Deflation Risk

Matthias Fleckenstein, Francis A. Longstaff, Hanno Lustig

UCLA Anderson

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Introduction

- Deflation has played a central role during the worst economic meltdowns in U.S. history.
  - Panic of 1837
  - Long Depression of 1874-1896
  - Great Depression of the 1930s
- Growing fears of deflation in the financial press.
  - "Nightmare scenario"
  - "Looming disaster"
  - "Growing threat"
- Mitigating risk of deflation is an explicit motivation behind many recent measures by the Federal Reserve such as the Quantitative Easing Programs.

Motivation

- Relatively little is known about the probability of deflation.
- Reason may be that the distribution of inflation is difficult to measure.
- Ang, Bekaert, and Wei (2007) show that econometric models perform poorly in estimating first moment of inflation.
- Survey data does better, but only looks at first moments, not tail probabilities.

This Paper

- We use a new market based approach to measure deflation risk.
  - First we solve for the risk-neutral density of inflation from the market prices of inflation calls and puts.
  - Then we solve for the inflation risk premium via MLE estimation of an affine term structure model using the term structure of inflation swap rates.
  - Finally, we solve for the objective density of inflation by inverting the risk-premium-adjusted characteristic function for the implied inflation density function.

Results

- Long run expected inflation is 2.50 percent.
- Inflation risk premium averages around 30 basis points for long horizons.
- Inflation volatility ranges from one to two percent.
- Inflation density is skewed to left, longer tails than for normal distribution.
- Probability of deflation sizable.
  - Bernanke, Aug 27, 2010, "Falling into deflation is not a significant risk."
  - On same date, market-implied probability of deflation was 15.11 percent for two-year horizon, 5.36 percent for five-year horizon, and 2.84 percent for a ten-year horizon.
- Tail risk of deflation priced similarly to other types of tail risk such as catastrophe insurance and corporate bond defaults.
- Deflation correlated with other types of financial and economic tail risk such as systemic credit risk, liquidity risk, and unemployment.
- Inflation risk is priced less severely by the market.

Dynamics of the Price Level

- Objective Measure

$$
\begin{aligned}
dI &= IXdt + I\sqrt{V}dZ_I \\
dX &= \kappa(Y - X)dt + \sigma dZ_X \\
dY &= (\alpha - \beta Y)dt + \eta dZ_Y \\
dV &= \mu dt + s dZ_V
\end{aligned}
$$

- Risk-Neutral Measure

$$
\begin{aligned}
dI &= IXdt + I\sqrt{V}dZ_I \\
dX &= \lambda(Y - X)dt + \sigma dZ_X \\
dY &= (\phi - \gamma Y)dt + \eta dZ_Y \\
dV &= \mu dt + s dZ_V
\end{aligned}
$$

Valuing Inflation Swaps

- The payoff on a T-year inflation swap paying f on the fixed leg is $(1+f)^{10} - I_{10}$.
- The value of the inflation swap at time zero is

$$F(X, Y, T) = \exp(-A(T) - B(T)X - C(T)Y)$$

where

$$A(T) = \frac{\sigma^2}{2\lambda^2}\left(T - \frac{2}{\lambda}\left(1 - \exp^{-\lambda T}\right) + \frac{1}{2\lambda}\left(1 - e^{-2\lambda T}\right)\right)\ldots$$

$$B(T) = \frac{-\left(1 - e^{\lambda T}\right)}{\lambda}$$

$$C(T) = \frac{\gamma\left(1 - e^{-\lambda T}\right) - \lambda\left(1 - e^{-\gamma T}\right)}{\gamma(\lambda - \gamma)}$$

Valuing Inflation Options

- The payoff on a European-style inflation cap at expiration date T is $\max\left(0, I_T - (1+K)^T\right)$. Similarly, for a floor $\max\left(0, (1+K)^T - I_T\right)$.
- The value of the call option at time zero is

$$C(X,Y,V,T) = D(T)\, E^{Q^*}\left[max(0, I_T - (1+K)^T\right],$$

- The expectation is taken with respect to the adjusted risk-neutral measure Q^* for inflation defined by the following dynamics

$$\begin{aligned}
dI &= IXdt + I\sqrt{V}dZ_I \\
dX &= \left(\lambda(Y-X) + \sigma^2 B(T-t) + \rho\sigma\eta C(T-t)\right)dt + \sigma dZ_X \\
dY &= \left(\alpha - \beta Y + \eta^2 C(T-t) + \rho\sigma\eta B(T-t)\right)dt + \eta dZ_Y \\
dV &= \mu dt + s dZ_V
\end{aligned}$$

The Distribution of the Price Level

- The log of the relative price level can be expressed as,

$$\ln I_T = \int_0^T X_s ds$$
$$- \frac{1}{2}\int_0^T V_s ds + \int_0^T \sqrt{V_s} dZ_V$$

- This can be expressed as

$$\ln I_T = u_T + w_T \text{ under the } F\text{-measure,}$$
$$\ln I_T = v_T + w_T \text{ under the } P\text{-measure.}$$

- u_T and v_T are normally distributed random variates.
- u_T and v_T are the value of the integral, w_T represents the term on the second line.
- u_T and v_T are independent of the value of w_T.

Data

- Inflation swaps
  - Maturities from one to 50 years for the period from July 23, 2004 to October 5, 2012.
- Inflation caps and floors
  - Strikes from -2% to 6 % in increments of 50bps, and maturities from 1 to 30 years for the period from October 5, 2009 to October 5, 2012.
- Inflation surveys
  - University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters, the Livingston Survey, Bloomberg one-year forecasts for the period from July 2004 to October 2012.
- Measures of financial tail risk
  - One-year Refcorp-Treasury yield spread, 10–15 % CDX IG index tranche prices, one-year Libor-Treasury spread, five-year swap spread, VIX index, Merrill Lynch MOVE index, Baa spread over the five-year Treasury rate, spread for a five-year CDS contract on the U.S. Treasury.
- Macroeconomic variables
  - U.S. industrial production, U.S. unemployment rate, University of Michigan consumer confidence index.

Summary Statistics for Inflation Swap Rates

Swap Maturity	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.758	1.369	−4.545	2.040	3.802	2141
2	1.930	1.087	−3.605	2.191	3.460	2141
3	2.088	0.853	−2.047	2.293	3.351	2141
4	2.219	0.688	−1.228	2.390	3.342	2141
5	2.324	0.575	−0.570	2.468	3.310	2141
6	2.403	0.492	−0.080	2.521	3.310	2141
7	2.472	0.424	0.402	2.575	3.229	2141
8	2.530	0.375	0.639	2.613	3.195	2141
9	2.577	0.331	0.904	2.651	3.135	2141
10	2.621	0.295	1.146	2.685	3.145	2141
12	2.677	0.278	1.280	2.738	3.160	2141
15	2.739	0.278	1.161	2.797	3.330	2141
20	2.801	0.290	1.069	2.865	3.360	2141
25	2.848	0.302	1.211	2.911	3.390	2141
30	2.903	0.300	1.454	2.959	3.500	2141
40	2.784	0.246	1.454	2.819	3.377	1016
50	2.781	0.261	1.465	2.830	3.500	842

Summary Statistics for Inflation Caps and Floors

Option Maturity	Average Floor Value by Strike								Average Cap Value by Strike									
	-2	-1	0	1	2	3	4	5	-1	0	1	2	3	4	5	6	Ave.	N
1	7	14	27	54	105	181	294	398	237	153	91	41	17	7	4	2	25.6	572
2	19	29	48	92	184	334	544	749	523	349	204	96	43	22	13	9	25.9	593
3	27	39	61	111	229	444	751	1050	831	572	342	162	72	38	24	17	26.5	500
5	36	49	76	134	282	599	1058	1565	1456	1045	658	325	154	70	42	29	25.9	577
7	32	46	76	144	311	697	1261	1954	2040	1518	1014	545	230	133	79	55	25.7	434
10	31	49	85	170	368	843	1561	2536	2752	2111	1478	846	413	198	120	76	25.7	566
12	28	44	80	164	376	900	1704	2862	3119	2424	1732	1005	482	236	126	75	26.4	531
15	24	39	73	156	376	967	1918	3332	3536	2793	2035	1191	560	274	142	84	26.2	569
20	21	36	69	157	408	1121	2278	3959	3985	3254	2467	1473	690	326	172	94	25.1	488
30	15	31	75	205	629	1807	2786	5812	4614	4117	3428	2163	1154	573	309	187	21.5	215

Risk-Neutral Distribution

- The density $h(z)$ of the continuously compounded inflation rate $z = \ln(I_T)/T$ under the (adjusted) risk-neutral measure is a member of the five-parameter class of generalized hyperbolic densities (Ghysels and Wang (2011)).
- The generalized hyperbolic density is given by

$$h(z) = \frac{\left(a^2 - b^2\right)^{q/2} d^{-q} e^{b(z-c)}}{\sqrt{2\pi} a^{q-1/2} K_q\left(d\sqrt{a^2 - b^2}\right)} \frac{K_{q-1/2}\left(a\sqrt{d^2 + (z-c)^2}\right)}{\left(\sqrt{d^2 + (z-c)^2}\right)^{1/2-q}}$$

  - a, b, c, d, and q are parameters, and $K_q(\cdot)$ denotes the modified Bessel function.

Maximum Likelihood Estimation

- We assume that the two-year and 30-year inflation swap rates are measured without error.
- Thus, given a parameter vector Θ, X and Y can be expressed as explicit linear functions of the two inflation swap prices $F(X, Y, 2)$ and $F(X, Y, 30)$.

$$\ln F(X, Y, 2) = -A(2) - B(2)X - C(2)Y,$$
$$\ln F(X, Y, 30) = -A(30) - B(30)X - C(30)Y,$$

- The log of the joint likelihood function LLK_t of the two-year and 30-year inflation swap prices is

$$= -\ln\left(2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}}\right) - \frac{1}{2\left(1-\rho_{XY}^2\right)}\left[\frac{\left(X_{t+\Delta t}-\mu_{X_t}\right)^2}{\sigma_X^2}\right.$$
$$\left. -2\rho_{XY}\left(\frac{X_{t+\Delta t}-\mu_{X_t}}{\sigma_X}\right)\left(\frac{Y_{t+\Delta t}-\mu_{Y_t}}{\sigma_Y}\right) + \frac{\left(Y_{t+\Delta t}-\mu_{Y_t}\right)^2}{\sigma_Y^2}\right]$$

- We maximize the log likelihood function over the 22-dimensional parameter vector

Solving for the Objective Distribution

- Let $\Phi(x; w)$ denote the characteristic function for the density function $h(x)$,

$$\Phi\left(x; w\right) = \int_{-\infty}^{\infty} e^{iwx} h\left(x\right) dx$$

- Using the properties of characteristic functions

$$\Phi\left(v_T + w_T; w\right) = \frac{\Phi\left(u_T + w_T; w\right)\Phi\left(v_T; w\right)}{\Phi\left(u_T; w\right)}$$

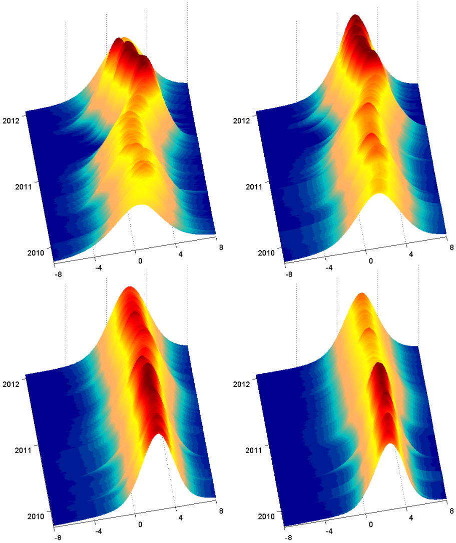
- Given this characteristic function $\phi(v_T + w_T)$, we recover the cumulative density function $\Psi(ln(I_T)/T)$ of the realized inflation rate using the Gil-Pelaez inversion integral,

$$\Psi\left(z\right) = \frac{1}{2} - \frac{1}{\pi}\int_{0}^{\infty} w^{-1}\mathrm{Im}\left[e^{-iwz}\phi\left(v_T + w_T; w\right)\right] dw$$

Maximum Likelihood Estimation of the Inflation Swap Model

Parameter	Value	Standard Error
κ	1.041346	0.477189
σ	0.037872	0.000544
α	0.089929	0.002182
β	3.540201	0.087404
η	0.006448	0.000007
λ	1.063634	0.005859
ϕ	0.000708	0.000001
γ	0.000001	0.000000
ρ	−0.1667560	0.004911
v_1	0.00001107	0.00000081
v_3	0.00000089	0.00000024
v_4	0.00000153	0.00000034
v_5	0.00000187	0.00000040
v_6	0.00000174	0.00000037
v_7	0.00000179	0.00000038
v_8	0.00000187	0.00000040
v_9	0.00000218	0.00000047
v_{10}	0.00000254	0.00000057
v_{12}	0.00000202	0.00000043
v_{15}	0.00000129	0.00000030
v_{20}	0.00000076	0.00000022
v_{25}	0.00000050	0.00000018

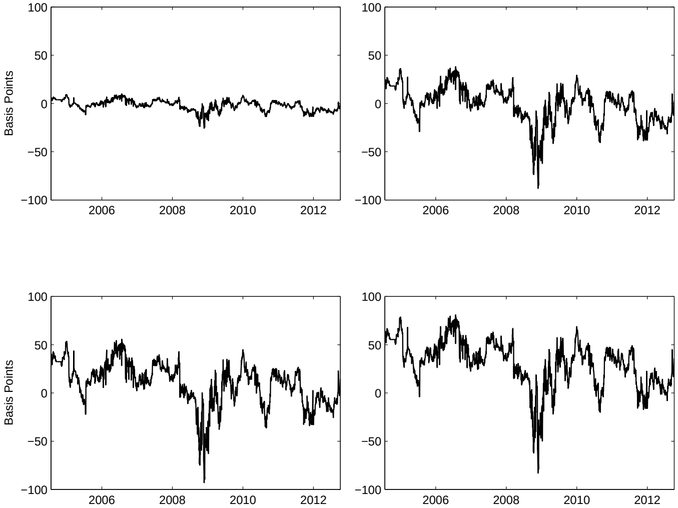
Inflation Densities



Summary Statistics for Inflation Risk Premia

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	−1.92	5.47	−25.66	−1.36	9.95	2141
2	−3.80	11.18	−54.46	−2.42	20.07	2141
3	−3.94	14.84	−72.08	−1.95	27.47	2141
4	−2.91	17.16	−82.27	−0.45	33.24	2141
5	−1.22	18.71	−88.07	1.50	38.05	2141
6	0.83	19.79	−91.26	3.78	42.27	2141
7	3.04	20.57	−92.86	6.11	46.05	2141
8	5.30	21.17	−93.48	8.44	49.50	2141
9	7.57	21.63	−93.46	10.77	52.70	2141
10	9.79	22.00	−93.04	13.09	55.67	2141
12	14.02	22.56	−91.04	17.44	61.10	2141
15	19.62	23.12	−88.63	23.11	67.72	2141
20	26.56	23.68	−84.40	30.20	75.78	2141
25	30.26	24.01	−82.32	33.95	80.15	2141
30	30.62	24.23	−83.04	34.37	80.96	2141

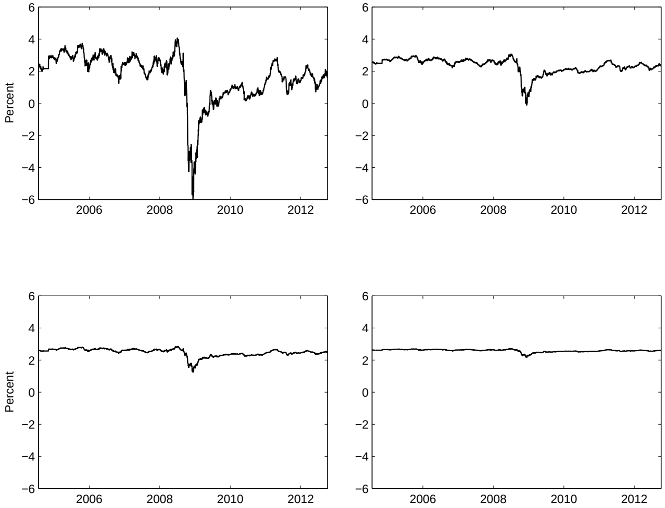
Inflation Risk Premia



Summary Statistics for Expected Inflation

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.776	1.348	−4.399	2.051	3.882	2141
2	1.968	1.031	−3.317	2.281	3.588	2141
3	2.127	0.775	−1.650	2.284	3.497	2141
4	2.248	0.597	−0.746	2.351	3.496	2141
5	2.336	0.475	−0.054	2.422	3.452	2141
6	2.395	0.386	0.496	2.468	3.427	2141
7	2.442	0.314	0.908	2.492	3.327	2141
8	2.477	0.259	1.143	2.514	3.199	2141
9	2.501	0.213	1.441	2.525	3.130	2141
10	2.523	0.177	1.595	2.536	3.143	2141
12	2.537	0.151	1.702	2.540	3.035	2141
15	2.543	0.134	1.652	2.547	2.960	2141
20	2.536	0.124	1.609	2.560	2.896	2141
25	2.545	0.117	1.761	2.571	2.776	2141
30	2.597	0.084	2.165	2.616	2.716	2141

Expected Inflation



Comparison of Survey Forecasts with Market-Implied Forecasts

Forecast Horizon	Survey	Survey Forecast	Market-Implied Forecast	N
1 Year	Michigan	2.91	1.79	99
	Bloomberg	2.53	1.79	99
	SPF	2.22	1.80	33
	Livingston	4.71	1.60	16
2 Years	SPF	2.32	1.99	33
	Livingston	3.53	1.67	8
3 Years	SPF	2.30	2.08	29
5 Years	Michigan	2.39	2.31	29
10 Years	Michigan	3.24	2.53	99
	SPF	2.49	2.53	33
	Livingston	2.44	2.52	16

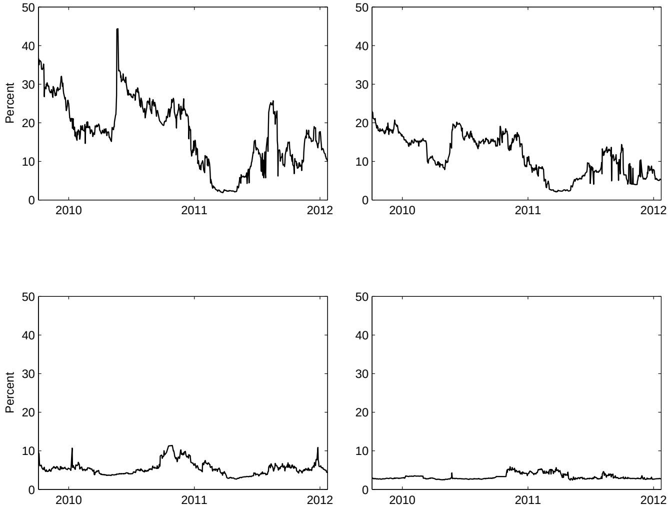
Summary Statistics for the Volatility, Skewness, and Kurtosis of the Inflation Distribution

Horizon	Average Volatility	Average Skewness	Median Excess Kurtosis	N
1	2.171	−2.376	3.095	572
2	2.258	−1.251	8.107	593
3	2.226	−1.115	13.390	500
5	2.165	−1.480	18.364	577
7	2.016	−1.230	9.789	434
10	1.831	−1.207	8.263	566
12	1.525	−0.913	6.165	531
15	1.241	−0.722	3.429	569
20	1.076	−0.513	1.873	488
30	0.693	−0.160	−0.006	215

Summary Statistics for Deflation Probabilities

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	17.25	8.75	1.88	17.57	44.37	572
2	11.44	5.29	2.07	11.33	23.04	593
3	4.28	1.94	1.33	3.63	8.89	500
5	5.34	1.60	2.67	5.17	11.39	577
7	2.93	0.62	1.89	2.80	5.93	434
10	3.29	0.77	2.40	2.90	5.86	566
12	2.28	0.20	1.95	2.25	3.31	531
15	2.97	0.28	2.62	2.93	4.61	569
20	2.32	0.12	1.98	2.32	3.31	488
30	2.33	0.17	1.85	2.32	3.35	215

Deflation Probabilities



Summary Statistics for the Pricing of Deflation Tail Risk

Horizon	Mean Ratio	Median Ratio
1	1.632	1.345
2	2.163	1.797
3	5.165	4.393
5	3.057	3.068
7	4.477	4.374
10	3.167	3.011
12	3.437	3.360
15	3.089	3.094
20	4.022	3.707
30	4.246	4.257
All	3.321	3.166

Regression of Daily Changes in Deflation Probabilities on Financial Tail Risk Variables

Horizon	Flight to Liquidity	Super Senior	Libor Spread	Swap Spread	VIX	Trsy Vol	SP 500 Return	Baa Spread	Trsy CDS	R^2	N
1	−0.19	1.89*	2.32**	−1.96**	−1.14	−1.31	−3.45**	1.96**	−1.79*	0.088	550
2	−0.41	2.75**	1.91*	−1.33	−2.02**	0.03	−3.16**	2.70**	−0.66	0.063	585
3	−1.04	0.30	1.40	−0.18	−0.21	−0.69	−0.67	−0.08	1.63	0.018	439
5	−0.95	1.55	−0.16	1.82*	−0.97	0.66	0.10	−1.02	1.05	0.034	563
7	−0.49	1.77*	−1.18	1.76*	−1.62	1.12	−1.39	−1.03	−0.70	0.091	364
10	−0.74	1.28	−0.05	2.33**	−1.04	−0.21	−0.74	−1.01	0.69	0.027	551
12	−0.31	0.75	0.32	1.25	0.80	−0.83	1.19	−1.54	1.37	0.015	495
15	0.25	1.05	0.26	0.61	0.26	−1.46	1.02	−1.69*	2.03**	0.024	554
20	2.01**	−0.54	−1.35	0.28	−0.69	1.09	−1.06	−1.57	1.47	0.030	439
30	0.77	−2.02**	−0.69	3.34**	−1.30	−0.10	−1.65*	−2.69**	0.55	0.078	203

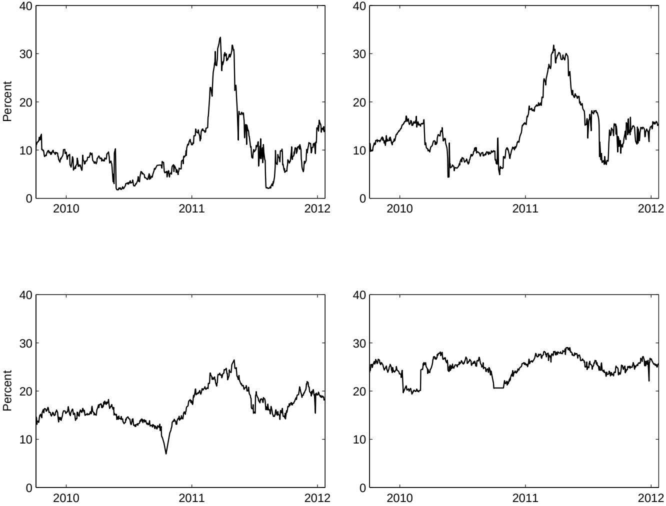
Regression of Monthly Changes in Deflation Probabilities on Financial and Macroeconomic Variables

Horizon	Super Senior	Swap Spread	Baa Spread	Indus Prod	Unempl	Cons Conf	R^2	N
1	2.24**	1.60	1.84*	0.76	1.63	0.70	0.540	28
2	2.10**	0.90	4.15**	0.65	1.88*	1.38	0.643	28
3	1.21	0.56	1.05	1.22	2.00*	1.04	0.432	26
5	−0.02	1.02	1.20	−0.90	−0.57	−0.95	0.336	28
7	1.07	1.49	0.67	0.02	1.10	0.99	0.239	26
10	−0.61	1.39	1.76*	0.64	0.27	−0.47	0.256	28
12	−0.71	1.64	2.03*	0.60	1.32	−1.02	0.423	26
15	−0.64	1.20	1.38	0.63	1.95*	0.88	0.385	28
20	1.95*	1.44	−0.71	−0.10	1.10	−0.06	0.388	26

Summary Statistics for the Probabilities of Inflationary Scenarios

	Probability Inflation > 4.00			Probability Inflation > 5.00			Probability Inflation > 6.00			
Horizon	Mean	Min.	Max	Mean	Min.	Max	Mean	Min.	Max	N
1	10.38	1.75	33.40	4.09	0.48	13.80	1.42	0.47	6.06	572
2	14.16	4.36	31.76	5.72	2.19	13.07	1.82	0.77	3.82	593
3	24.69	16.23	40.92	10.33	6.49	19.53	2.88	1.62	6.67	500
5	17.02	6.95	26.47	6.75	3.07	10.72	1.83	1.01	4.04	577
7	26.54	20.81	33.52	10.94	8.36	14.07	2.78	1.95	4.37	434
10	25.21	19.36	29.03	10.28	8.16	12.21	2.54	2.01	3.36	566
12	30.19	25.13	32.98	12.43	10.13	13.90	2.97	2.36	3.76	531
15	24.23	17.11	26.90	9.74	6.86	10.92	2.27	1.80	2.55	569
20	29.49	22.38	32.53	12.06	8.91	13.41	2.81	2.08	3.13	488
30	29.45	22.79	33.69	12.04	9.10	13.97	2.81	2.12	3.30	215

Inflation Probabilities



Summary Statistics for the Pricing of Inflation Tail Risk

Horizon	Mean Ratio	Median Ratio
1	1.386	1.349
2	1.462	1.420
3	1.137	1.239
5	1.741	1.722
7	1.399	1.393
10	1.359	1.377
12	1.362	1.401
15	1.505	1.556
20	1.665	1.700
30	1.758	1.749
All	1.463	1.441

Conclusion

- We solve for the objective distribution of inflation using the market prices of inflation swap and option contracts and study the nature of deflation risk.
- Market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.5 percent for horizons of up to 30 years.
- Deflation risk is priced by the market in a manner similar to that of other major types of tail risk such as catastrophic insurance losses or corporate bond defaults.
- Deflation risk is significantly related to measures capturing stress in the financial system and credit risk in the economy.
- Our results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major systemic shocks in the financial market.