Soft Shareholder Activism

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Abstract

This paper studies informal communications (voice) and exit as alternative ways through which shareholders can influence managers when obtaining control is not feasible or too costly. By focusing on the interaction between financial markets and the incentives of activist investors to communicate with managers, the analysis relates the effectiveness of voice and exit to market liquidity; entrenchment and compensation structure of managers; and expertise, liquidity and ownership size of activist investors. Importantly, I characterize the circumstances under which activist investors prefer to voice themselves publicly rather than engaging with managers behind the scenes.

Keywords: Shareholder Activism, Voice, Exit, Corporate Governance, Communication, Transparency, Cheap-Talk.

JEL Classification Numbers: D74, D82, D83, G23, G32, G34

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Introduction

Shareholder activism has many faces. Most notably, investors can seek control by accumulating a significant number of voting shares or by winning board seats in contested director elections. With control the activist investor can force the manager to take actions or replace him if needed. However, the implementation of these tactics can be very costly and their success is not guaranteed.[1] As a practical matter, launching a proxy fight or a hostile takeover is out of reach for many investors.

What else can investors do? They can always sell their shares if they are dissatisfied with the way the firm is managed. The threat of exit can in and of itself pressure managers to be more accountable.[2] However, in many cases, investors choose to voice their opinion about what firms should do. They can convey their message privately, trying to work the issues with the company "behind-the-scenes"; or publicly, informing the market about their criticism and demand. Whether these communications are private or public, they seem widespread. For example, Brav, Jiang, Partnoy and Thomas (2008) find that in 48.3% of the cases activist hedge funds declare that they "...intend to *communicate* with the board/management on a regular basis with the goal of enhancing shareholder value".[3] By sending letters, making phone calls or even meeting face to face with senior executives or board members, investors can express their view how to unlock what they believe is a hidden value. This can include the common activist goals of spinning off a division or a share buyback, but it can also be a recommendation on strategic or operational changes in the firm. Consistent with this view, the former U.S. SEC Chairman, Mary L. Schapiro, pointed in her speech from 2010 that "...boards can also benefit from access to the ideas and the concerns investors may have. Good communications can build credibility with shareholders and potentially enhance corporate strategies".[4]

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[1] The cost of intervention includes the fees of hiring lawyers, proxy advisors, investment banks, public relations, and advertising firms. Gantchev (2013) estimates that the average cost of a US public activist campaign ending in a proxy fight is $10.5 millions. Since most blockholders hold small stakes (La Porta, Lopez de Silanes, and Shleifer (1999)) these expenses are even more significant.

[2] Bharath, Jayaraman, and Nagar (2013), Edmans, Fang, and Zur (2013), and Parrino, Sias, and Starks (2003) find evidence that are consistent with exit as a form of governance.

[3] Becht, Franks, and Grant (2010), Becht, Franks, Mayer, and Rossi (2009), Carleton, Nelson, and Weisbach (1998), and McCahery, Sautner, and Starks (2011) also find evidence consistent with informal communications between investors and managers.

[4] See SEC website for "Speech by SEC Chairman: Remarks at the NACD Annual Corporate Governance

Informal communications and exit are alternative ways through which shareholders can influence managers,[5] and since these mechanisms do not require formal control and are relatively inexpensive, I refer to the combination of communication and exit as *soft shareholder activism*. The goal of this paper is to study the conditions under which soft shareholder activism can be an effective form of corporate governance.

Previous studies have analyzed models of communication and exit in isolation, however, to the best of my knowledge, the interaction between the two has not been studied. Apart from providing a realistic description of the toolkit that is at the disposal of many investors, studying these two mechanisms in the same framework can shed light on the interaction between financial markets and the ability of investors to influence managers by simply talking to them. As I argue below, this interaction can explain why managers would listen to investors and follow their recommendations even when the conflict of interest is significant. Importantly, by accounting for the market reaction, this framework allows me to study the choice of investors between private and public communications, a topic which was overlooked by the existing literature. As a whole, the analysis of this paper provides novel predictions about how the effectiveness of communications is related to various characteristics of the market, the activist investor and the target company.

To study this topic I develop a model of a publicly held firm with the following key ingredients. The manager of the firm is not necessarily maximizing shareholder value. While no shareholder can obtain control, there is an activist investor with private information that complements the manager's knowledge. The activist can communicate with the manager and advise him about the optimal course of action. To capture the informal nature of this interaction, communication (hereafter, voice) is modeled as cheap talk a la Crawford and Sobel (1982). In particular, the amount of information that can be communicated by the activist is determined endogenously. Given the manager's reaction to her recommendation, the activist decides whether to keep her shares and wait until the long-term value is realized, or exit by

Conference" October 19, 2010. Related, in her speech to the International Corporate Governance Network in July 2009, Chairman Schapiro noted that "Regulation FD does not restrict communications between companies and their shareholders" nor does it "prevent companies from seeking out and listening to the views of investors."

[5] The idea that exit and voice are alternative means of dealing with conflicts was originated in Hirschman (1970).

selling her stake in the open market. The price in the interim period is set fairly by a market maker, reflecting the available public information.

The first result shows that when the communication with the manager is *private* (that is, the recommendation is not observed by the market maker), voice and exit exhibit complementarity. In other words, voice is more effective with exit than without it. To understand this result, note that inevitably the activist will manipulate some of her information in order to overcome the conflict of interest between shareholders and the manager. Worrying about the credibility of the activist, the manager will often ignore her recommendations. However, with the option to exit, the activist can sell her shares at times she believes the share is over-priced. Since the share is likely to be over-priced when the manager makes bad decisions, exit reduces the sensitivity of the activist to this opportunistic behavior, and thereby increases her credibility from the manager's perspective. By relaxing the tension between the manager and the activist, exit enhances the activist's ability to influence the manager. This result can explain why in spite of agency problems, and even though managers do not have to listen, communications between investors and managers are common.

The complementarity between voice and exit as described above is *endogenously* determined by the market. Indeed, in equilibrium, the price the activist gets for her shares when she exits incorporates the rational expectations of the market about the communication between the activist and the manager, and in particular, any inefficiencies in the manager's decision. In other words, the activist's willingness to tolerate managerial inefficiencies is priced. Lower prices, however, reduce the incentives of the activist to exit, and therefore reduce her willingness to compromise with the manager. Through this channel the market endogenously puts constraints on the extent of complementarity between voice and exit. Importantly, because of this feedback from the market, parameters such as liquidity and the activist's expertise or stake size, have a non-trivial effect on voice. For example, the influence of the activist on the manager can *decrease* with the quality of the activist's private information. Essentially, higher quality of information increases the amount of information that can be potentially communicated by the activist, but it also intensifies the adverse selection when the activist exits. Adverse selection makes exit less attractive from the activist's point of view. Since voice and exit complement each other, the latter effect can dominate the former and overall harm the activist's credibility.

Different from the existing literature on exit (see the literature review below), here exit is a powerful form of shareholder activism even when managers are not sensitive to short-term stock performances. The channel through which exit works in this model is by enhancing the credibility of the activist's recommendation, and thereby her influence on managerial decisions. Nevertheless, in practice, the structure of executive compensation and career concerns often make managers sensitive to short-term performances. It turns out that the sensitivity of managers to prices interacts with voice and exit in a non-trivial way. In Section 3 I show that without voice, the effect of this sensitivity on shareholder value is ambiguous and can be negative. By contrast, when the ability of the activist to communicate with the manager is considered, the sensitivity of the manager to short-term prices unambiguously benefits shareholders. Intuitively, since exit has a negative impact on the stock price (it signals overvaluation), the manager tries to minimize the likelihood the activist exits. This is true with and without voice. However, with voice, this objective is translated into stronger incentives to follow the activist's recommendation, which in turn, enhances the incentives of the activist to communicate her private information to the manager. This intuition also demonstrates that the sensitivity of managers to short-term performances creates another novel channel through which voice and exit exhibit complementarity.

The framework of this paper emphasizes the importance of the two-way feedback between financial markets and the quality of direct communications between investors and managers. Importantly, the feedback from the market depends on the information that is publicly available at the time prices are determined. For example, the transparency of managerial decisions can change the formation of prices, and thereby, the activist's influence on the manager. Moreover, by choosing between private and public communications, the activist can change the information that is available to the market. The strategic disclosure of the activist's private information will not only affect prices, but also the ability of the activist to influence the manager's decision.

The second set of results of the paper relates to the role of transparency. In Section 4 I show that public communications are more effective than private communication if and only if the manager is sensitive to short-term stock performances and his decisions are observed by the market. How can transparency weaken the effectiveness of voice? Intuitively, when the communication with the manager is public, the activist loses credibility since both the market maker and the manager understand that the activist cannot resist the temptation to misrepresent her information in order to get the highest price possible when she decides to exit. For this reason, when communication is public, voice and exit may in fact exhibit substitution. However, public communications can also be beneficial. When the manager's decisions are observable by the market, the market maker can compare these decisions with the activist's recommendations. If the manager ignores the activist's recommendation, whether or not the activist exits, the price drops. Thus, ignoring the activist's recommendations imposes additional cost on the manager when communication is public. In those cases, if the manager is sensitive to short-term stock performances, public communications enhance the influence of the activist on the manager. Overall, another contribution of this paper is the characterization of investors' choice between private and public communications.

Finally, the analysis of this paper relates the effectiveness of soft shareholder activism to the characteristics of the target firm, the manager and the activist. In Section 5 I discuss the novel empirical predictions that follow from this analysis. In particular, I argue that the ability of the activist to influence the manager, and hence, the frequency of engagement between investors and managers or directors, depends on the liquidity of the market, the entrenchment and compensation structure of the manager, as well as on the expertise, liquidity and ownership size of the activist. Moreover, the analysis characterizes the circumstances under which activist investors prefer to voice themselves publicly rather than engaging with managers behind-the-scenes, and relates this choice to some of the characteristics that are mentioned above.

Relation to the Literature

Traditional models of shareholder activism focus on the benefits of corrective actions through direct intervention (for example, Shleifer and Vishny (1986), Kyle and Vila (1991), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Maug (1998), Kahn and Winton (1998), Bolton and von Thadden (1998), Noe (2002), Aghion, Bolton, and Tirole (2004), and Faure-Grimaud and Gromb (2004)). These studies share the idea that large shareholders are able to exercise *formal control* and either force the company's management to improve the value of the firm or do it themselves. By contrast, here I focus on *real control*. I show that even when formal control cannot be obtained or exercised by shareholders, activist investors can improve the value of the firm by communicating with managers and advising them how to make better decisions.[6]

Levit and Malenko (2011) also consider communication as a form of shareholder activism. They study non-binding voting for shareholder proposals as a mechanism through which shareholders can aggregate and communicate their opinions. Related, Harris and Raviv (2010) study the optimal allocation of control between shareholders and managers when strategic communication and strategic delegation are possible. In Cohn and Rajan (2013) the activist can make recommendations to the board of directors, and the board arbitrates between the activist and the manager. However, they do not consider strategic communication. Importantly, the three studies that are mentioned above do not consider the possibility of exit or trade, a theme which is central in the present paper.

By relating liquidity and exit to voice, the present paper also contributes to the literature on governance and liquidity. Coffee (1991), Bhide (1993) and Aghion, Bolton, and Tirole (2004) argue that because voice and exit are mutually exclusive, liquidity is harmful as it allows a shareholder to exit rather than intervene. By contrast, Kyle and Vila (1991), Kahn and Winton (1998), and Maug (1998) show that liquidity can encourage voice by enabling a block to form in the first place. Maug (1998) and Faure-Grimaud and Gromb (2004)) show that liquidity can encourage voice by allowing the blockholder to earn trading gains from intervention. None of these studies, however, consider exit as a form of governance and voice as a form of communication, and for this reason the source of complementarity between voice and liquidity is inherently different in the present paper.

Admati and Pfleiderer (2009) and Edmans (2009) point out that exit can be an effective form of governance in and of itself. However, unlike the present study, their argument crucially relies on the sensitivity of managers to short-term stock performances. Building on these two models, Dasgupta and Piacentino (2011) and Edmans and Manso (2011) consider exit and voice in the

[6] In this respect, this paper is also related to the literature on formal versus real authority in organizations (e.g., Aghion and Tirole (1997)).

same framework. Dasgupta and Piacentino (2011) also point out that exit can complement voice. However, both of these studies assume that investors have formal control and can affect the value of the firm through direct interventions.

More broadly, the existing literature on the real effects of financial markets identifies two channels through which the share price affects managerial decisions: compensation and learning (see Bond, Edmans, and Goldstein (2012) for a survey). In this respect, the present paper contributes to this literature by identifying a new channel. Even though the manager's compensation does not depend on the short-term stock price, and there is no learning from prices (the activist exits only after the manager makes his decision), still there is an effect: the option of the activist to exit by selling her shares for the endogenously-determined price enhances the activist's credibly when she communicates with the manager, and thereby affects managerial decisions and firm value.

Finally, the analysis of this paper is related to the literature of strategic transmission of information. Unlike Crawford and Sobel's (1982) canonical model of cheap talk, here the sender (activist) has an outside option whose value is determined endogenously by the decision maker (manager) and a third party (market maker). Che, Dessein, and Kartik (2013) and Shimizu (2008) also study models of strategic communication with outside options. However, in both cases, the outside option is exogenous. As was suggested above and is explained in more details below, the endogenous nature of the outside option in the present model (short-term prices) not only determines the amount of information that can be communicated, but also the channel (private versus public) through which information can be most effectively communicated.

1 Baseline Model - Setup

A public firm has to choose between two business strategies, L and R. If strategy $a \in \{L, R\}$ is implemented, the long-term shareholder value is given by

$$v(\theta, a) = \theta \cdot 1_{\{a=R\}} - \theta \cdot 1_{\{a=L\}}. \tag{1}$$

Random variable θ has a continuous probability density function f and full support over $\left[\underline{\theta}, \overline{\theta}\right]$ where $\underline{\theta} < 0 < \overline{\theta}$. According to (1), the value of each strategy depends on θ, and shareholder value is maximized if strategy R is implemented when $\theta > 0$ and strategy L is implemented when $\theta < 0$.

While shareholders own the cash flows rights of the firm, the manager has the formal authority to decide on the strategy. Throughout the analysis I do not distinguish between the board of directors and the manager. The preferences of the manager are given by

$$u_M = v\left(\theta + \beta, a\right), \tag{2}$$

where $\beta \in \left(\max\left\{0, -\mathbb{E}\left[\theta\right]\right\}, -\underline{\theta}\right)$. According to (2), the manager's policy does not necessarily maximize shareholder value. The manager implements strategy R if and only if he believes $\theta \geq -\beta$. When $\theta \in [-\beta, 0)$ shareholders strictly prefer strategy L, but there is a risk that the manager chooses strategy R. Parameter β captures the manager's bias toward strategy R, and a higher β means a higher bias. As I show below, the assumption $\beta > -\mathbb{E}\left[\theta\right]$ guarantees that without more information the manager always chooses $a = R$. The challenge of shareholders is convincing the manager to choose strategy L when $\theta < 0$.

The ownership structure of the firm consists of dispersed shareholders and an activist investor whose stake in the firm is given by $\alpha > 0$. The activist investor corresponds to blockholders such as hedge funds or other institutional investors. Dispersed shareholders have no ability or incentives to discipline the manager and hence remain passive. The focus of the analysis is on the ability of the activist to influence the manager's decision. The activist, however, does not have and cannot obtain formal control, and thus, the manager cannot be forced to take actions. Presumably, the cost of initiating and executing proxy contests or hostile takeovers is too high. Instead, I study the ability of the activist to communicate her own view to the manager and persuade him to follow her recommendation.

The key assumption of the model is that the activist has a piece of information that the manager does not have. To emphasize the channel of communication and for simplicity, I assume that the activist privately observes θ while the manager is uninformed about θ. In Section

2.2.1, I relax the assumption that the manager has no private information, and in Section 2.2.2, I relax the assumption that the activist has perfect information. The assumption about the informational advantage of the activist is related to a broad literature on how corporate insiders may learn value-relevant information from outsiders.[7] This assumption, however, does not mean that the activist knows more than the manager on every aspect. It only requires that the activist can add to the knowledge of the manager or the directors (who are typically less informed than the manager) in one area with a significant impact on the value of the firm. For example, activist hedge funds often use their expertise in strategy, operations and financing to advise their portfolio companies on these issues. Consistent with this view, Becht, Franks, and Grant (2010) show that even when obtaining control is not possible activist funds realize significant returns by advising their portfolio companies.

After the activist observes θ and before the manager decides on the strategy, the activist can communicate with the manager by sending him a *private* message $m \in \left[\underline{\theta}, \overline{\theta}\right]$. The activist's private information is non-verifiable and her recommendation m cannot be backed-up. Moreover, the content of m does not affect the activist's payoff directly but only through its effect on the manager's decision. This assumption captures the informal nature of the communication between the activist and the manager, and it implies that there are no private benefits or costs from communication per-se. I denote by $m(\theta)$ the message the activist sends conditional on θ, and by $a(m) \in \{R, L\}$ the manager's decision conditional on observing message m. Formally, communication is modeled as cheap talk a la Crawford and Sobel (1982).[8]

After communicating with the manager the activist can trade with a competitive and risk neutral market maker. With probability $\delta \in [0, 1]$ the activist is hit by a liquidity shock and she is forced to sell her entire stake in the firm. The activist's needs for liquidity are independent of θ. If the activist has no liquidity shortage, she is free to choose whether to exit or keep her

[7] For example, Holmstrom and Tirole (1993) argue that stock prices provide information about the manager's actions and are therefore useful for managerial incentive contracts. Levit and Malenko (2011) analyze nonbinding voting for shareholder proposals and show that the information that is conveyed by voting outcome can affect corporate decision makers. Marquez and Yilmaz (2008) examine tender offers where shareholders have information about the firm value that the raider does not have. In Dow and Gorton (1997), Foucault and Gehrig (2008), and Goldstein and Guembel (2008), firms use information in stock prices to make investment decisions.

[8] For simplicity, the analysis considers only pure strategy equilibria.

holdings in the firm. The important assumption is that the activist can exit by selling her entire stake in the firm before the long-term value is realized. Allowing the activist to buy shares or to gradually sell her stake will not change the main results of the paper. I denote by $s = 1$ the decision of the activist to exit and sell her shares and by $s = 0$ her decision to keep them. When the activist exits even though she does not have to, we say that the activist strategically exits. To save on notation, I assume $s = 1$ whenever the activist is subject to a liquidity shock.

The decision of the activist to exit is observed by the market maker. The market maker, however, is uninformed about θ and he does not observe the manager's decision, the message the activist sent the manager or the activist's needs for liquidity. In Section 4 I relax some of these assumptions and consider different modes of transparency. Based on the available public information, and in particular, the decision of the activist to exit, the market maker sets the short-term price of the share to be the expected value of the firm. I denote this price by $p(s)$. Overall, the activist's preferences are given by

$$u_A = \alpha \times [sp(s) + (1 - s) v(\theta, a)] . \tag{3}$$

To summarize, there are four periods in the model. Initially, before the activist observes her liquidity needs but after she becomes informed about θ, the activist sends the manager a private message m. At period 1, after observing message m, the manager decides between strategy R and strategy L. At period 2, the activist observes her liquidity needs and the manager's decision, and then decides whether to exit. The market maker observes the decision of the activist to exit and determines the stock price accordingly. Finally, at period 3, the outcome is realized and becomes public. All agents are risk neutral and preferences are common knowledge.

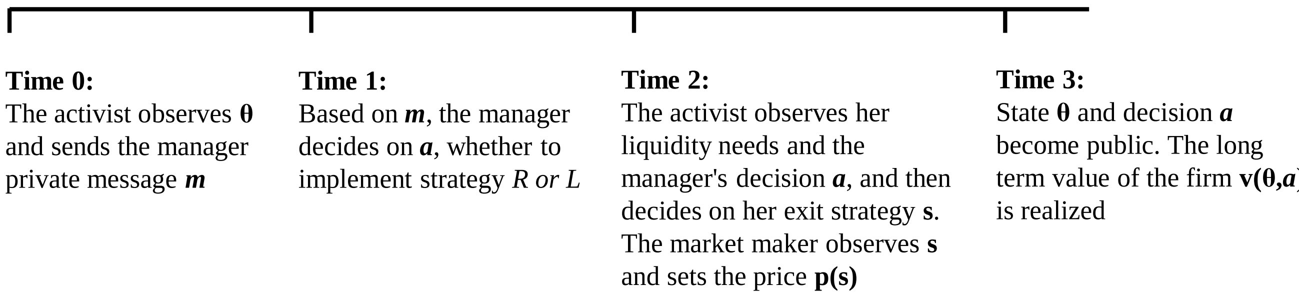

Figure 1 - Timeline

2 Analysis

Consider the set of Perfect Bayesian equilibria of the game. According to (2), the manager has no direct utility from the short-term price $p(s)$. For this reason, the manager's decision is not directly affected by the activist's decision to exit.[9] Let $\mathbb{E}[\theta|m]$ be the manager's expectations of θ conditional on observing message m. Note that $\mathbb{E}[\theta|m] = \mathbb{E}[\theta|\theta \in \varepsilon(m)]$ where $\varepsilon(m) = \{\theta : m(\theta) = m\}$. It follows from (2) that in any equilibrium the manager implements strategy L if and only if,

$$\beta \leq -\mathbb{E}[\theta|m] \tag{4}$$

Condition (4) is central for the analysis that follows. Importantly, according to (4), the share price has no *direct* effect on the manager's decision. However, as I show below, it will have an *indirect* effect through the channel of communication. This feature is one aspect by which this model departs from the existing literature.

To study the conditions under which voice is an effective form of shareholder activism, I focus on equilibria in which the activist reveals information about θ and the manager condition his decision on this information. I name equilibria with this property as *influential.* When

[9] Since the activist observes the manager's decision, off-equilibrium events are possible. However, the manager's utility is invariant to $p(s)$, and hence, the activist's off-equilibrium beliefs or actions cannot change the set of equilibria.

the equilibrium is influential, there are at least two different messages the activist sends the manager, these messages convey different information and trigger different decisions by the manager. If the equilibrium is not influential, the manager ignores all messages from the activist, and the activist cannot influence the manager's decision by communication.[10]

Definition 1 *An equilibrium is **"influential"** if there exist $m_1 \neq m_2$ such that $\varepsilon(m_1)$ and $\varepsilon(m_2)$ are not empty, $\mathbb{E}[\theta|m_1] \neq \mathbb{E}[\theta|m_2]$ and $a(m_1) \neq a(m_2)$.*

In our context, an equilibrium is considered more efficient if it generates a higher ex-ante shareholder value. Note that since the share price is set fairly by the market maker, even though the activist decides strategically when to exit, in any equilibrium the activist's expected utility equals the expected shareholder value. The analysis will focus on the most efficient equilibrium. For this purpose it is useful to define a *threshold equilibrium*. An influential equilibrium is also a threshold equilibrium if the manager chooses strategy R if and only if $\theta \geq \tau$, where $\tau \in (\underline{\theta}, \overline{\theta})$. Hereafter, we focus the analysis on threshold equilibria. The next lemma shows that in the search for efficiency the focus on threshold equilibria is without the loss of generality. All the omitted proofs, including the proof of Lemma 1, are collected in the Appendix.

Lemma 1 *For any influential equilibrium there is a threshold equilibrium which is more efficient.*

2.1 Benchmarks

To study the interaction between exit and voice I start by considering two benchmarks. In the first benchmark the activist can exit but she cannot voice herself. In the second benchmark the activist can voice herself but she cannot exit.

[10] Equilibria in which the manager responds to different messages that convey the same information are not considered influential according to Definition 1.

2.1.1 Benchmark I - Exit without Voice

Suppose by assumption the activist cannot communicate with the manager. The manager has no information about θ before he makes a decision. Since $\beta \geq -\mathbb{E}[\theta]$, according to (4), the manager chooses $a = R$. The activist understands that without her recommendation the value of the firm is θ. The activist exits either because she needs liquidity or because $\theta \leq p(1)$. In the latter case, the activist sells her shares since the shares are over-priced. The market maker correctly anticipates the behavior of the manager and the behavior of the activist, and sets the share price accordingly. Therefore, in any equilibrium of this benchmark the price upon exit, $p(1)$, must be a solution of

$$p = \frac{\delta \mathbb{E}[\theta] + (1-\delta) \Pr[\theta \leq p] \mathbb{E}[\theta | \theta \leq p]}{\delta + (1-\delta) \Pr[\theta \leq p]} \tag{5}$$

Intuitively, if the activist exits because she needs liquidity then the fair value of the firm is $\mathbb{E}[\theta]$. If the activist exits because the share is over-priced then the fair value of the firm is $\mathbb{E}[\theta | \theta \leq p]$. The right hand side of (5) is the weighted average of these valuations. Because of the adverse selection in the activist's decision to exit, the market maker forms the "worst case" beliefs when $s = 1$. For this reason, the solution of (5) always exists, it is unique and is given by the global minimum of the right hand side of (5). Note that the activist keeps her shares if and only if she does not need liquidity and $\theta > p(1)$. Therefore, $p(0) = \mathbb{E}[\theta | \theta > p(1)]$. Hereafter, whenever there is no risk of confusion I refer to $p(1)$ as p.

The equilibrium of this benchmark also corresponds to the non-influential equilibrium of the game. As in any cheap talk game, this equilibrium always exists even when communication is allowed. The next lemma characterizes this equilibrium.

Lemma 2 *A non-influential equilibrium always exists. In any non-influential equilibrium the manager chooses strategy R and the activist exits if and only if she needs liquidity or $\theta \leq p(1)$, where $p(1)$ is unique and given by the global minimum of the right hand side of (5).*

2.1.2 Benchmark II - Voice without Exit

Suppose by assumption the activist cannot exit ($s = 1$ is not allowed). According to (3), without the option to exit the activist's utility is proportional to $v(\theta, a)$. Consistent with maximizing long-term shareholder value, the activist would like the manager to choose strategy R if and only if $\theta \geq 0$. However, the activist cannot force the manager to follow this decision rule, and hence, the manager must have the incentives to do it. Can the activist use her private information to convince the manager to follow her recommendation?

Lemma 3 *An influential equilibrium without exit exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < 0]$. Moreover, in any influential equilibrium without exit, the activist recommends on strategy R if $\theta \geq 0$ and on strategy L otherwise; the manager follows these recommendations.*

Since the manager is biased toward strategy R, the manager always follows the activist's recommendation to choose strategy R. The challenge of the activist is persuading the manager to implement strategy L. Recall that if the manager is following the activist's recommendation, the activist would recommend on strategy L if and only if $\theta < 0$. According to (4), if the manager learns that $\theta < 0$ then choosing strategy L is optimal from his perspective if and only if $\beta \leq -\mathbb{E}[\theta|\theta < 0]$. It follows, when $\beta \leq -\mathbb{E}[\theta|\theta < 0]$ there is an equilibrium in which the manager follows the activist's recommendation.

Interestingly, when $\beta \leq -\mathbb{E}[\theta|\theta < 0]$ information is not fully revealed by the activist in equilibrium. If on the contrary information is fully revealed, the manager would implement strategy R if and only if $\theta \geq -\beta$. Instead, the activist has incentives to introduce noise into the communication with the manager. The activist can reveal whether θ is greater or smaller than zero, but not the exact value of θ. By pooling very low realizations of θ with intermediate realizations of θ, the activist is able to persuade the manager to choose strategy L even when $\theta \in [-\beta, 0)$.[11]

When $\beta > -\mathbb{E}[\theta|\theta < 0]$ the activist's insistence on the implementation of the first best decision rule results in a complete breakdown of communication. The manager expects the

[11] More generally, in the Appendix I show that with voice and exit, if a fully revealing equilibrium exists then there is a threshold equilibrium which is strictly more efficient.

activist to recommend on strategy L if and only if $\theta < 0$, and hence, when the manager receives a recommendation to implement strategy L he believes that θ is on average $\mathbb{E}[\theta|\theta < 0]$. Since $\beta > -\mathbb{E}[\theta|\theta < 0]$ the manager prefers strategy R over strategy L in those cases. It follows, when $\beta > -\mathbb{E}[\theta|\theta < 0]$ there is no equilibrium in which the manager follows the activist's recommendation.

As one might expect, the manager follows the activist's recommendation in equilibrium only if the conflict of interest between them is relatively small. Importantly, the threshold below which an influential equilibrium exists is determined by the activist's desire to implement the first best decision rule, $\tau = 0$. In the next section I show that with exit the activist does not necessarily insist on implementing the first best, and for this reason, the activist is able to exercise more influence on the manager by using her voice.

2.2 Voice and Exit

The focus of this section is on the interaction between voice and exit. The consideration of voice does not change the existence and nature of the non-influential equilibrium that is characterized by Lemma 2. However, the possibility of exit does change the existence and nature of the influential equilibrium that is described by Lemma 3.

Suppose in equilibrium the manager follows threshold τ, where τ can be different from zero. Let $v(\theta, \tau)$ be the expected value of the firm under decision rule τ. That is, $v(\theta, \tau) = \theta$ if $\theta < \tau$ and $v(\theta, \tau) = -\theta$ otherwise. Given the implementation of threshold τ, the activist exits if and only if she needs liquidity or the share is over-priced, that is, $v(\theta, \tau) \leq p(1)$. Since the market maker has rational expectations about the manager's decision rule and the activist's exit strategy, the price of the share upon exit must be the solution of $p = \varphi(p, \tau)$ where

$$\varphi(p, \tau) = \frac{\delta \mathbb{E}[v(\theta, \tau)] + (1-\delta) \Pr[v(\theta, \tau) \leq p] \mathbb{E}[v(\theta, \tau) | v(\theta, \tau) \leq p]}{\delta + (1-\delta) \Pr[v(\theta, \tau) \leq p]}. \tag{6}$$

The interpretation of (6) is similar to the interpretation of (5). The only difference is that in (6) the conditional expectations are taken with respect to random variable $v(\theta, \tau)$ instead of θ. Indeed, the right hand side of (5) is a special case of (6) when $\tau = \underline{\theta}$. Following a reasoning similar to the one behind Lemma 2, the unique solution of $p = \varphi(p,\tau)$, denoted by $\pi(\tau)$, is the global minimum of $\varphi(\cdot,\tau)$. Thus, the price upon exit is given by $\pi(\tau)$. Note that the inefficiency of the manager's decision rule increases with the distance of τ from zero. Since the share price reflects the fair value of the firm under the expectations that the manager implements threshold τ, $\pi(\tau)$ is increasing with τ if and only if $\tau < 0$.

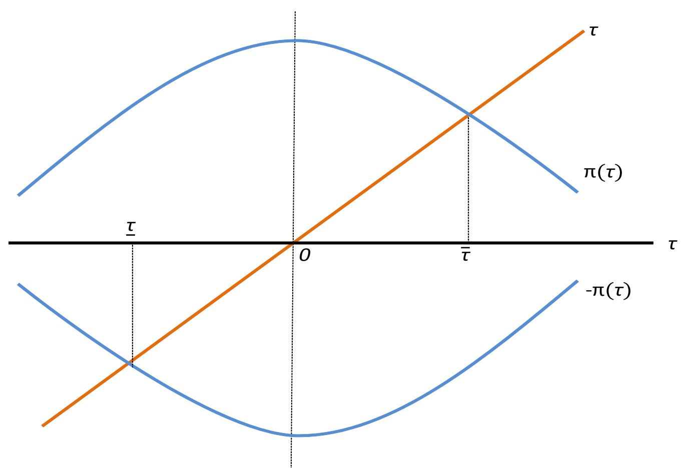

Figure 2

The discussion above demonstrates that prices are affected by the (private) communication between the activist and the manager. This is reflected by the dependence of $\pi(\tau)$ on τ. However, market prices also affect the incentives of the activist to communicate with the manager and persuade him to implement threshold τ. To understand this channel, consider the conditions under which the activist is willing to recommend the manager to follow threshold τ (which does not have to equal zero), if she expects the manager to follow her recommendations. Note that by exiting the activist can guarantee herself a payoff of p, where p is the price upon exit. Therefore, if $p < |\theta|$ the activist has strict incentives to keep her shares (unless she needs liquidity): when $\theta > \max\{0,p\}$ the activist recommends on strategy R and gets $v(\theta,R) = \theta > p$, and when $\theta < \min\{0,-p\}$ the activist recommends on strategy L and gets $v(\theta,L) = -\theta > p$. However, if $|\theta| \leq p$ then no matter which strategy the manager implements, the activist is better off by exiting. In other words, the manager's decision does not affect the activist's payoff when $|\theta| \leq p$. For this reason, the activist is willing to recommend on threshold

τ as long as $-p \leq \tau \leq p$.

The feedback between the determination of prices by the market maker and the communication between the activist and the manager constrains the set of feasible outcomes in equilibrium. In particular, a threshold $\tau \neq 0$ can be supported in equilibrium only if

$$-\pi(\tau) \leq \tau \leq \pi(\tau). \tag{7}$$

As was argued above, when $-\pi(\tau) \leq \theta \leq \pi(\tau)$ the activist decides to exit whether or not she needs liquidity, and hence, she is willing to recommend on a threshold that deviates from shareholders' first best long-term strategy.[12] Figure 2 illustrates that there are unique thresholds $\underline{\tau} < 0 < \overline{\tau}$ such that condition (7) is equivalent to

$$\underline{\tau} \leq \tau \leq \overline{\tau}. \tag{8}$$

Finally, threshold τ can be supported in equilibrium only if the manager has incentives to follow the activist's recommendation to implement strategy L when $\theta < \tau$ and strategy R when $\theta \geq \tau$. Recall the binding constraint is convincing the manager to implement strategy L when $\theta < \tau$. Similar to discussion that follows Lemma 3, if the manager expects the activist to recommend on the implementation of threshold τ, he will follow her recommendations if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \tau]$. Importantly, higher bias β requires a lower threshold τ in order to persuade the manager to follow the activist's recommendation to choose strategy L. Overall, threshold τ can be implemented in equilibrium only if

$$\beta \leq -\mathbb{E}[\theta|\theta < \tau] \tag{9}$$

Combined, threshold τ can be supported in equilibrium if and only if both conditions (8) and (9) hold. The next proposition summarizes the observations above and provides necessary and sufficient conditions for the existence of an influential equilibrium with voice and exit.

[12] Note that condition (7) also requires $\pi(\tau) \geq 0$. Indeed, in any influential equilibrium the price upon exit must be positive. Otherwise, the activist is better off recommending the manager to implement threshold $\tau = 0$ and never exit unless she needs liquidity. However, $\tau = 0$ implies that the value of the firm is positive for any θ, and therefore, $p < 0$ is inconsistent with the fair value of the firm.

Proposition 1 *Let $\underline{\tau}$ be the unique negative solution of $-\pi(\tau) = \tau$. An influential equilibrium exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$.*

Proposition 1 implies that when $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$ an influential equilibrium exists. According to Lemma 1, if an influential equilibrium exists then a threshold equilibrium must also exist. Let $z(\beta)$ be the solution of $\beta = -\mathbb{E}[\theta|\theta < -z]$ and note that condition (9) holds if and only if $\tau \leq -z(\beta)$.[13] The discussion that precedes Proposition 1 implies that when $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$ any threshold in $[\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$ can be supported in equilibrium, and the most efficient influential equilibrium is an equilibrium with threshold $\min\{0, -z(\beta)\}$.[14] As expected, shareholder value under the most efficient equilibrium decreases with β.

The comparison between Proposition 1 and Lemma 3 reveals that while the definition of an influential equilibrium is invariant to the exit strategy of the activist, due to the interaction between voice and exit, the existence of an influential equilibrium does depend on the ability of the activist to exit.

In particular, Figure 3 shows that voice is more effective with exit than without it. In this respect, voice and exit exhibit *complementarity.* When $\beta \leq -\mathbb{E}[\theta|\theta < 0]$ the manager's bias is relatively small and the first best decision rule can be obtained in equilibrium whether or not the activist can exit. When $-\mathbb{E}[\theta|\theta < \underline{\tau}] < \beta$ the manager's bias is relatively large and an influential equilibrium does not exist regardless of the activist's ability to exit. In both cases, shareholder expected value in the most efficient equilibrium is invariant to the ability of the activist to exit. However, when $-\mathbb{E}[\theta|\theta < 0] < \beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$ an influential equilibrium exists if and only if the activist can exit, and when the activist can exit, shareholder expected value in the most efficient equilibrium is given by $\mathbb{E}[v(\theta, -z(\beta))] \in (\mathbb{E}[\theta], \mathbb{E}[|\theta|])$.

[13] The function $z(x)$ is defined on $[-\mathbb{E}[\theta], \infty)$ and it has the following properties: (i) $z(x) < x$; (ii) $z(x)$ strictly increases in x; (iii) $\lim_{x \to -\underline{\theta}} z(x) = -\underline{\theta}$; (iv) $z^{-1}(\tau) = -\mathbb{E}[\theta|\theta < -\tau]$.

[14] Note that $\min\{0, -z(\beta)\} \in [-\beta, 0]$ and a threshold equilibrium is Pareto Efficient if and only if $\tau \in [-\beta, 0]$.

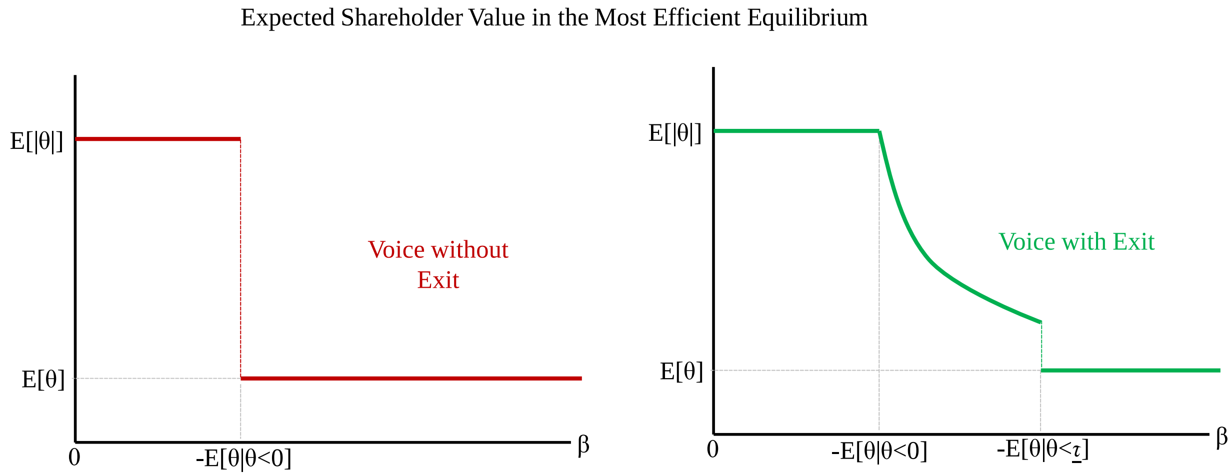

Figure 3

What is the intuition behind the complementarity between voice and exit? When the activist can exit, her payoff becomes less sensitive to the long-term performances of the firm. The activist is more willing to tolerate managerial inefficiencies. Instead of insisting on the implementation of the first best, the activist is also willing to support thresholds in the range $[\underline{\tau}, 0)$. For this reason, the manager views the recommendations of the activist as being more credible and he is more likely to follow them. Overall, more information can be communicated in equilibrium.[15]

The complementarity between voice and exit can also be seen by noting that shareholder value under the most efficient equilibrium increases with δ, the frequency of the activist's liquidity shock. Essentially, when δ is high the activist is less likely to exit because the share is over-valued, and the negative price impact of exit is diminished. Favorable terms of trade

[15] Alternatively, one can focus on the Pareto Efficient equilibrium that maximizes the expected utility of the manager instead of shareholders value. It can be shown that under this alternative selection an influential equilibrium is still more likely to exist with exit than without it. In this sense, voice and exit continue to exhibit complementarity. However, in terms of welfare, shareholder value is higher with exit than without exit if and only if $-\mathbb{E}[\theta|\theta < 0] < \beta$. The intuition is similar expect from cases where $\beta \leq -\mathbb{E}[\theta|\theta < 0]$. In those cases, the activist's insensitivity due to exit can be exploited by the manager to advance her agenda on the expense of shareholders. Note, however, this can happen only when the inherent conflict of interest β is relatively small.

make exit more attractive, and similar to the intuition above, voice becomes more effective. In the Appendix I show that as $\delta \to 0$ the conditions under which an influential equilibrium exists converge to the conditions of the benchmark case of voice without exit, that is, $\underline{\tau} \uparrow 0$ and $\overline{\tau} \downarrow 0$.

2.2.1 Informed Manager

The key assumption of the model is that the activist has a piece of information that the manager does not have. For simplicity, it is assumed that the manager is uninformed. However, the main results extend to a setup where the manager also has private information.

For example, suppose the manager privately observes θ with probability η, and whether the manager is informed or uninformed is his own private information. As in the baseline model, the uninformed manager follows the activist's recommendation to implement threshold τ only if $\beta \leq -\mathbb{E}\left[\theta | \theta < \tau\right]$. By contrast, the informed manager ignores the activist's recommendation and always implements strategy R if and only if $\theta \geq -\beta$. While the activist does not know whether the manager is informed, she understands she cannot change the decision of the informed manager. Effectively, the activist communicates as if she is facing just the uninformed manager. For this reason, most of the results of the baseline model extend to this setup, and in particular, voice and exit continue to exhibit complementarity.

When an influential equilibrium exists, the informed manager implements threshold $-\beta$ and the uninformed manager implements threshold τ. Recall that the most efficient threshold that can be implemented by the uninformed manager in equilibrium is $\min\left\{0, -z\left(\beta\right)\right\}$ where $-z\left(\beta\right) > -\beta$. It follows, shareholder value and the share price in the most efficient influential equilibrium decrease with the probability that the manager is informed. Since low prices increase the activist's sensitivity to the long-term performances of the firm, we conclude that an influential equilibrium is less likely to exist when the manager is privately informed.[16]

[16] It can be shown that in equilibrium the price upon exit p is a weighted average of $\varphi\left(p, -\beta\right)$ and $\varphi\left(p, \tau\right)$, and the weight on $\varphi\left(p, -\beta\right)$ increases with η.

2.2.2 Activist's Expertise and Block Size

Communication of private information and trading based on private information are the key channels through which activism is exercised in this paper. Thus, the quality of the activist's private information plays an important role in the analysis. In the Appendix I show that when $-\mathbb{E}[\theta|\theta < \underline{\tau}] < \beta < -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]]$ there is an inverted U-shape relation between the quality of the activist's private information and the effectiveness of voice. In other words, the ability of the activist to communicate with the manager and thereby increase the value of the firm can *decrease* with the quality of her private information.

Seemingly, with a higher quality of private information the activist has more opportunities to persuade the manager to take decisions that maximize shareholder value. This intuition is correct. However, higher quality of private information can also intensify the adverse selection between the activist and the market maker. When the activist is perceived to be better informed, the market maker interprets exit as a stronger indiction that the share is over-priced, and consequently, exit becomes less attractive from the activist's point of view. Since exit and voice complement each other, the latter effect can dominate the former, and in those cases, higher quality of private information harms the ability of the activist to influence the manager.[17]

In order to get private information the activist may need to invest resources. In the Appendix I show that when information is costly, the amount of information the activist acquires in equilibrium and the effectiveness of voice can *increase* with the cost of information. Without a commitment mechanism, a low cost of acquiring information exacerbates the adverse selection problem since the activist is tempted to acquire a significant amount of information. As was explained above, the expectations that the activist will be well informed inevitably harms her ability to influence the manager. Since information is less valuable if the activist cannot use it to influence the manager, the activist ends up acquiring less private information in equilibrium.

The cost of acquiring information (per share) tends to decrease with the number of shares the activist owns, α. Thus, the analysis suggests that small blockholders can be more effective than large blockholders when voicing themselves. Essentially, small share-holdings is a commitment

[17] In a model of cheap talk communication without outside options, Fischer and Stocken (2001) show that higher quality of sender information can provide the sender with more opportunities to mislead the decision maker, and thereby, decrease the amount of information that can be revealed in equilibrium.

tool to remain relatively uninformed, which increases the effectiveness of voice due to a weaker adverse selection in exit. This could be another explanation of why some investors choose to limit the size of their initial holdings in the firm.

3 Managerial Sensitivity to Short-Term Performances

Managers are often concerned about the short-term performances of their company's stock. Short-term concerns arise when the executive compensation package includes stocks and options that can be cashed out at an interim period, or when managers try to demonstrate executive talent and thereby increase the likelihood of keeping their job or being promoted. In this section I study how these concerns affect the interaction between exit and voice. For this purpose, I modify the preferences of the manager as follows,

$$u_M = \omega p\left(s\right) + v\left(\theta + \beta, a\right). \tag{10}$$

Parameter $\omega \geq 0$ is the relative weight the manager puts on the short-term stock price.

Recall the activist exits whenever she believes the share is over-valued. For this reason, exit always conveys bad news for the company. When $\omega > 0$, the share price has a direct effect on the manager, and therefore, the possibility of exit can pressure the manager to be more accountable to shareholders.

Lemma 4 *A non-influential equilibrium exists for any $\omega \geq 0$. A non-influential equilibrium in which the manager chooses strategy R with probability one exists if and only if* $\Pr\left[\theta \geq \pi\left(\underline{\theta}\right)\right] \geq \Pr\left[\theta \leq -\pi\left(\underline{\theta}\right)\right]$ *or $\omega \leq \hat{\omega}$ where $\hat{\omega} \in (0, \infty)$.*

The set of non-influential equilibria changes with ω. In particular, when $\omega > 0$ a non-influential equilibrium does not have to be unique, and in contrast to Lemma 2, the manager may choose strategy L with a strictly positive probability. The reason for this change is that the manager tries to support the short-term share price by minimizing the probability the activist exits. Recall the activist observes the manager's decision before she decides whether to exit.[18]

[18] Since in equilibrium the activist can perfectly predict the manager's decision, assuming that the activist

When ω is large, the desire to minimize the likelihood the activist exits can outweigh the bias toward strategy R, and a non-influential equilibrium in which the manager chooses strategy R with probability one does not exist. In those cases, the sensitivity of the manager to short-term performances increases shareholder value if and only if $\mathbb{E}[\theta] < 0$. We conclude that in the absence of voice ω has an ambiguous effect on shareholder value.

The next result shows that with voice the sensitivity of the manager to short-term performances has a positive effect on shareholder value.

Proposition 2 *The set of influential equilibria increases with ω. Moreover, unless the first best is obtained in equilibrium, shareholder expected value under the most efficient equilibrium strictly increases with ω.*

According to Proposition 2, when $\omega > 0$ the manager has stronger incentives to follow the activist's recommendation and voice is more effective. If the manager ignores the activist's recommendation, the activist is strictly more likely to exit. From the point of view of the manager, the best way to convince the activist not to sell her shares is simply following her recommendations. Thus, when $\omega > 0$ the threat of exit increases the influence the activist has on the the manager thorough voice, and an influential equilibrium is more likely to exist.

In conclusion, the sensitivity of the manager to short-term performances increases the effectiveness of voice and shareholder value, but only if the activist can exit. Thus, the analysis of this section demonstrates another novel channel through which voice and exit complement each other. In Section 2 the complementarity arises from the direct effect of exit on the incentives of the activist to communicate, and here the complementarity arises from the direct effect of exit on the incentives of the manager to follow the activist's recommendation.

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does not observe the manager's decision before she exits will not change the results of Section 2. However, with this assumption, ω has no effect on the equilibrium, and the analysis of Section 3 coincides with Section 2. Note that the existing literature on exit always assumes that $\omega > 0$ and the manager's decision is observed by the activist before she exits, and without these two assumptions, exit is ineffective as a governance mechanism. The analysis of Section 2 contrasts this observation by demonstrating that exit can be effective in other ways.

4 Transparency

In the baseline model the market maker does not observe the manager's decision or the activist's message. Under these assumptions there is *"No-Transparency" (NT)*. In this section I study the effect of transparency on voice and exit. I distinguish between three regimes that differ with respect to the information set of the market maker: under *"Action-Transparency" (AT)* the market maker observes the manager's decision, under *"Voice-Transparency" (VT)* the market maker observes the activist's message, and under *"Full-Transparency" (FT)* the market maker observes both.[19] I conclude with a discussion on the activist's choice between private and public voice.

4.1 Transparency of Actions

Under Action-Transparency the market maker uses two pieces of information to price the share: the activist's decision to exit and the manager's decision to implement strategy $a \in \{L, R\}$. When the equilibrium is non-influential, the activist cannot change the manager's decision and she takes prices as given. By contrast, when the equilibrium is influential the manager follows the activist's recommendations, and since the share price depends on the manager's decision, the activist can use her influence on the manager to change prices. Through this channel transparency changes the effectiveness of voice.

Proposition 3

(*i*) *In any influential equilibrium under Action-Transparency $p_R = p_L$, where p_a is the share price conditional on exit and the manager's decision. Moreover, the threshold equilibrium under Action-Transparency is unique and the threshold is given by $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$.*

(*iii*) *If $\omega = 0$ then any equilibrium under Action-Transparency is also an equilibrium under No-Transparency, and an influential equilibrium under Action-Transparency exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \tau^*_{AT}]$.*

[19] Under either of these regimes, an influential equilibrium is an equilibrium in which the activist can change the manager's decision by communication. It can be shown that for each of these regimes and for any $\omega \geq 0$ there is β^* such that a threshold equilibrium exists if and only if $\beta \leq \beta^*$.

Part (i) of Proposition 3 demonstrates that Action-Transparency imposes constraints on the set of influential equilibria. In particular, in any influential equilibrium the price upon exit must be invariant to the decision of the manager. Intuitively, if the share price conditional on exit depends on the manager's decision, the activist can arbitrage the difference between these prices by sending the appropriate message. The activist has incentives to inflate prices in order to secure better terms of trade if and when she exits. The desire to inflate the stock price distorts the communication with the manager, and consequently, the activist's recommendations lose credibility. It turns out that there is a unique threshold $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$ that satisfies this constraint, and this threshold is independent of ω. Therefore, if a threshold equilibrium exits, the threshold must be τ^*_{AT}.

Part (ii) of Proposition 3 shows that the constraints in part (i) have adverse consequences: an influential equilibrium may not exist, and when an influential equilibrium exists, it may be less efficient. Indeed, according to Proposition 1, under No-Transparency an influential equilibrium exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$. Since $\underline{\tau} < \tau^*_{AT}$ there are instances where an influential equilibrium exists under No-Transparency but it does not exist under Action-Transparency. In those cases, the most efficient equilibrium under Action-Transparency is strictly less efficient than the most efficient equilibrium under No-Transparency. Moreover, if $0 < \tau^*_{AT}$ then an influential equilibrium under Action-Transparency does not exist even if $\beta \leq -\mathbb{E}[\theta|\theta < 0]$. Based on Lemma 3, when $\beta \leq -\mathbb{E}[\theta|\theta < 0]$ an influential equilibrium under No-Transparency exists even if the activist cannot exit. It follows, when $0 < \tau^*_{AT}$ voice and exit exhibit *substitution* under Action-Transparency. Overall, voice can be less effective as a form of soft shareholder activism under Action-Transparency.[20]

4.2 Transparency of Voice

Under Voice-Transparency the market maker observes the communication between the activist and the manager. If the equilibrium is non-influential, the manager ignores any message from the activist. But the activist may still try to influence the market maker directly in order to

[20] In Admati and Pfleiderer (2009), transparency of actions can also be undesirable. However, the channel in their model is fundamentally different: transparency distorts the blockholder's exit strategy.

get the highest price possible when she exits. However, the incentive to inflate the short-term stock price destroys the activist's credibility and her messages are ignored by the market maker as well. Thus, the characterization of a non-influential equilibrium under Voice-Transparency coincides with Lemma 2.

When the equilibrium is influential the market maker uses the activist's message to learn about θ as well as about the decision of the manager. However, since the activist also has incentives to inflate the share price, any information beyond what is necessary to persuade the manager to choose one strategy over the other cannot be revealed in equilibrium. For this reason, the influential equilibria under Voice-Transparency and Action-Transparency regimes are equivalent, and Proposition 3 applies to Voice-Transparency as well.[21]

Proposition 4 *For all $\omega \geq 0$ the set of influential equilibria under Action-Transparency and Voice-Transparency are equivalent.*

4.3 Full Transparency

Transparency presents the activist with the opportunity to inflate the share price, and consequently, reduces her credibility and limits her ability to influence the manager. However, relative to Action-Transparency, Full-Transparency can enhance the effect of voice.

Proposition 5 *Any influential equilibrium under Action-Transparency is also an equilibrium under Full-Transparency. Moreover, if $\omega > 0$ there are $\beta^*_{AT} < \beta^*_{FT}$ such that an influential equilibrium exists under Full-Transparency but not under Action-Transparency if and only if $\beta \in (\beta^*_{AT}, \beta^*_{FT}]$.*

Under Full-Transparency the market maker observes the activist's recommendation and the manager's decision, and thus, he can compare between the two. If the manager ignores the activist's recommendation, whether or not the activist exits, the share price drops. Effectively, the manager's decision to ignore the activist's recommendation signals the market maker that

[21] Two equilibria are *equivalent* if the manager's decision rule and the activist's exit strategy are identical across equilibria.

the manager is not choosing the efficient strategy. Consequently, the market maker will update her expectations about the long-term value of the firm downward, and prices will drop. Therefore, when $\omega > 0$ and the market maker observes both the activist's recommendation and the manager's decision, ignoring the activist's recommendation imposes additional cost on the manager. In other words, Full-Transparency adds another layer of discipline on managers and it amplifies the effect of voice. Overall, Proposition 5 shows that when the decisions of managers are observable by the market, activists can have a greater influence on managers if they voice themselves publicly.

4.4 Choosing Between Private and Public Voice

In many cases, the activist can *choose* whether to publicize her message or to keep her communication with the manager private. As the analysis above suggests, the benefit from a public voice depends on whether or not the actions of the manager are observed by the market. If the activist can commit to the channel of communication before she observes her private information, she will choose the channel that maximizes the value of the firm. In particular, based on Proposition 4 and under the conditions of Proposition 3, when managerial decisions are not observed by the market, the activist will choose private communication. By contrast, based on Proposition 5, when managerial decisions are observed by the market, the activist will choose public communication.

Alternatively, suppose the activist chooses between private and public communication (or both) only after she observes her private information. Conditional on the transparency of the manager's actions, the set of feasible equilibria is the union of all equilibria under public and private voice. Indeed, if the actions of the manager are observed (not observed) by the market maker, an equilibrium under Full-Transparency (Voice-Transparency) can be implemented as follows: the manager ignores any private messages from the activist, and the activist always sends the same private message. Similarly, an equilibrium under Action-Transparency (No-Transparency) can be implemented as follows: the manager and the market maker ignore any public message sent by the activist, and the activist always sends the same public message. Overall, if the actions of the manager are observed by the market, and under the conditions of

Proposition 3, the most efficient equilibrium will be implemented by public communication. If the actions of the manager are not observed by the market, the most efficient equilibrium will be implemented by private communication.[22]

5 Empirical Implications

Informal communications between investors and the company's senior executives or board members are relatively common. Apart from Brav, Jiang, Partnoy and Thomas (2008),[23] other studies also find evidence in support of this view. Becht, Franks, and Grant (2010) use proprietary data collected from five activist funds and show that private and informal engagements are extensive and profitable. They give example of a successful private engagement between an activist fund and the management of a company whose largest shareholder was a family holding over 50% of the voting rights. While the fund owned less than 2% of the company, it was able to significantly change the strategy of the company and consequently realized significant abnormal returns on its investment. Becht, Franks, Mayer, and Rossi (2009) provide evidence on "behind the scenes" communication as a form of (profitable) shareholder activism of the Hermes UK Focus Fund. Carleton, Nelson, and Weisbach (1998) study letters TIAA-CREF sends to their portfolio companies and find that they are usually successful at inducing firms to make governance related changes. Finally, McCahery, Sautner, and Starks (2011) survey institutional investors and find that 55% of them would be willing to engage in discussions with the firm's executives. They also conclude that behind-the-scenes shareholder activism may be more prevalent than previously thought.

Informal engagement between investors and managers is expected to be intensive when the activist can effectively voice herself and influence decision making via communication. The extent of informal engagement can be measured by the number of meetings, emails, letters, or phone calls between investors and the firm's management/board. The model offers new

[22]Farrell and Gibbons (1989) consider a model of cheap talk model with multiple audiences and compare public and private communication. However, in their model the multiple audiences do not interact as they do in the current model.

[23]Interestingly, Brav, Jiang, Partnoy and Thomas (2008) do not observe more aggressive tactics in events in which the hedge funds declare that they intend to communicate with the management/board.

predictions about the expected frequency of these activities.

Prediction 1 *The frequency of engagement between the activist and the management/board of the firm decreases with the longevity of the activist and increases with market liquidity and activist's ability to trade anonymously.*

Prediction 1 is a reflection of the complementarity between voice and exit: when exit has a lower price impact, the activist becomes less sensitive to the long-term value of the firm and more effective when communicating with a biased management. The price impact is a factor of market liquidity, and the activist's longevity and ability to trade anonymously. In the model, longevity (or the length of the investment horizon) is the likelihood the activist is not subject to a short-term liquidity shock, $1-\delta$. Longevity can be measured by the length of lock-up and redemption notification periods, or by the turnover in the investor's holdings. Prediction 1 is consistent with Solomon and Soltes (2012) who study the frequency of meetings between senior management and investors and show that investors who have greater turnover in their holdings gain greater access to management.

While market liquidity or the activist's ability to trade anonymously are not explicitly modeled, they have a similar effect on the price impact of exit.[24] In this respect, Prediction 1 is also consistent with Edmans, Fang, and Zur (2013) who show that filing of a Schedule 13G leads to a positive market reaction and an improvement in operating performance, and these effects are stronger in more liquid firms and firms with high managerial sensitivity to the stock price. Since Schedule 13G (as opposed to Schedule 13D) is filed when the investor intends to remain passive and has no planes to seek control, the authors interpret these results as evidence for governance through exit rather than voice. However, Briggs (2007) argues that "... it seems that merely making suggestions to management about what it should be doing is perfectly permissible [under Schedule 13G] ..." Under this interpretation, the evidence that Edmans, Fang, and Zur (2013) find are also consistent with the current model.

[24] For example, a model with anonymous trade and noise traders a la Kyle (1985) would deliver similar results. The main difference is that in the alternative setup the activist makes profits on the expense of noise traders and in the current setup the activist effectively trades against herself (when she needs liquidity).

Proposition 1 and Proposition 2, respectively, predict that voice is more effective when the conflicts of interest between shareholders and the manager are small (low β) and the short-term component in the manager's equity-based incentives package is relatively large (high ω). In those cases, one would expect to see more frequent engagement between investors and management.

Prediction 2 *The frequency of engagement between the activist and the management/board of the firm decreases with the conflict of interest between shareholders and the management/board, and the duration of the manager's equity-based incentives package.*

The analysis of Section 4 considers both private and public communication, and concludes that public communication is more effective if and only if the manager is sensitive to the stock price and his decision can be observed by the market before its long-term effect on the firm's performances is made public. Corporate decisions that are subject to mandatory disclosure (for example, divestitures, spin-offs or changes to payout policy) fall into this category. Thus, for decisions that do no require disclosure or are in the gray area of what has to be disclosed (for example, changes of product the market strategies or the supply chain), the model predicts that the activist is more likely to engage with management behind the scenes.[25]

Prediction 3 *The relative frequency of public to private engagement between the activist and the management/board of the firm is higher when the corporate decision is subject to mandatory disclosure and the duration of the manager's equity-based incentives package is high.*

In Section 2.2.2 I argue that high quality of activist's private information (or low cost of acquiring private information) can limit her ability to effectively use her voice. In particular, In Appendix C I show that when the conflict of interest between the manager and shareholders is moderate, there is an inverted U-shape relation between the activist's quality of private

[25] If the activist optimally decides between public and private engagements, on average, there should be no differences in the observed returns for these two strategies. Nevertheless, Becht, Franks, and Grant (2010) find that for board structure and payout changes, and restructuring events other than takeovers, the returns to the activist are higher when the engagement is private.

information and the effectiveness of voice. The quality of activist's private information, which reflects the investors's expertise, can be proxied by the business experience or formal education of the hedge fund manager.

Prediction 4 *The frequency of engagement between the activist and the management/board of the firm has an inverted U-shape relation with the expertise of the activist.*

An alternative interpretation for the results in Section 2.2.2 regrades the size of the activist's holdings α. The model predicts that even small investors who are mainly active in middle markets and do not have the capacity of obtaining control through hostile takeovers or proxy fights (and in particular, those with holdings below 5%) can have a significant effect on the value of the firm. Moreover, when the firm has a controlling shareholder and a change of control is practically impossible, the model predicts that smaller blockholders can still play an active role and enhance the value of the firm. These predictions are in contrast with other models of intervention which builds on the ability of investors to obtain formal control and force management to take actions.

Finally, the communication between investors and the manager of the firm is often informal and private. It is therefore difficult to measure the magnitude and quality of the informal engagement using publicly available data. To the extent that the frequency of engagement or its quality are not observed, the analysis of this paper offers *indirect* empirical predictions. The model predicts that the effectiveness of soft shareholder activism generally decreases with the longevity of the activist. That is, there is an inverse relationship between the longevity of the activist and the value of the firm. This prediction in contrast to other models of shareholder activism (for example, Admati and Pfleiderer (2009) predict exactly the opposite). Moreover, different from other models of exit, soft shareholder activism can be effective even when managers are not sensitive to the short-term performances of the firm. Thus, by studying the effect of exit on the performances of firms with a negligible amount of short-term executive compensation, one can indirectly identify the effect or prevalence of private engagement and communication.

6 Concluding Remarks

This paper offers a new perspective on shareholder activism by focusing on what can be achieved when costly formal control cannot be obtained or exercised by shareholders. Two primary mechanisms are analyzed, voice and exit. Departing form the majority of the existing literature on shareholder activism, voice is modeled as a strategic transmission of information. This form of informal communication is a reflection of investors' attempt to exercise activism by sending letters, calling senior executives, and meeting with board members, thereby providing their input and ultimately changing the strategic course of the company. The paper analyzes the conditions under which soft shareholder activism is an effective form of corporate governance. It highlights the interaction between voice and exit, and relates their effectiveness to market liquidity; entrenchment and compensation structure of the manager; and the expertise, liquidity and ownership size of the activist. The paper also characterizes the conditions under which activist investors prefer public communications over behind the scenes interactions.

The analysis of this paper leaves out several important issues. First, activist investors such as hedge funds are often blamed for being opportunistic and pursuing short-term goals. How does the analysis change when the activist is biased? Interestingly, a bias can be helpful. When the activist and other shareholders do not share the same agenda, the biased activist may exit even if the share is under-valued from the perspective of other shareholders, and keep her stake when the share is over-valued. This dynamic relaxes the adverse selection when the activist exits and pushes prices upward. When voice and exit exhibit complementarity, this effect enhances the activist's ability to voice herself credibility and create value to other shareholders as well.

Second, the analysis abstracts from the possibility that some investors may have the capacity to exercise both soft and hard activism, for example, by initiating a proxy fight to replace the board if it refuses to follow their recommendations. I conjuncture that soft and hard shareholder activism will complement each other, but a formal analysis is needed to fully understand the interaction between the two, and therefore, is left for future research.[26]

[26] In the context of strategic voting, Levit and Malenko (2011) show the presence of an activist investor who can launch a proxy fight to replace a biased manager enhances the advisory role of non-binding voting, but only if the activist herself is biased.

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Appendix A

The next lemma provides auxiliary results for the proofs that follow.

Lemma A.1 *For any influential equilibrium define* $\Theta \equiv \left\{\theta \in \left[\underline{\theta}, \overline{\theta}\right] : a\left(m\left(\theta\right)\right) = R\right\}$ *and let* p_Θ *be the price upon exit. Then:*

(i) $p_\Theta > 0$.

(ii) The set Θ *satisfies* $\Theta^c \supseteq \left[\underline{\theta}, -p_\Theta\right]$ *and* $\Theta \supseteq \left[p_\Theta, \overline{\theta}\right]$.

(iii) The activist exits strategically if and only if $\theta \in \Upsilon \equiv \left[-p_\Theta, p_\Theta\right]$.

(iv) The price upon exit is unique and given by the global minimum of $\varphi\left(p, \Theta\right)$ *where*

$$\varphi\left(p, \Theta\right) = \frac{\delta \mathbb{E}\left[v\left(\theta, \Theta\right)\right] + \left(1 - \delta\right) \Pr\left[v\left(\theta, \Theta\right) \leq p\right] \mathbb{E}\left[v\left(\theta, \Theta\right) | v\left(\theta, \Theta\right) \leq p\right]}{\delta + \left(1 - \delta\right) \Pr\left[v\left(\theta, \Theta\right) \leq p\right]}, \tag{11}$$

and $v\left(\theta, \Theta\right) = \theta$ *if* $\theta \in \Theta$ *and* $v\left(\theta, \Theta\right) = -\theta$ *otherwise.*

(v) If Θ' *and* Θ'' *are such that* $v\left(\theta, \Theta'\right) \geq (>)\, v\left(\theta, \Theta''\right)$ *for all* θ *then* $p_{\Theta'} \geq (>)\, p_{\Theta''}$.

(vi) When $s = 0$ *the price is given by* $p_\Theta\left(0\right) = \mathbb{E}\left[v\left(\theta, \Theta\right) | v\left(\theta, \Theta\right) > p_\Theta\right]$.

Proof. Consider part (i) and suppose on the contrary $p_\Theta \leq 0$. Since the equilibrium is influential, by definition, there are $m_L \neq m_R$ such that $a\left(m_R\right) = R$ and $a\left(m_L\right) = L$. Recall action a and message m are not observable by the market maker. Thus, in equilibrium, the activist takes p_Θ as given. Based on (3), if $\theta \geq 0$ the activist maximizes her utility by sending message m_R and choosing $s = 0$. If $\theta < 0$ the activist maximizes her utility by sending message m_L and choosing $s = 0$. Either way, the activist never exits strategically. Thus, if $s = 1$ the market maker attributes the activist's decision to exit to a liquidity shock. Moreover, the market maker expects the manager to choose $a = R$ if and only if $\theta > 0$. Therefore, the fair value of the firm, and the price upon exit, must equal to $\mathbb{E}\left[|\theta|\right] > 0$, a contradiction.

Consider part (ii). Suppose on the contrary there is $\theta \in \Theta^c \cap \left[p_\Theta, \overline{\theta}\right]$. If the activist chooses $m = m_R$ and $s = 0$ she gets $\theta > p_\Theta$. If she chooses $m = m_L$ she gets $\max\left\{p_\Theta, -\theta\right\}$. Since

$p_\Theta > 0$, the activist is strictly better off sending message $m \in m_R$. This contradicts the presumption that $\theta \in \Theta^c \Rightarrow m = m_L$. A similar argument proves that if on the contrary there is $\theta \in \Theta \cap [\underline{\theta}, -p_\Theta]$ then the activist has incentives to send message m_L which contradicts the presumption that $\theta \in \Theta \Rightarrow m = m_R$.

Consider parts (iii) and (iv). If in equilibrium the manager implements strategy R if and only if $\theta \in \Theta$, then for any θ the value of the firm is $v(\theta, \Theta)$. The activist exits if and only if she is subject to a liquidity shock or if the share is over-priced, that is, $v(\theta, \Theta) \leq p_\Theta$. Note that under the specification of part (i), $v(\theta, \Theta) \leq p_\Theta \Leftrightarrow \theta \in [-p_\Theta, p_\Theta]$. Therefore, $\Upsilon \equiv [-p_\Theta, p_\Theta]$. The market maker has rational expectations about the manager's decision rule and the activist exit strategy, and therefore, the price upon exit must be the solution of $p = \varphi(p, \Theta)$. Extending Proposition 1 in Acharya et al. (2011) for a random variable $v(\theta, \Theta)$, one can show that the global minimum of $\varphi(p, \Theta)$ is the unique solution of $p = \varphi(p, \Theta)$.

Consider part (v). Since $v(\theta, \Theta') \geq (>) \, v(\theta, \Theta'')$ for all θ then $\varphi(p, \Theta') \geq (>) \, \varphi(p, \Theta'')$ for any p. Since p_Θ is the global minimum of $\varphi(p, \Theta)$, and it is the solution of $p = \varphi(p, \Theta)$, it follows, $p_{\Theta'} \geq (>) \, p_{\Theta''}$. ■

Proofs of Section 2

Proof of Lemma 1. Consider an influential equilibrium where Θ is non-threshold. There are $\theta_1 < \theta_2$ such that $\theta_1 \in \Theta$ and $\theta_2 \notin \Theta$. Recall that in any influential equilibrium the activist can dictate a. Based on (3), if $\theta \in [\theta_1, \theta_2]$ the activist is indifferent between R and L. Hence, the activist exits with probability one if $\theta \in [\theta_1, \theta_2]$ and $[\theta_1, \theta_2] \subset \Upsilon$.

Let $m_i = m(\theta_i)$ for $i \in \{1, 2\}$. Note that $\theta_i \in \varepsilon(m_i)$ and $\mathbb{E}[\theta | m_i] = \mathbb{E}[\theta | \theta \in \varepsilon(m_i)]$. According to (4)

$$\mathbb{E}[\theta | \theta \in \varepsilon(m_1)] \geq -\beta \geq \mathbb{E}[\theta | \theta \in \varepsilon(m_2)] \tag{12}$$

Consider an alternative equilibrium with $\hat{\Theta}$. In this equilibrium, the communication strategy is identical to the original equilibrium, with the sole exception $\hat{\varepsilon}(m_i) \equiv \varepsilon(m_i) \cup \{\theta_j\} \setminus \{\theta_i\}$. Under the new strategy, the activist sends message m_i when he observes θ_j, $i \neq j$. Since $\theta_1 < \theta_2$ then $\mathbb{E}[\theta | \theta \in \hat{\varepsilon}(m_1)] \geq \mathbb{E}[\theta | \theta \in \varepsilon(m_1)]$ and $\mathbb{E}[\theta | \theta \in \varepsilon(m_2)] > \mathbb{E}[\theta | \theta \in \hat{\varepsilon}(m_2)]$. Thus, given condition (4) and (12), the manager has incentives to follow the activist's recommendation under the new strategy. Note that $v(\theta,\hat{\Theta}) \geq v(\theta,\Theta)$ for any θ (the difference in proportional to $(\theta_2-\theta_1)-(\theta_1-\theta_2)>0$). Based on Lemma A.1 part (v), $p_{\hat{\Theta}} \geq p_{\Theta}$. Based on Lemma A.1, $\Upsilon=[-p_{\Theta},p_{\Theta}] \subset [-p_{\hat{\Theta}},p_{\hat{\Theta}}]=\hat{\Upsilon}$. The activist finds it weakly optimal to follow the new communication strategy, yielding an equilibrium which is strictly more efficient. One can repeat this procedure as long as the equilibrium is non-threshold, eventually, converging to a threshold equilibrium. ■

Proof of Lemma 2. Apart from the characterization of $p(1)$, the proof of the Lemma is given in the main text. The characterization of $p(1)$ follows from Proposition 1 in Acharya et al. (2011). ■

Proof of Lemma 3. By Definition 1, if an influential equilibrium exists then the activist can dictate the action taken by the manager. Since the activist cannot exit, she has strict incentives to persuade the manager to choose R when $\theta>0$ and to choose L when $\theta<0$.

Suppose $\beta \leq -\mathbb{E}[\theta|\theta<0]$. Consider an equilibrium in which the activist sends message m_R if $\theta \geq 0$ and message $m_L \neq m_R$ otherwise. Conditional on $m=m_R$ the manager believes $\theta \geq 0$. According to (4), the manager chooses strategy R if and only if $\mathbb{E}[\theta|\theta \geq 0]+\beta \geq 0$. Since $\beta > -\mathbb{E}[\theta] > \mathbb{E}[\theta|\theta \geq 0]$, this condition always holds. Conditional on $m=m_L$ the manager believes that $\theta<0$. According to (4), the manager chooses strategy L if and only if $\mathbb{E}[\theta|\theta<0]+\beta \leq 0$, which by assumption holds. Thus, the manager follows the activist's recommendation. Given the manager's expected behavior, it is in the best interest of the activist to follow the proposed communication strategy, so this is indeed an influential equilibrium.

To see the other direction, consider an influential equilibrium. Define M_L be the set of all messages such that $a(m)=L$. Similarly, define M_R. Since the equilibrium is influential, neither set is empty. According to (4), if $a(m)=L$ then $\mathbb{E}[\theta|m]+\beta \leq 0$. Therefore, integrating over all $m \in M_L$ it follows that $\mathbb{E}[\theta|m \in M_L]+\beta \leq 0$. Similarly, integrating over all $m \in M_R$ implies $\mathbb{E}[\theta|m \in M_R]+\beta \geq 0$. Recall the activist has incentives to send $m \in M_L$ if and only if $\theta \leq 0$. For this reason, $\mathbb{E}[\theta|m \in M_L]=\mathbb{E}[\theta|\theta \leq 0]$ and $\mathbb{E}[\theta|m \in M_R]=\mathbb{E}[\theta|\theta>0]$. Overall,

$\beta \leq -\mathbb{E}[\theta|\theta < 0]$ holds as required. ■

Proof of Proposition 1. I prove that there are unique $\underline{\tau} < 0 < \overline{\tau}$ such that $-\pi(\tau) \leq \tau \leq \pi(\tau)$ if and only if $\underline{\tau} \leq \tau \leq \overline{\tau}$. Consider several properties of $\pi(\tau)$. First, based on Lemma A.1 part (v), if $|\tau'| < |\tau''|$ then $\pi(\tau') \geq \pi(\tau'')$. Second, for any p and τ (6) can be rewritten as

$$\varphi(p,\tau) = \frac{\delta\left[-\int_{\underline{\theta}}^{\tau}\theta dF(\theta) + \int_{\tau}^{\overline{\theta}}\theta dF(\theta)\right] + (1-\delta)\left[-\int_{\min\{-p,\tau\}}^{\tau}\theta dF(\theta) + \int_{\tau}^{\max\{p,\tau\}}\theta dF(\theta)\right]}{\delta + (1-\delta)\Pr[\theta \in [\min\{-p,\tau\}, \max\{p,\tau\}]]} \tag{13}$$

Thus, $\varphi(p,\tau)$ is continuous in $\tau \in [\underline{\theta}, \overline{\theta}]$. Since $\pi(\tau)$ is the unique minimum of $\varphi(p,\tau)$, it is continuous in τ as well. Third, note that $\pi(0) > 0$. Overall, $\pi(\tau) - \tau$ decreases in τ when $\tau > 0$. By the intermediate value theorem, there is $\overline{\tau} \in [0, \overline{\theta}]$ such that if $\tau \in [0, \overline{\tau})$ then $\pi(\tau) - \tau > 0$ an if $\tau \in (\overline{\tau}, \overline{\theta}]$ then $\pi(\tau) - \tau < 0$. Similarly, $\pi(\tau) + \tau$ increases in τ when $\tau < 0$. By the intermediate value theorem, there is $\underline{\tau} \in [\overline{\theta}, 0]$ such that if $\tau \in (\underline{\tau}, 0]$ then $\pi(\tau) + \tau > 0$ an if $\tau \in [\underline{\theta}, \underline{\tau})$ then $\pi(\tau) + \tau < 0$. This completes the argument.

Next, suppose an influential equilibrium exists. According to Lemma 1, if a non-threshold influential equilibrium exists then a threshold equilibrium must also exist. Let that threshold be τ. Based on the discussion that precedes Proposition 1, the activist is willing to recommend on threshold τ if and only if $\underline{\tau} \leq \tau \leq \overline{\tau}$. Similar to the arguments in the proof of Lemma 3, it has to be $\beta \leq -\mathbb{E}[\theta|\theta \leq \tau]$. It follows, $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$. If $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}]$ then the discussion that precedes Proposition 1 implies that an equilibrium with threshold $\underline{\tau}$ exists (the activist sends message m_R if $\theta \geq \underline{\tau}$ and message $m_L \neq m_R$ otherwise). ■

Proofs of Section 3

Proof of Lemma 4. Consider a non-influential equilibrium in which the manager chooses $a = R$ with probability $x \in [0,1]$. Recall a is observed by the activist but not by the market maker. The activist chooses $s = 1$ if and only if $v(\theta, a) < p(1, x)$ and the market maker sets the price on

$$p(s,x) = \begin{cases} x\pi(\underline{\theta}) + (1-x)\pi(\overline{\theta}) & \textit{if } s=1 \\ x\mathbb{E}[\theta|\theta > p(1,x)] + (1-x)\mathbb{E}[-\theta|-\theta > p(1,x)] & \textit{if } s=0 \end{cases}$$

In this equilibrium, the manager's expected utility is

$$u_M(a,x) = \omega p(1,x) + \omega(1-\delta)\Pr[v(\theta,a) \geq p(1,x)][p(0,x) - p(1,x)] + \mathbb{E}[v(\theta+\beta,a)]$$

The manager chooses $a = R$ if and only if $\Delta(x) \geq 0$ where $\Delta(x) \equiv u_M(R,x) - u_M(L,x)$ and is given by

$$\Delta(x) = \omega(1-\delta)(\Pr[\theta \geq p(1,x)] - \Pr[\theta \leq -p(1,x)])[p(0,x) - p(1,x)] + 2\mathbb{E}[\theta+\beta]$$

Note that $\Delta(x)$ is continuous in $x \in [0,1]$. A non-influential equilibrium must satisfy either $\Delta(1) \geq 0$, $\Delta(0) \leq 0$, or $\Delta(x) = 0$ for $x \in (0,1)$. Therefore, if $\Delta(1) < 0 < \Delta(0)$, by the intermediate value theorem there is always $x^* \in (0,1)$ such that $\Delta(x^*) = 0$. A non-influential equilibrium always exists.

Recall $p(0,x) - p(1,x) > 0$ for all x and by assumption $\mathbb{E}[\theta+\beta] > 0$. Therefore, if $\Pr[\theta \geq \pi(\underline{\theta})] \geq \Pr[\theta \leq -\pi(\underline{\theta})]$ then $\Delta(1) > 0$. If $\Pr[\theta \geq \pi(\underline{\theta})] < \Pr[\theta \leq -\pi(\underline{\theta})]$ then $\Delta(1) \geq 0$ if and only if $\omega \leq \hat{\omega}$ where

$$\hat{\omega} \equiv \frac{1}{1-\delta} \frac{2\mathbb{E}[\theta+\beta]}{\mathbb{E}[\theta|\theta > \pi(\underline{\theta})] - \pi(\underline{\theta})} \frac{1}{|\Pr[\theta \leq -\pi(\underline{\theta})] - \Pr[\theta \geq \pi(\underline{\theta})]|}$$

This completes the argument. ■

Proof of Proposition 2. Consider an influential equilibrium and let M_a be the set of messages that yields action a. In an influential equilibrium neither set is empty. Without the loss of generality, suppose $M_R \cup M_L = [\underline{\theta}, \overline{\theta}]$. Since the market maker does not observe a or m, $p(s)$ is given by (11). Regardless of the message, the activist observes the actual decision a and strategically exits if and only if $v(\theta,a) < p(1)$. In equilibrium, the manager's expected utility from action a conditional on message m is:

$$u_M(a,m) = \omega p(1) + \omega(1-\delta)\Pr[v(\theta,a) > p(1)|m][p(0) - p(1)] + \mathbb{E}[v(\theta+\beta,a)|m]$$

The manager follows the recommendation of the activist if and only if $u_M(R,m) \geq u_M(L,m)$ for all $m \in M_R$ and $u_M(R,m) \leq u_M(L,m)$ for all $m \in M_L$. Integrating over all message in M_A, there is an equivalent equilibrium such that the activist only reveals whether $m \in M_R \Leftrightarrow \theta \in \Theta$ or $m \in M_L \Leftrightarrow \theta \in \Theta^c$. Thus, it is sufficient to consider an equilibrium with two messages. Let these messages be m_R and m_L. Note that if $m = m_R$ then $v(\theta,L) \leq p_\Theta(1)$ for sure and if $m = m_L$ then $v(\theta,R) \leq p_\Theta(1)$ for sure. Therefore, the manager follows the recommendation of the activist if and only if

$$\Psi_R(\Theta,\omega) \leq \beta \leq \Psi_L(\Theta,\omega)$$

where

$$\Psi_R(\Theta,\omega) = -\mathbb{E}[\theta|\theta \in \Theta] - \frac{\omega}{2}(1-\delta)\Pr[\theta > p_\Theta(1)|\theta \in \Theta][p_\Theta(0) - p_\Theta(1)] \tag{14}$$

and

$$\Psi_L(\Theta,\omega) = -\mathbb{E}[\theta|\theta \in \Theta^c] + \frac{\omega}{2}(1-\delta)\Pr[\theta < -p_\Theta(1)|\theta \in \Theta^c][p_\Theta(0) - p_\Theta(1)] \tag{15}$$

Condition $\Psi_R(\Theta,\omega) \leq \beta$ requires $u_M(R,m_R) \geq u_M(L,m_R)$ and $\beta \leq \Psi_L(\Theta,\omega)$ requires $u_M(R,m_L) \leq u_M(L,m_L)$. Since $p_\Theta(s)$ is independent of ω and $p_\Theta(0) - p_\Theta(1) > 0$, $\Psi_R(\Theta,\omega)$ strictly decreases in ω and $\Psi_L(\Theta,\omega)$ strictly increases with ω. Therefore, the manager finds it optimal to follow Θ when $\omega' \geq \omega$. If $p_\Theta(1)$ does not change, the activist has the same incentives to recommend on Θ and exit when $\theta \in \Upsilon$. In this case, the market maker does not change the price. It follows, the same equilibrium can be supported for any $\omega' > \omega$.

To show the second part, suppose an influential equilibrium with decision rule Θ exists.

Suppose Θ is not the first best. For any $t \in [-p_\Theta(1), -p_\Theta(1)]$ define

$$\Gamma(t) \equiv \begin{cases} \Theta \cap [t, \infty) & \textit{if } t \in [-p_\Theta(1), 0) \\ \Theta \cap [0, \infty) \cup [p_\Theta(1) - t, p_\Theta(1)] & \textit{if } t \in [0, p_\Theta(1)] \end{cases}$$

and note that the efficiency of decision rule $\Gamma(t)$ increases with t, where $\Gamma(-p) = \Theta$ and $\Gamma(p) = \left[0, \overline{\theta}\right] \neq \Theta$. Indeed, in every step with remove $\theta < 0$ from $\Gamma(t)$ or add $\theta > 0$. Note also that $\Gamma(t)$ is smooth, and hence, $\Psi_L(\Gamma(t), \omega)$ and $\Psi_R(\Gamma(t), \omega)$ are continuous in t and ω. Since Θ can be supported in equilibrium then $\Psi_R(\Theta, \omega) \leq \beta \leq \Psi_L(\Theta, \omega)$. It follows, $\Psi_R(\Theta, \omega + \varepsilon) < \beta < \Psi_L(\Theta, \omega + \varepsilon)$ for any $\varepsilon > 0$. We conclude, for any $\varepsilon > 0$ there is $t_\varepsilon \in (-p_\Theta(1), -p_\Theta(1)]$ such that $\Gamma(t_\varepsilon)$ is strictly more efficient than Θ, and $\Psi_R(\Gamma(t_\varepsilon), \omega + \varepsilon) \leq \beta \leq \Psi_L(\Gamma(t_\varepsilon), \omega + \varepsilon)$. Note that $\Gamma(t)$ satisfies the condition in Lemma A.1, and based on Lemma A.1 part (iv), $p_{\Gamma(t)}(1)$ is increasing with t as well. Therefore, an equilibrium with decision rule $\Gamma(t_\varepsilon)$ exists when the sensitivity of the manager is $\omega + \varepsilon$.. $\blacksquare$

Proofs of Section 4

Proof of Proposition 3. Let $p_a(s)$ be the price of the share if the manager decides on a and the activist chooses s. I prove that any influential equilibrium under Action-Transparency satisfies $p_R(1) = p_L(1)$. Suppose on the contrary the equilibrium is influential and $p_L \neq p_R$. Note that M_L and M_R are not empty. Let $p_{\max} = \max\{p_L(1), p_R(1)\}$ and note that $p_{\max} > 0$. Otherwise, the activist sends $m \in M_R$ if and only if $\theta > 0$ and never exits strategically. This implies $p_R = \mathbb{E}[\theta|\theta > 0] > 0$, a contradiction. Consider the activist's exist strategy. If $\theta > p_{\max}$ the activist sends $m \in M_R$ and chooses $s = 0$. If $\theta < -p_{\max}$ the activist sends $m \in M_L$ and chooses $s = 0$. If $\theta \in [-p_{\max}, p_{\max}]$ the activist chooses $s = 1$ and sends $m \in M_R$ if and only if $p_R > p_L$. Thus, if $p_R > p_L$ the manager chooses $a = L$ if and only if $\theta < -p_R$. Moreover, if $a = L$ the activist never exits unless she needs liquidity. Therefore, $p_L = \mathbb{E}[-\theta|\theta < -p_R] > p_R$, a contradiction. Similarly, if $p_R < p_L$ the manager chooses $a = R$ if and only if $\theta > p_L$. Moreover, if $a = R$ the activist never exits unless she needs liquidity. Therefore, $p_R = \mathbb{E}[\theta|\theta > p_L] > p_L$, a contradiction. Hereafter, I denote $p_L(1)$ and $p_R(1)$ by

$p(1)$.

The rest of the proof is done in several steps. First, I prove that if $\omega = 0$ then any equilibrium under Action-Transparency is an equilibrium under No-Transparency. Suppose an influential equilibrium under Action-Transparency with decision rule Θ exists. Based on Lemma A.1, it is necessary $p(1) > 0$, $\left[p(1), \overline{\theta}\right] \subset \Theta$ and $[\underline{\theta}, -p(1)] \subset \Theta^c$. Moreover, $p(1)$ must be the solution of both $p = \varphi_R(p, \Theta)$ and $p = \varphi_L(p, \Theta)$ where

$$\varphi_a(p,\Theta) \equiv \begin{cases} \frac{\delta \mathbb{E}[\theta|\theta\in\Theta]+(1-\delta)\Pr[\theta<p|\theta\in\Theta]\mathbb{E}[\theta|\theta\in[-p,p]\cap\Theta]}{\delta+(1-\delta)\Pr[\theta<p|\theta\in\Theta]} & \text{if } a = R \\ \frac{\delta \mathbb{E}[-\theta|\theta\in\Theta^c]+(1-\delta)\Pr[-\theta<p|\theta\in\Theta^c]\mathbb{E}[-\theta|\theta\in[-p,p]\cap\Theta^c]}{\delta+(1-\delta)\Pr[-\theta<p|\theta\in\Theta^c]} & \text{if } a = L \end{cases} \tag{16}$$

Note that according to (11) and (16), $\varphi(p, \Theta)$ can be rewritten as

$$\varphi(p,\Theta) = \varphi_R(p,\Theta)\, c(p,\Theta) + (1 - c(p,\Theta))\, \varphi_L(p,\Theta)$$

where

$$c(p,\Theta) \equiv \frac{\delta \Pr[\theta \in \Theta] + (1-\delta)\Pr[\theta \in [-p,p] \cap \Theta]}{\delta + (1-\delta)\Pr[\theta \in [-p,p]]}$$

Therefore, if $p(1)$ is a solution of $p = \varphi_R(p,\Theta)$ and $p = \varphi_L(p,\Theta)$, it is also a solution of $p = \varphi(p,\Theta)$. Therefore, given Θ and Υ the price upon exit under Action-Transparency and No-Transparency is the same. Since $\omega = 0$, this completes the argument.

Second, I show that there is unique $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$ that satisfies both $p = \varphi_R\left(\left[\tau, \overline{\theta}\right], p\right)$ and $p = \varphi_L\left(\left[\tau, \overline{\theta}\right], p\right)$. Fix τ, and let $p_a(\tau)$ be the solution of $p = \varphi_a(\tau, p)$. As in Lemma A.1, $p_a(\tau)$ exists and it is unique. Note also that $p_L(\tau)$ strictly decreases with τ and $p_R(\tau)$ strictly increases with τ. Therefore, if there is a solution to $p_L(\tau) = p_R(\tau)$ in $[\underline{\tau}, \overline{\tau}]$, it is unique. To show that a solution exists, note that $p_L(\tau) > -\tau$. Since $\pi(\underline{\tau}) = -\underline{\tau}$ and $\pi(\tau)$ is a weighted average of $p_R(\tau)$ and $p_L(\tau)$, then $p_L(\underline{\tau}) > -\underline{\tau} = \pi(\tau)$ implies $-\underline{\tau} > p_R(\underline{\tau})$. Similarly, note that $p_R(\tau) > \tau$. Since $\pi(\overline{\tau}) = \overline{\tau}$ then $p_R(\overline{\tau}) > \overline{\tau} = \pi(\tau)$ implies $\overline{\tau} > p_L(\overline{\tau})$. Overall, $p_L(\underline{\tau}) > p_R(\underline{\tau})$ and $p_R(\overline{\tau}) > p_L(\overline{\tau})$ implies that a solution exists in $(\underline{\tau}, \overline{\tau})$, as required.

Third, suppose a threshold equilibrium exists, where $\omega \geq 0$. As in Proposition 1, the activist has incentives to recommend on threshold τ if and only if $-p \leq \tau \leq p$. Since the price must be a solution of $p = \varphi(p, \tau)$, the threshold must satisfy $\tau \in [\underline{\tau}, \overline{\tau}]$. From the previous arguments, it follows that the threshold must satisfy $\tau = \tau^*_{AT}$. This completes the proof part (i).

Last, it is left to show that when $\omega = 0$ then an influential equilibrium under Action-Transparency exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \tau^*_{AT}]$. Based on Lemma 1 we can focus on threshold equilibrium, and based on part (i), we can focus on threshold equilibrium with threshold τ^*_{AT}. Since $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$ then the only step that is needed is to require that the manager has incentives to follow threshold τ^*_{AT}. This is satisfied by the condition $\beta \leq -\mathbb{E}[\theta|\theta < \tau^*_{AT}]$. ∎

Proof of Proposition 4. Consider an influential equilibrium under Action-Transparency with decision rule Θ. I argue that there is an equilibrium under Voice-Transparency in which the activist sends message m_R if $\theta \in \Theta$, message $m_L \neq m_R$ otherwise, and any other message is ignored by the market maker and the manager. The market maker observes m and infers that if $m = m_R$ ($m = m_L$) then $\theta \in \Theta$ ($\theta \in \Theta^c$) and $a = R$ ($a = L$). Therefore, if $m = m_a$ the price must be a solution to $p = \varphi_a(\Theta, p)$. Since Θ is an influential equilibrium under Action-Transparency, the solution exists, and for the same reason as in the Action-Transparency regime, it has to be $p_R = p_L > 0$. Since $p_R = p_L > 0$ and any other message is ignored, the incentives of the activist to send message m_A or m_R are solely determined by his incentives to change the manager's decision. Since Θ is an equilibrium under Action-Transparency, the manager has incentives to follow the recommendation of the activist, and the activist has incentives to make this recommendation.[27]

Consider an influential equilibrium under Voice-Transparency. Let M_a be the set of all (public) messages that are sent with a strictly positive probability in equilibrium and yield decision a. Also, let $p_m(s)$ be the share price conditional on $s = 1$ and message $m \in M$. By definition, M_R and M_L are not empty. Since $\delta > 0$ there is a strictly positive probability that the activist exits. Therefore, sending $m \notin \arg\max_{m \in M_R} p_m(1) \cup \arg\max_{m \in M_L} p_m(1)$ is a strictly dominated strategy. This implies that there are exactly two different prices conditional on exit, p_R for $m \in M_R$ and p_L for $m \in M_L$. In fact, there are exactly two types messages:

[27]Note that in (14) and (15) the prices are conditioned on a when the regime is Action-Transparency and on m when it is Voice-Transparency. Since in equilibrium $p_a(1)$ is invariant to a, these conditions are equivalent.

one that yields strategy R and price p_R, and one that yields strategy L and price p_L. As in the proof of Proposition 3, it must be that $p_A = p_R > 0$. Given this argument, it is immediate to see that any set Θ and Υ that emerge as equilibrium under Voice-Transparency can also emerge as equilibrium under Action-Transparency. ■

Proof of Proposition 5. Given Proposition 4, we focus on transparency of type $\varsigma \in \{AT, FT\}$. Let $p_{I_\varsigma(a,m)}(s)$ be the price conditional on s and $I_\varsigma(a,m)$ where $I_{AT}(a,m) = \{a\}$ and $I_{FT}(a,m) = \{a,m\}$. As in the proof of Proposition 2, and for the reasons in Proposition 3, it is sufficient to consider an equilibrium with exactly two messages: m_R when $\theta \in \Theta$ leading to $a = R$ and m_L when $\theta \in \Theta^c$, leading to $a = L$. For the reasons in Proposition 3, in all of these cases $p_{I_\varsigma(R,m_R)}(1) = p_{I_\varsigma(L,m_L)}(1)$.

The manager follows the recommendation to choose $a = R$ if and only if

$$\begin{aligned}
& \mathbb{E}[\theta+\beta|\theta\in\Theta] + \omega p_{I_\varsigma(R,m_R)}(1) \\
& +\omega(1-\delta)\Pr\left[\theta > p_{I_\varsigma(R,m_R)}(1)\,|\theta\in\Theta\right]\left[p_{I_\varsigma(R,m_R)}(0) - p_{I_\varsigma(R,m_R)}(1)\right] \\
\geq\ & -\mathbb{E}[\theta+\beta|\theta\in\Theta] + \omega p_{I_\varsigma(L,m_R)}(1) \\
& +\omega(1-\delta)\Pr\left[-\theta > p_{I_\varsigma(L,m_R)}(1)\,|\theta\in\Theta\right]\left[p_{I_\varsigma(L,m_R)}(0) - p_{I_\varsigma(L,m_R)}(1)\right]
\end{aligned} \tag{17}$$

Note that $0 < p_{I_\varsigma(R,m_R)}(1) < \mathbb{E}[\theta|\theta\in\Theta]$ and since $\beta > 0$, then $\mathbb{E}[\theta+\beta|\theta\in\Theta] > 0$. Suppose $\varsigma = AT$. Based on Proposition 3, $p_{I_\varsigma(R,m_R)}(1) = p_{I_\varsigma(L,m_R)}(1)$. Moreover, since $\theta\in\Theta \Rightarrow \theta > -p_{I_\varsigma(R,m_R)}(1)$ then $\Pr\left[-\theta > p_{I_\varsigma(L,m_R)}(1)\,|\theta\in\Theta\right] = 0$. Thus, condition (17) never binds when $\varsigma = AT$. Suppose $\varsigma = FT$. Note that

$$\begin{aligned}
(1-\delta)\Pr[\theta > p_{R,m_R}(1)\,|\theta\in\Theta][p_{R,m_R}(0) - p_{R,m_R}(1)] &= \mathbb{E}[\theta|\theta\in\Theta] - p_{R,m_R}(1) \\
(1-\delta)\Pr[-\theta > p_{L,m_R}(1)\,|\theta\in\Theta][p_{L,m_R}(0) - p_{L,m_R}(1)] &= \mathbb{E}[-\theta|\theta\in\Theta] - p_{L,m_R}(1)
\end{aligned}$$

Thus, condition (17) can be rewritten as

$$(1+\omega)\,\mathbb{E}[\theta|\theta\in\Theta] + \beta \geq 0$$

since $\mathbb{E}[\theta|\theta \in \Theta] > 0$, this condition never binds for $\varsigma = FT$ as well.

The manager follows the recommendation to choose $a = L$ if and only if

$$\begin{aligned}
& -\mathbb{E}[\theta+\beta|\theta \in \Theta^c] + \omega p_{I_\varsigma(L,m_L)}(1) \\
& +\omega(1-\delta)\Pr\left[-\theta > p_{I_\varsigma(L,m_L)}(1)\,|\theta \in \Theta^c\right]\left[p_{I_\varsigma(L,m_L)}(0) - p_{I_\varsigma(L,m_L)}(1)\right] \\
\geq\ & +\mathbb{E}[\theta+\beta|\theta \in \Theta^c] + \omega p_{I_\varsigma(R,m_L)}(1) \\
& +\omega(1-\delta)\Pr\left[\theta > p_{I_\varsigma(R,m_L)}(1)\,|\theta \in \Theta^c\right]\left[p_{I_\varsigma(R,m_L)}(0) - p_{I_\varsigma(R,m_L)}(1)\right]
\end{aligned} \tag{18}$$

Suppose $\varsigma = AT$ and note that $p_{I_\varsigma(L,m_L)}(1) = p_{I_\varsigma(R,m_L)}(1)$. Note also that $\theta \in \Theta^c \Rightarrow \theta < p_{I_\varsigma(R,m_L)}(1)$, and hence, $\Pr\left[\theta > p_{I_\varsigma(R,m_L)}(1)\,|\theta \in \Theta^c\right] = 0$. Last note that

$$(1-\delta)\Pr\left[-\theta > p_{I_\varsigma(L,m_L)}(1)\,|\theta \in \Theta^c\right]\left[p_{I_\varsigma(L,m_L)}(0) - p_{I_\varsigma(L,m_L)}(1)\right] = \mathbb{E}[-\theta|\theta \in \Theta^c] - p_{I_\varsigma(L,m_L)}(1)$$

Therefore, when $\varsigma = AT$ condition (18) can be rewritten as

$$\beta \leq -\left(1+\frac{\omega}{2}\right)\mathbb{E}[\theta|\theta \in \Theta^c] - \frac{\omega}{2}p_{I_\varsigma(L,m_L)}(1) \tag{19}$$

Suppose $\varsigma = FT$ and note that

$$\begin{aligned}
(1-\delta)\Pr[-\theta > p_{L,m_L}(1)\,|\theta \in \Theta^c][p_{L,m_L}(0) - p_{L,m_L}(1)] &= \mathbb{E}[-\theta|\theta \in \Theta^c] - p_{L,m_L}(1) \\
(1-\delta)\Pr[\theta > p_{R,m_L}(1)\,|\theta \in \Theta^c][p_{R,m_L}(0) - p_{R,m_L}(1)] &= \mathbb{E}[\theta|\theta \in \Theta^c] - p_{R,m_L}(1)
\end{aligned}$$

therefore, when $\varsigma = FT$ condition (18) can be rewritten as

$$\beta \leq -(1+\omega)\,\mathbb{E}[\theta|\theta \in \Theta^c] \tag{20}$$

Note, however, that

$$0 < p_{I_\varsigma(L,m_L)}(1) < -\mathbb{E}[\theta|\theta \in \Theta^c]$$

Therefore, the RHS of (19) is small than the RHS of (20) if and only if $p_{I_\varsigma(L,m_L)}(1) > \mathbb{E}[\theta|\theta \in \Theta^c]$. It follows, if the manager has incentives to follow decision rule Θ under $\varsigma = AT$, it has incentives to follow decision rule Θ when $\varsigma = FT$. Note that in both regimes the on-equilibrium path prices and the activist incentives are identical given Θ. We conclude, if Θ is an equilibrium when $\varsigma = AT$, it is also an equilibrium when $\varsigma = FT$. This analysis also proves that for each $\varsigma \in \{AT, FT\}$ and any $\omega \geq 0$ there is β^*_ς such that an influential equilibrium exists if and only if $\beta \leq \beta^*_\varsigma$, where $\beta^*_{AT} < \beta^*_{FT}$. ■

Appendix B

Lemma B.1 *For any $\omega > 0$ there is β^* such that a threshold equilibrium under No-Transparency exists if and only if $\beta \leq \beta^*$.*

Proof. When Θ is a threshold decision rule, then $-\mathbb{E}[\theta|\theta \in \Theta] = -\mathbb{E}[\theta|\theta > \tau]$. Since $\mathbb{E}[\theta + \beta|\theta \geq \tau] \geq \mathbb{E}[\theta + \beta|] > 0$ the condition $\Psi_R\left(\left[\tau, \overline{\theta}\right], \omega\right) \leq \beta$ never binds. It follows, a threshold equilibrium exists if and only if $\beta \leq \beta^* \equiv \max_{\tau \in [\underline{\tau}, \overline{\tau}]} \Psi_L\left(\left[\tau, \overline{\theta}\right], \omega\right)$ ■

Lemma B.2 *A fully revealing equilibrium exists if and only if $\beta \leq -\underline{\tau}$. If a fully revealing equilibrium exists, there is a threshold equilibrium which is strictly more efficient.*

Proof. If the equilibrium is fully revealing, the effective threshold is $-\beta$. According to Proposition 1, such equilibrium exists if and only if $-\beta \in [\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$. Since $-\beta < -z(\beta)$, a fully revealing equilibrium exists if and only if $-\beta \in [\underline{\tau}, \overline{\tau}]$. For the same reason, if $-\beta \in [\underline{\tau}, \overline{\tau}]$ then $\min\{-z(\beta), 0\} \in [\underline{\tau}, \overline{\tau}]$ as well. Therefore, if a fully revealing equilibrium exists, there also exists more efficient equilibrium with threshold $-\beta < \min\{-z(\beta), 0\} \leq 0$. ■

Lemma B.3 *$\underline{\tau}$ decreases with δ and $\overline{\tau}$ increases with δ. Moreover, $\lim_{\delta \to 0} \underline{\tau} = \lim_{\delta \to 0} \overline{\tau} = 0$.*

Proof. I argue $\underline{\tau}$ decreases with δ. To see why, note that $\varphi(p, \tau)$ increases with δ, and hence, $\pi(\tau)$ increases with δ as well. Moreover, $\pi(\tau)$ increases with τ when $\tau < 0$. Since $\pi(\underline{\tau}) + \underline{\tau} = 0$ and $\underline{\tau} < 0$, applying the implicit function theorem on $\pi(\underline{\tau}) + \underline{\tau} = 0$ concludes this argument. Note that for any τ, $\lim_{\delta \to 0} \varphi(p, \tau) = \lim_{\delta \to 0} \pi(\tau) = -|\tau|$. Also recall $\pi(\underline{\tau}) + \underline{\tau}$ for any δ. If on the contrary $\lim_{\delta \to 0} \underline{\tau} < 0$ then $\lim_{\delta \to 0} (\pi(\underline{\tau}) + \underline{\tau}) = -|\lim_{\delta \to 0} \underline{\tau}| + \lim_{\delta \to 0} \underline{\tau} < 0$, a contradiction. A similar reasoning shows that $\overline{\tau}$ increases with δ and $\lim_{\delta \to 0} \overline{\tau} = 0$. ■

Lemma B.4 [Voice and Exit Exhibit Substitution Under Transparency] *If $\mathbb{E}[\theta|\theta > 0] + \mathbb{E}[\theta|\theta < 0] < 0$ there exists δ such that $\tau^*_{AT} > 0$.*

Proof. Suppose $\mathbb{E}[\theta|\theta > 0] + \mathbb{E}[\theta|\theta < 0] < 0$. Let τ_0 be the unique solution of $\mathbb{E}[\theta|\theta > x] + \mathbb{E}[\theta|\theta < x] = 0$ and note that $\tau_0 > 0$. Suppose on the contrary $\tau^*_{AT}(\delta) \leq 0$ for any δ. Thus, for any δ and $\beta < -\mathbb{E}[\theta|\theta < 0]$ a threshold equilibrium under Action-Transparency exists. Based on Proposition 3, the threshold is $\tau^*_{AT}(\delta)$. Recall, the price upon exit must satisfy $\varphi_L(\tau^*_{AT}(\delta), p) = \varphi_R(\tau^*_{AT}(\delta), p) = p$ for any δ. It follows from (16) that

$$\lim_{\delta \to 1} \mathbb{E}[\theta|\theta \geq \tau^*_{AT}(\delta)] + \mathbb{E}[\theta|\theta < \tau^*_{AT}(\delta)] = 0$$

Therefore, $\lim_{\delta \to 1} \tau^*_{AT}(\delta) = \tau_0 > 0$. This contradicts the assumption that $\tau^*_{AT}(\delta) \leq 0$ for any δ. ■

Appendix C - Activist's Expertise and Block Size

Consider a variant of the baseline model in which the activist perfectly observes θ with probability $\lambda \in (0,1]$, and with the complement probability the activist is uninformed about θ. Whether or not the activist is informed is her own private information. Parameter λ captures the quality of the activist's private information. To simplify the analysis, I assume throughout this section that $\mathbb{E}[\theta] > 0$.

Similar to the baseline model, the informed activist recommends on threshold τ only if $-p \le \tau \le p$, and she exits when $v(\theta,\tau) \le p$. By contrast, since $\mathbb{E}[\theta] > 0$, the uninformed activist advises the manager to choose strategy R and the manager follows this recommendation. If $p < \mathbb{E}[\theta]$ the uninformed activist exits only when she needs liquidity, and if $p \ge \mathbb{E}[\theta]$ the uninformed activist exits for sure. It follows, the price upon exit p must satisfy $p = \varphi(p,\tau,\lambda)$ where

$$\varphi(p,\tau,\lambda) = \begin{cases} \frac{\lambda\delta\mathbb{E}[v(\theta,\tau)]+\lambda(1-\delta)\Pr[v(\theta,\tau)\le p]\mathbb{E}[v(\theta,\tau)|v(\theta,\tau)\le p]+\delta(1-\lambda)\mathbb{E}[\theta]}{\lambda\delta+\lambda(1-\delta)\Pr[v(\theta,\tau)\le p]+\delta(1-\lambda)} & \text{if } p < \mathbb{E}[\theta] \\ \frac{\lambda\delta\mathbb{E}[v(\theta,\tau)]+\lambda(1-\delta)\Pr[v(\theta,\tau)\le p]\mathbb{E}[v(\theta,\tau)|v(\theta,\tau)\le p]+(1-\lambda)\mathbb{E}[\theta]}{\lambda\delta+\lambda(1-\delta)\Pr[v(\theta,\tau)\le p]+(1-\lambda)} & \text{if } p \ge \mathbb{E}[\theta] \end{cases} \tag{21}$$

Note that (21) is a special case of (6) when $\lambda = 1$.

Lemma C.1 *For any $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$ and $x \ge 0$ define*

$$\hat{\varphi}(p,\tau,x) \equiv \frac{\mathbb{E}[v(\theta,\tau)] + \frac{1-\delta}{\delta}\Pr[v(\theta,\tau) \le p]\,\mathbb{E}[v(\theta,\tau)\,|v(\theta,\tau) \le p] + x\mathbb{E}[\theta]}{1 + \frac{1-\delta}{\delta}\Pr[v(\theta,\tau) \le p] + x} \tag{22}$$

The solution of $p = \hat{\varphi}(p,\tau,x)$ exists, it is unique, and is given by $\hat{\pi}(\tau,x) = \min_{p \ge \min\{-\tau,\tau\}} \hat{\varphi}(p,\tau,x)$. Moreover, there are unique $\underline{\tau}(x) < 0 < \overline{\tau}(x)$ such that:

(i) $-\hat{\pi}(\tau,x) \le \tau \le \hat{\pi}(\tau,x)$ *if and only if* $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$.

(ii) $\frac{\partial \hat{\pi}(\tau,x)}{\partial x} > 0$ *if and only if* $\hat{\pi}(\tau,x) < \mathbb{E}[\theta]$.

(iii) *If* $\underline{\tau}(0) > (=,<) - \mathbb{E}[\theta]$ *then for all* $x \ge 0$: $\underline{\tau}(x) > (=,<) - \mathbb{E}[\theta]$ *and* $\frac{\partial \underline{\tau}(x)}{\partial x} < (=,>)\, 0$.

Proof. Note that (22) is a special case of (6) when $x = 0$. Similar to the arguments in the proof of Proposition 1, $p = \hat{\varphi}(p, \tau, x)$ has a unique solution, and there are $\underline{\tau}(x) < 0 < \overline{\tau}(x)$ with the properties given by part (i).

Consider part (ii). Since $\hat{\pi}(\tau, x)$ is the unique minimum of $\hat{\varphi}(p, \tau, x)$, then $\frac{\partial \hat{\pi}(\tau,x)}{\partial x} = \frac{\partial \hat{\varphi}(p,\tau,x)}{\partial x}|_{p=\hat{\pi}}$. Note that $\hat{\varphi}(p, \tau, x)$ is a weighted average of $\hat{\varphi}(p, \tau, 0)$ and $\mathbb{E}[\theta]$, and the weight on $\mathbb{E}[\theta]$ increases with x. Therefore, $\frac{\partial \hat{\varphi}(p,\tau,x)}{\partial x} > 0 \Leftrightarrow \hat{\varphi}(p, \tau, x) < \mathbb{E}[\theta]$. Since $\hat{\varphi}(\hat{\pi}, \tau, x) = \hat{\pi}$, it follows that $\frac{\partial \hat{\varphi}(p,\tau,x)}{\partial x}|_{p=\hat{\pi}} > 0 \Leftrightarrow \hat{\pi} < \mathbb{E}[\theta]$ as required.

Consider part (iii). Note that $\underline{\tau}(x)$ is defined by the unique negative solution of $\hat{\pi}(\underline{\tau}(x), x) + \underline{\tau}(x) = 0$. By the implicit function theorem, $\frac{\partial \underline{\tau}(x)}{\partial x} = -\frac{\frac{\partial \hat{\pi}(\tau,x)}{\partial x}|_{\tau=\underline{\tau}(x)}}{\frac{\partial \hat{\pi}(\tau,x)}{\partial \tau}|_{\tau=\underline{\tau}(x)}+1}$. Since $\underline{\tau}(x) < 0$ then $\frac{\partial \hat{\pi}(\tau,x)}{\partial \tau}|_{\tau=\underline{\tau}(x)} > 0$. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} < 0 \Leftrightarrow \frac{\partial \hat{\pi}(\tau,x)}{\partial x}|_{\tau=\underline{\tau}(x)} > 0$. Suppose $\underline{\tau}(x) > (=,<) - \mathbb{E}[\theta]$. Then $\hat{\pi}(\underline{\tau}(x), x) < (=,>) \mathbb{E}[\theta]$. Based on part (ii), $\frac{\partial \hat{\pi}(\tau,x)}{\partial x}|_{\underline{\tau}=\underline{\tau}(x)} > (=,<)0$. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} < (=,>) 0$. We conclude, $\underline{\tau}(x) > -\mathbb{E}[\theta] \Leftrightarrow \frac{\partial \underline{\tau}(x)}{\partial x} < 0$. Note that since $\hat{\pi}(\tau, x)$ is continuous in x then $\underline{\tau}(x)$ is continuous in x as well. Thus, the observation $\underline{\tau}(x) > -\mathbb{E}[\theta] \Leftrightarrow \frac{\partial \underline{\tau}(x)}{\partial x} < 0$ implies that if $\underline{\tau}(0) > (=,<) - \mathbb{E}[\theta]$ then for all $x \geq 0$ we have $\underline{\tau}(x) > (=,<) - \mathbb{E}[\theta]$ and $\frac{\partial \underline{\tau}(x)}{\partial x} < (=,>) 0$ as required. ■

Proposition C.1 *An influential equilibrium exists if and only if*

$$\beta \leq -\mathbb{E}\left[\theta|\theta < \max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right\}\right] \tag{23}$$

In equilibrium with threshold τ, the price upon exit is given by

$$\pi(\tau, \lambda) = \min\left\{\hat{\pi}\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), \hat{\pi}\left(\tau, \frac{1-\lambda}{\lambda}\right)\right\} \tag{24}$$

Proof. To ease notation we let $x_1 = \frac{1-\lambda}{\lambda}$ and $x_2 = \frac{1}{\delta}\frac{1-\lambda}{\lambda}$. Note that $x_1 < x_2$. If an influential equilibrium with threshold τ exists, the price upon exit must be the solution of $\varphi(p, \tau, \lambda) = p$. Based on Lemma C.1, the unique solution of $\hat{\varphi}(p, \tau, x_1) = p$ is $\hat{\pi}(\tau, x_1)$, and the unique solution of $\hat{\varphi}(p, \tau, x_2) = p$ is $\hat{\pi}(\tau, x_2)$. According to part (ii) of Lemma C.1, if $\hat{\pi}(\tau, x_2) < \mathbb{E}[\theta]$ then $\hat{\pi}(\tau, x_1) < \hat{\pi}(\tau, x_2)$ and if $\mathbb{E}[\theta] < \hat{\pi}(\tau, x_2)$ then $\hat{\pi}(\tau, x_2) < \hat{\pi}(\tau, x_1)$. Therefore, based on (21), the unique solution of $\varphi(p, \tau, \lambda) = p$ is (24).

There are two cases to consider. First, suppose $\underline{\tau}(x_1) \leq \underline{\tau}(x_2)$. According to Lemma C.1 part (iii), this implies $\underline{\tau}(0) \leq \underline{\tau}(x) \leq -\mathbb{E}[\theta]$ for all $x \geq 0$. Since $\underline{\tau}(x) = -\hat{\pi}(\underline{\tau}(x), x)$ then $\hat{\pi}(\underline{\tau}(x), x) \geq \mathbb{E}[\theta]$ for all $x \geq 0$. Since $\hat{\pi}(\tau, x)$ increases in τ when $\tau < 0$ then $\hat{\pi}(\tau, x) \geq \mathbb{E}[\theta]$ for all $x \geq 0$ and $\tau \in [\underline{\tau}(0), 0]$. According to part (ii) of Lemma C.1, $\pi(\tau, \lambda) = \hat{\pi}(\tau, x_2)$. Note that $-\hat{\pi}(\tau, x_2) \leq \tau \Leftrightarrow \underline{\tau}(x_2) \leq \tau$. Similar to Proposition 1, an influential equilibrium exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}(x_2)]$.

Second, suppose $\underline{\tau}(x_2) < \underline{\tau}(x_1)$. Based on Lemma C.1 part (iii), this implies $-\mathbb{E}[\theta] < \underline{\tau}(x) < \underline{\tau}(0)$ for all $x \geq 0$, and hence, $\hat{\pi}(\underline{\tau}(x), x) < \mathbb{E}[\theta]$ for all $x \geq 0$. Based on Lemma C.1, if $\hat{\pi}(\sigma, x) = \mathbb{E}[\theta]$ for some x, then $\hat{\pi}(\sigma, x) = \mathbb{E}[\theta]$ for all x. Recall $\hat{\pi}(\tau, x)$ increases in τ when $\tau < 0$. Let $\sigma < 0$ be the unique solution of $\hat{\pi}(\sigma, 0) = \mathbb{E}[\theta]$ when $\hat{\pi}(0, 0) > \mathbb{E}[\theta]$ and let $\sigma = 0$ if $\hat{\pi}(0, 0) \leq \mathbb{E}[\theta]$. Note that since $\hat{\pi}(\underline{\tau}(0), 0) < \mathbb{E}[\theta]$ then $\underline{\tau}(0) < \sigma$. If $\tau \leq \sigma$ then $\hat{\pi}(\tau, x) \leq \mathbb{E}[\theta]$ for all x. According to part (ii) of Lemma C.1, the price upon exit is $\pi(\tau, \lambda) = \hat{\pi}(\tau, x_1)$. Threshold $\tau \leq \sigma$ can be supported in equilibrium only if $-\hat{\pi}(\tau, x_1) \leq \tau \Leftrightarrow \underline{\tau}(x_1) \leq \tau$. If $\sigma < \tau \leq 0$ then $\hat{\pi}(\tau, x) > \mathbb{E}[\theta]$ for all x. Based on Lemma C.1, the price upon exit is $\pi(\tau, \lambda) = \hat{\pi}(\tau, x_2)$. Threshold $\sigma < \tau \leq 0$ can be supported in equilibrium only if $-\hat{\pi}(\tau, x_2) \leq \tau \Leftrightarrow \underline{\tau}(x_2) \leq \tau$. Note that $\underline{\tau}(x_2) < \underline{\tau}(0)$ and $\underline{\tau}(0) < \sigma$. Thus, $\sigma < \tau \leq 0$ implies $\underline{\tau}(x_2) \leq \tau$, and the constraint $\underline{\tau}(x_2) \leq \tau$ does not bind. Overall, threshold τ can be supported in equilibrium only if $\underline{\tau}(x_1) \leq \tau$. Similar to Proposition 1, an influential equilibrium exists if and only if $\beta \leq -\mathbb{E}[\theta|\theta < \underline{\tau}(x_1)]$. ■

Proposition C.2 *Let λ^* be the (highest) level of λ that maximizes the expected value of the firm in equilibrium. Then, $\lambda^* < 1$ if and only if*

$$-\mathbb{E}[\theta|\theta < \underline{\tau}] < \beta < -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]] \tag{25}$$

Proof. Consider the most efficient influential equilibrium. Based on Proposition C.1, an influential equilibrium exists if and only if (23) holds. If (23) holds then an equilibrium with threshold $\min\{-z(\beta), 0\}$ exists, and this is the most efficient threshold that can be supported in equilibrium. In this equilibrium, the value of the firm is $\lambda\mathbb{E}[v(\theta, \min\{0, -z(\beta)\})] + (1-\lambda)\mathbb{E}[\theta]$, and note that it is greater than $\mathbb{E}[\theta]$ and it strictly increases with λ.

There are three cases to consider. First, suppose $\beta \le -\mathbb{E}[\theta|\theta < \underline{\tau}(0)]$. Based on Proposition 1, if $\lambda = 1$ an influential equilibrium with threshold $\min\{-z(\beta), 0\}$ exists and the value of the firm is $\mathbb{E}[v(\theta, \min\{0, -z(\beta)\})]$. Therefore, $\lambda^* = 1$.

Second, suppose $\beta \ge -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]]$. If $\underline{\tau}(0) \le -\mathbb{E}[\theta]$ then, based on Lemma C.1 part (iii), $\frac{\partial \underline{\tau}(x)}{\partial x} \ge 0$. It follows, the upper bound in (23) increases in λ. Thus, both the likelihood that an influential equilibrium exists and the value of the firm in an influential equilibrium increase with λ. For these reasons, $\lambda^* = 1$. If $-\mathbb{E}[\theta] < \underline{\tau}(0)$ then, based on Lemma C.1 part (iii), $-\mathbb{E}[\theta] < \max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right\} < \underline{\tau}(0)$. Since $\beta \ge -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]]$, based on Proposition C.1, an influential equilibrium does not exist for any λ, and hence, $\lambda^* = 1$.

Third, suppose $-\mathbb{E}[\theta|\theta < \underline{\tau}(0)] < \beta < -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]]$. Since $-\mathbb{E}[\theta] < \underline{\tau}(0)$, based on Lemma C.1 part (iii), $-\mathbb{E}[\theta] < \max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right\} < \underline{\tau}(0)$, and the upper bound in (23) decreases in λ. In particular, if $\lambda = 1$ then an influential equilibrium does not exists, and if $\lambda \to 0$ then $\max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right\} \to -\mathbb{E}[\theta]$. Since $\beta < -\mathbb{E}[\theta|\theta < -\mathbb{E}[\theta]]$, there is $\lambda_0 \in (0, 1)$ such that an influential equilibrium exists if and only if $\lambda < \lambda_0$. Since conditional on the existence of an influential equilibrium the value of the firm strictly increases with λ, we have $\lambda^* = \lambda_0$. ■

6.1 Acquisition of Information

Suppose initially the activist is uninformed about θ, but she has the option to observe θ by investing $c \ge 0$. The activist's decision to acquire information is unobserved by the market maker and the manager. I assume that c is the activist's private information, where c is distributed according to a cumulative distribution function G with full support over $[0, \infty)$, and it is independent of all other random variables.

Suppose in equilibrium the manager implements threshold τ and the price upon exit is p. Consider the decision of the activist to acquire information. If the activist acquires information her expected value per share is

$$\delta p + (1 - \delta)\mathbb{E}[\max\{v(\theta, \tau), p\}] - c/\alpha \tag{26}$$

The benefit from the acquisition of information is twofold: the activist can use the information in order to advise the manager to follow threshold τ, but the information can also be used in order to exit when the share is over-priced. If the activist decides to remain uninformed she gets

$$\delta p + (1-\delta)\max\{\mathbb{E}[\theta], p\} \tag{27}$$

It follows, in any equilibrium there is $c^* > 0$ such that the activist acquires information if and only if $c \leq c^*$. The probability the activist is informed in equilibrium is given by $\lambda = G(c^*)$. Based on the analysis above, for any threshold τ and λ the price upon exit is given by $\pi(\tau, \lambda)$. However, here, λ is endogenous. In particular, c^* is given by the solution of

$$\frac{c}{\alpha(1-\delta)} = \mathbb{E}[\max\{v(\theta,\tau), \pi(\tau, G(c))\}] - \max\{\mathbb{E}[\theta], \pi(\tau, G(c))\} \tag{28}$$

It can be shown that the right hand side of (28) decreases with c, and therefore, for any τ the solution of (28) exists and is unique.

As in the baseline model, a non-influential equilibrium always exists, and the level of information acquisition in this equilibrium is given by the solution of (28) when $\tau = \underline{\theta}$. Similar to the analysis when λ is exogenous, an influential equilibrium exists if and only if β is below some critical value which depends on λ. Different from that analysis, here λ is given by $G(c^*)$ where c^* is the solution of (28) when $\tau = \min\{0, -z(\beta)\}$.

Proposition C.3 *Suppose condition (25) holds. There are* $G_1 \overset{FDSD}{\prec} G_2$ *such that* $G_1(c_1^*) < G_2(c_2^*)$ *and an influential equilibrium exists if* $c \sim G_2$ *but it does not exist when* $c \sim G_1$.

Proof. I start by arguing that for a given τ, (28) has a unique solution given by $c^*(\tau)$. For this purpose I prove that the RHS of (28) is decreasing in c. To see why, note that if $\pi(\tau, G(c)) \leq \mathbb{E}[\theta]$ then the right hand side of (28) increases in $\pi(\tau, G(c))$, and according to Lemma C.1 part (ii) and Proposition C.1, $\pi(\tau, G(c))$ decreases in c. If $\pi(\tau, G(c)) > \mathbb{E}[\theta]$ then the right hand side of (28) decreases in $\pi(\tau, G(c))$, and according to Lemma C.1 part (ii) and Proposition C.1, $\pi(\tau, G(c))$ increases in c. In both cases, the RHS of (28) decreases in c. Also note that the RHS is positive and it is bounded from below and from above. This completes the argument.

Recall the manager follows threshold τ if and only if $\tau \leq -z(\beta)$. Thus, the most efficient equilibrium is weakly smaller than $\min\{-z(\beta), 0\}$. Given the beliefs about her decision to acquire information, the informed activist recommends on threshold τ if and only if

$$-\pi(\tau, G(c^*(\tau))) \leq \tau \leq \pi(\tau, G(c^*(\tau)))$$

I argue that $\pi(\tau, G(c^*(\tau)))$ increases with τ, when $\tau < 0$. Recall $\frac{\partial \hat{\pi}(\tau,x)}{\partial \tau} > 0 \Leftrightarrow \tau < 0$. If $\pi(\tau, G(c)) < \mathbb{E}[\theta]$ then the RHS of (28) increases in $\pi(\tau, G(c))$, and hence, $c^*(\tau)$ increases in τ. Since the LHS of (28) is higher, it must be that the RHS of (28) is higher as well. This implies that $\pi(\tau, G(c^*(\tau)))$ increases with τ. Similarly, if $\pi(\tau, G(c)) > \mathbb{E}[\theta]$ then the RHS of (28) decreases in $\pi(\tau, G(c))$ and hence $c^*(\tau)$ decreases in τ. Since the LHS of (28) is smaller, it must be that the RHS of (28) is smaller as well. This implies that $\pi(\tau, G(c^*(\tau)))$ increases with τ. Since $\pi(\tau, G(c^*(\tau)))$ increases with τ it is sufficient to consider the highest threshold below $\min\{-z(\beta), 0\}$. It follows, an influential equilibrium with threshold $\min\{-z(\beta), 0\}$ exists if and only if $-\pi(\tau_\beta, \lambda_\beta) \leq \tau_\beta$ where $\lambda_\beta \equiv G(c^*(\min\{-z(\beta), 0\}))$ and $\tau_\beta = \min\{-z(\beta), 0\}$. Based on Proposition C.1, this condition holds if and only if

$$\beta \leq -\mathbb{E}\left[\theta | \theta < \max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda_\beta}{\lambda_\beta}\right), \underline{\tau}\left(\frac{1-\lambda_\beta}{\lambda_\beta}\right)\right\}\right] \tag{29}$$

The proof follows in three steps. First, since $\underline{\tau}(0) > -z(\beta) > -\mathbb{E}[\theta]$ and $\lim_{x\to\infty} \underline{\tau}(x) = -\mathbb{E}[\theta]$ there is $x_2 > 0$ such that $\underline{\tau}(x_2) = -z(\beta)$. Define c_2 such that

$$\frac{c_2}{\alpha(1-\delta)} = \mathbb{E}[\max\{v(\theta, -z(\beta)), z(\beta)\}] - \mathbb{E}[\theta]$$

and let G_2 be such that $G_2(c_2) = \frac{1}{1+x_2}$. It follows, $\underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right) = \underline{\tau}(x_2)$ and since $\underline{\tau}(x_2) = -z(\beta)$ then $\underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right) = -z(\beta)$. Recall $\hat{\pi}(\underline{\tau}(x), x) = -\underline{\tau}(x)$ for all $x \geq 0$, therefore, $\hat{\pi}\left(-z(\beta), \frac{1-G_2(c_2)}{G_2(c_2)}\right) = z(\beta) < \mathbb{E}[\theta]$. Based on Proposition C.1 and Lemma C.1 part (iii), $\hat{\pi}\left(-z(\beta), \frac{1-G_2(c_2)}{G_2(c_2)}\right) = \pi(-z(\beta), G_2(c_2))$. It follows, c_2 is the unique solution of (28) when

$\tau = -z(\beta)$ and $G = G_2$. We conclude that for $G = G_2$ there is an influential equilibrium with threshold $\tau = -z(\beta)$ and $c_2^* = c_2$.

Second, consider a non-influential equilibrium when $G = G_2$. Denote by c_{NR} the level of information acquisition in this case, and note that it is given by the unique solution of (28) when $\tau = \underline{\theta}$. Recall that for a given c, $\pi(\tau, G_2(c))$ increases with τ when $\tau < 0$. Therefore, $\mathbb{E}[\theta] > \pi(-z(\beta), G_2(c_2^*)) > \pi(\underline{\theta}, G_2(c_2^*))$. It follows, when the RHS of (28) is evaluated at $\tau = \underline{\theta}$, $G = G_2$, and $c = c_2^*$ it is lower than when it is evaluated at $\tau = -z(\beta)$, $G = G_2$, and $c = c_2^*$. This implies that $c_{NR} < c_2^*$.

Third, consider any G_1 such that $G_1 \overset{FDSD}{\prec} G_2$. That is, $G_2(c) < G_1(c)$ for all c. I argue that an influential equilibrium does not exist when $G = G_1$. Suppose on the contrary an influential equilibrium exists. By assumption, $G_2(c_2^*) < G_1(c_2^*)$. Also, recall $\pi(-z(\beta), G_2(c_2^*)) < \mathbb{E}[\theta]$. Based on Lemma C.1 part (ii) and Proposition C.1, $\pi(-z(\beta), \lambda)$ is decreasing in λ. Therefore,

$$\mathbb{E}[\max\{v(\theta, -z(\beta)), \pi(-z(\beta), \lambda)\}] - \mathbb{E}[\theta]$$

is decreasing in λ. It follows, $G_2(c_2^*) < G_1(c_1^*)$. Since $\underline{\tau}(0) > -\mathbb{E}[\theta]$, based on Lemma C.1 part (iii), $\underline{\tau}\left(\frac{1-G_2(c_2^*)}{G_2(c_2^*)}\right) < \underline{\tau}\left(\frac{1-G_1(c_1^*)}{G_1(c_1^*)}\right)$. Since $-z(\beta) = \underline{\tau}\left(\frac{1-G_2(c_2^*)}{G_2(c_2^*)}\right)$ then $-z(\beta) < \underline{\tau}\left(\frac{1-G_1(c_1^*)}{G_1(c_1^*)}\right)$. This contradicts the condition in Proposition C.1. Thus, an influential equilibrium does not exist when $G = G_1$. Note that in a non-influential equilibrium, if G_1 is sufficiently close to G_2 then c_1^* converges to $c_{NR} < c_2^*$ and it is possible to find G_1 sufficiently close to G_2 such that $G_1(c_1^*) \approx G_2(c_{NR}) < G_2(c_2^*)$. This completes the proof. ■