. The first and second columns report data for the cross-section of firms for which we have customer information from Compustat. The sample is 1980-2009. The variance in Column 1 is calculated based on daily stock return data, while the variance in Column 2 is based on 20 quarters worth of annual sales growth (in the current and the next four years). Column 3 reports the corresponding moments for the benchmark model (M1). Column 4 shows the first-degree approximation of the benchmark model (M2). Columns 5-7 show three variations of the full-degree network model for different parameter values, described in the main text. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

Table 14: DOWNSTREAM TRANSMISSION: NETWORK MOMENTS

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	Model
Panel A: Out-degree Moments					
median K^{out}	1.00	1.00	–	–	3.88
99 th % K^{out}	3.38	3.32	–	–	6.15
median H^{out}	0.05	0.03	–	–	0.25
99 th % H^{out}	0.95	0.54	–	–	1.00
$Corr(K_t^{out}, S_t)$	0.01	–0.07	–	–	0.14
$Corr(H_t^{out}, S_t)$	–0.30	–0.09	–	–	–0.09
$Corr(H_t^{out}, V_{t+1})$	0.17	0.11	0.25	0.09	0.05
Panel B: In-degree Moments					
median K^{in}	1.00	1.12	–	–	1.90
99 th % K^{in}	31.61	16.80	–	–	7.39
median H^{in}	0.00	0.00	–	–	0.11
99 th % H^{in}	0.28	0.24	–	–	0.85
$Corr(K_t^{in}, S_t)$	0.37	0.49	–	–	0.59
$Corr(H_t^{in}, S_t)$	–0.44	–0.28	–	–	–0.39
$Corr(H_t^{in}, V_{t+1})$	0.25	0.19	0.09	0.12	0.32

Notes: All reported moments are time-series averages unless explicitly mentioned otherwise. The first and second columns report data for the cross-section of firms for which we have customer information from Compustat. The sample is 1980-2009. The variance in Column (1)-(2) are calculated based on daily stock return data, while the variance in Column (3)-(4) are based on 20 quarters worth of annual sales growth (in the current and the next four years). Column (5) reports the corresponding moments for the benchmark model (M1). Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

strong (see Table 3) unlike that between volatility and in-Herfindahl. In addition, Table 4 showed that out-Herfindahl is a robust determinant of firm volatility after controlling for other variables, while the relation between supplier Herfindahls and volatility is not.