

# A Model of Monetary Policy and Risk Premia

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## Abstract

We present a model in which monetary policy drives risk premia and the cost of capital. Risk tolerant agents (banks) borrow from risk averse agents (depositors) and invest in risky assets, but they are subject to a reserve requirement. The central bank uses open market operations to set the nominal interest rate, which is the opportunity cost of holding reserves, and in this way it controls the cost of leverage and the level of risk taking in the economy. There are no nominal rigidities. Lower nominal rates result in lower risk premia and higher volatility. Unexpected policy shocks are amplified via balance sheet effects. We use the model to examine the implications of various conventional and unconventional policies for asset prices and stability.

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# I. Introduction

The textbook model of monetary policy operates through the link between nominal and real interest rates induced by nominal rigidities. At the same time, recent experience has led to a broad understanding that monetary interventions, through their impact on financial institutions, also affect the risk premium component of the cost of capital. This mechanism is particularly salient during periods of economic stress, and it has prompted central banks around the world to devise policies aimed at stabilizing the prices of risky assets. In calmer years, its presence is felt more subtly: did the low interest rate environment of the mid 2000s encourage excessive risk taking by financial institutions? Phenomena as diverse as the “Greenspan put,” large scale asset purchases, “Operation Twist”, across-the-board guarantees, and lender-of-last-resort interventions can all be brought under the umbrella of a risk premium channel of monetary policy.

The canonical New Keynesian framework does not speak to risk premia. Since the risk premium channel runs through financial institutions, it has surfaced in the banking literature alternatively as the bank lending channel, the credit channel, or financial stability policy more broadly. Yet risk is the province of asset pricing.

In this paper, we provide an asset pricing framework for studying the risk premium channel of monetary policy. The central bank in the model varies the nominal interest rate in order to regulate the effective risk aversion of the marginal investor in the economy. It does so by influencing financial institutions’ cost of taking leverage, and hence also the aggregate level of risk taking. Lowering the nominal rate reduces the cost of taking leverage and increases aggregate risk taking, resulting in a decrease in equilibrium risk premia.

More concretely, we model an endowment economy populated by two types of agents, those with low risk aversion and those with high risk aversion. We think of the more risk tolerant agents as pooling their wealth in the form of the equity capital of financial intermediaries, which we identify as banks. Since banks invest on behalf of the risk tolerant agents, in equilibrium they take leverage. They do so by borrowing from the more risk averse agents, or taking deposits. Our view of banks and deposits is purposely simplified, abstracting from other features such as screening and monitoring in order to focus on risk taking and risk premia.

The central bank requires banks to hold a fraction of the deposits that they raise as reserves. Reserves are a liability of the central bank and they enter circulation as a result of

open market operations.<sup>1</sup> Aside from this restriction, the model is frictionless. In particular, there are no nominal rigidities, which allows us to focus exclusively on the risk premium channel. The single state variable of the model is the share of bank capital in total wealth.

The difference between the (endogenous) return on reserves and the return on risk-free bonds represents the opportunity cost of holding reserves, and hence the cost of taking leverage. This difference equals the nominal interest rate. Hence, the central bank regulates banks' demand for leverage by inducing changes in the nominal rate. An increase in the nominal rate represents an increase in the cost of leverage and so it reduces banks' demand for leverage. As banks are the risk tolerant investors in the economy, this causes aggregate risk taking to fall and the economy's effective risk aversion to rise, driving up the equilibrium risk premium.

The solution to the model shows that the nominal rate equals the shadow price of banks' reserves requirement. When the reserve requirement binds tightly, the nominal rate is high. When the reserves requirement is slack, banks' demand for leverage is satiated, their holdings of the risky endowment claim are at the unconstrained optimum, and the nominal rate is zero. A zero nominal rate therefore implies that further easing cannot increase banks' risk taking. Indeed, any further attempt to lower the nominal rate results in banks holding excess reserves. As a result, the nominal rate in the model is bounded below by zero.

We show that banks' problem can be rewritten as an unconstrained portfolio-choice problem by replacing the real interest rate (the return on risk-free bonds or deposits) with the real Fed Funds rate, the rate that banks would charge to lend reserves to each other in an interbank market. The spread between these two rates can be referred to as the external finance spread. Therefore, a valid interpretation of how monetary policy drives bank leverage is that it alters the external finance spread. The model predicts that this spread is in fact proportional to the nominal rate. As we show in Figure 1 and discuss in Section IV., there is in fact a very strong relationship over a long sample period between the level of the nominal rate and spread between Fed Funds and Treasury Bills. This evidence is highly suggestive of a positive relationship between the nominal interest rate and banks cost of obtaining leverage, which is the essential reduced form of our model.

Our model allows the central bank to specify the nominal interest rate policy as a function

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<sup>1</sup>We do not micro-found the reserve requirement, but this can be done in several ways. An important rationale for reserves is that deposit insurance severs the link between banks' risk taking and the rate they pay on deposits. This creates a role for the government in regulating banks' risk taking, and a reserves requirement provides a flexible way of doing so. In turn, the standard rationale for government-run deposit insurance is the need to avoid costly banks runs (Diamond and Dybvig 1983).

of the state variable, the net worth share of the banking sector. We solve for the dynamics of reserves required for the central bank to support its target nominal rate. The solution shows that the nominal rate depends on the dynamics of total reserves, but not on the quantity of reserves. The reason is that the return to holding reserves does not depend on their level, but on their growth rate over time.<sup>2</sup> We take reserves to be the numeraire in the model, so inflation is the change in the price of consumption expressed in reserves, or minus the capital gain on reserves.

To analyze the full implications of the model, we solve for the equilibrium using projection methods. To demonstrate the impact of monetary policy, we compare prices and quantities between a high and a low nominal rate regime. We show that when nominal rates are high, bank leverage is low, the Sharpe ratio and risk premium of the endowment claim are high, and the valuation of the endowment claim is low. An additional interesting finding is that volatility is decreasing in the nominal rate. Volatility in the model is endogenously stochastic; it depends on banks' net worth. Since low interest rates increase bank leverage, they also increase the volatility of the state variable, the volatility of discount rates, and hence the volatility of returns.

We further examine two dynamic interest rate policies. One captures a “Greenspan put” by decreasing the nominal rate as bank net worth falls in part of the state space. We find that this policy stabilizes prices locally since after a negative endowment shock banks' leverage rises and discount rates fall. However, as leverage rises and eventually becomes satiated, there is no further room for such accommodation so that further negative shocks cause prices to fall rapidly. At this point volatility becomes significantly higher than it would have been otherwise. Thus in our framework a Greenspan put policy stabilizes prices in the short run at the expense of greater instability in the long run.

The second dynamic policy we consider shows the impact of forward guidance. Under forward guidance, the central bank commits to keeping nominal rates low even after the economy recovers (the wealth share of the banking sector rises). We show that by reducing future discount rates this policy is able to induce an additional increase in price valuations even when current nominal rates are at the zero lower bound.

Finally, we extend the model to incorporate unexpected shocks to the central bank's nominal rate policy. We show that policy shocks lead to a second round amplification effect on risk premia and other equilibrium quantities. This effect is akin to a financial accelerator:

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<sup>2</sup>One way to see this is to consider a one-time doubling of total reserves. This would halve the value of each unit of reserves (i.e. double the price level), but it would not affect the holding return of reserves going forward, and so it would leave the nominal rate unchanged.

when nominal rates fall, the assets on bank balance sheets rise more than their liabilities, which raises banks' net worth and enables them to expand their balance sheets, pushing risk premia down even further.

The rest of this paper is organized as follows: Section 2 reviews the literature, Section 3 presents the model, Section 4 characterizes the equilibrium, Section 5 presents the results for a benchmark economy, Section 6 examines dynamic policies, Section 7 introduces an extension with policy shocks, and Section 8 concludes.

## II. Related literature

Our paper is most closely related to the literature on the bank lending channel of monetary policy initiated by Bernanke (1983) and formalized by Bernanke and Blinder (1988) and Kashyap and Stein (1994). The bank lending channel relies on an imperfect substitutability between bank loans and unintermediated bonds so that a contraction in bank lending affects the overall availability of funding and spills over to the macroeconomy. The transmission link from monetary policy runs through bank reserves: a drop in the supply of reserves forces banks to shrink their balance sheets.

Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) develop the broader balance sheet channel of monetary policy, which emphasizes the impact of policy shocks on the net worth of borrowers. A drop in interest rates strengthens borrower balance sheets, enabling more borrowing and investment. Against the backdrop of the 2008 financial crisis, recent models shift attention from firms to financial intermediaries (e.g. Adrian and Shin 2010, Gertler and Kiyotaki 2010, Cúrdia and Woodford 2009, Brunnermeier and Sannikov 2013). In these models, a maturity or liquidity mismatch between intermediary assets and liabilities causes interest rate shocks to affect intermediary net worth, driving the supply of credit.

Our principal contribution to the literatures on the bank lending and balance sheet channels is to focus on the risk premium component of the cost of capital within a canonical asset pricing framework. Specifically, we model an economy populated by agents with different levels of risk aversion, which gives rise to a credit market as in Dumas (1989), Wang (1996) and Longstaff and Wang (2012). Risk tolerant agents deploy their wealth in levered portfolios which we interpret as banks. They raise funds by selling bonds to risk averse households, or depositors. The key friction is a reserve requirement that imposes a cost on leverage. A reserve requirement is similar to a margin requirement (e.g. Gromb and Vayanos 2002, Brun-

nermeier and Pedersen 2009, Gârleanu and Pedersen 2011) with the key distinction that its cost is set by monetary policy. Specifically, the opportunity cost of holding reserves is equal to the nominal interest rate. By controlling the nominal interest, the central bank controls the cost of leverage and the amount of risk taking in the economy.

A changing financial landscape has called into question the centrality of bank reserves as a transmission mechanism (Woodford 2010). Our approach is to adopt the reserves framework as a convenient way of arriving at the essential reduced form of our model, a positive relationship between the nominal interest rate and banks' external finance spread. We present empirical support for this reduced form and focus on its implications for risk premia.

Stein (1998) and Stein (2012) also study the ability of the central bank to control bank leverage by adjusting the supply of reserves. In Stein (2012), leverage entails a potential negative externality resulting from fire sales. Reserves function as "pollution permits" whose price, the nominal rate, provides a useful signal that enables regulators to maintain financial stability. We build on Stein's (1998, 2012) insights by providing a fully specified dynamic model of this mechanism. For example, we find that in an intertemporal setting it is the growth rate of reserves rather than their quantity that determines the nominal rate and controls risk taking. This observation is consistent with the observed weak relationship between interest rate changes and the quantity of reserves over short horizons (the so-called "liquidity puzzle," e.g. Friedman and Kuttner 2010).

In contrast to the literature, our model does not require any nominal rigidities.<sup>3</sup> In fact, it exhibits neutrality with respect to the quantity of reserves but not their dynamics. Whereas a one-off increase in the quantity of reserves is offset by a proportional increase in the price level, a sustained increase in the expected growth rate of reserves results in higher expected depreciation, which makes holding reserves more costly. Besides the growth rate, the central bank can also adjust the stochastic exposure of reserves growth to economic shocks, which also affects the opportunity cost of holding reserves by altering the inflation risk premium. In this way, our asset pricing framework allows us to abstract from nominal rigidities and their concomitant real effects and focus exclusively on risk taking. This differs from other papers on monetary policy and bank risk taking including Stein (2012), Adrian and Shin (2010), and Dell'Ariccia, Laeven, and Marquez (2011).

Our paper is also related to the literature on the role of government liabilities as sources

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<sup>3</sup>Kashyap and Stein (1994) write that absent nominal rigidities, "there can be no real effects of monetary policy through either the lending channel of the conventional money channel".

of liquidity for the financial sector (Woodford 1990, Gertler and Karadi 2011, Caballero and Farhi 2013, Krishnamurthy and Vissing-Jorgensen 2012, Greenwood, Hanson, and Stein 2012). In our model, reserves provide liquidity to banks and crowd in investment in risky assets just as government debt does in Woodford (1990).

On the empirical side, Bernanke and Gertler (1995) and Bernanke and Blinder (1992) are early papers that find support for the bank lending and balance sheet channels of monetary policy. Bernanke and Blinder (1992) show that monetary tightening as reflected in a shock to the Fed Funds rate, leads banks to shrink their balance sheets. Kashyap, Stein, and Wilcox (1993) show that bank funding is also sensitive to policy shocks. Kashyap and Stein (2000) show that smaller banks are particularly sensitive. More recently, Jiménez, Ongena, Peydró, and Saurina Salas (2011) and Landier, Sraer, and Thesmar (2013) provide further corroborating evidence on the links between monetary policy and bank balance sheets.

Our model generates a positive relationship between nominal interest rates and risk premia, a phenomenon often called “reaching for yield”. This relationship has been documented in several event studies, including Bernanke and Kuttner (2005), Duffee (1998), and Hanson and Stein (2012). We note that in our model this is purely a nominal phenomenon since in the absence of price rigidities the nominal and real interest rates do not move in parallel.

### III. Model

We model an infinite-horizon exchange economy in continuous time  $t \geq 0$ , with aggregate endowment  $D_t$  that follows a geometric Brownian motion:

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D dB_t. \quad (1)$$

The economy is populated by a continuum of agents with total mass one. There are two types of agents,  $A$  and  $B$ . Both types have recursive preferences as in Duffie and Epstein (1992), the continuous-time analog to the discrete-time formulation of Epstein and Zin (1989). Duffie-Epstein preferences allow us to vary the elasticity of intertemporal substitution (EIS) independently of the risk aversion coefficient. An EIS greater than one ensures that valuations are decreasing in risk aversion so that when nominal rates are high resulting in high effective risk aversion, asset prices are low.

To ensure stationarity, we assume that agents die at a rate  $\kappa$ . New agents are also born with intensity  $\kappa$  with a fraction  $\bar{\omega}$  as type  $A$  and  $1 - \bar{\omega}$  as type  $B$ . Gârleanu and Panageas

(2008) show that under these assumptions, the lifetime utility  $V_0^i$  of an agent of type  $i = A, B$  can be expressed as the recursion

$$V_0^i = E_0 \left[ \int_0^\infty f^i (C_t^i, V_t^i) dt \right] \quad (2)$$

$$f^i (C_t^i, V_t^i) = \left( \frac{1 - \gamma^i}{1 - 1/\psi^i} \right) V_t^i \left[ \left( \frac{C_t^i}{[(1 - \gamma^i) V_t^i]^{1/(1-\gamma^i)}} \right)^{1-1/\psi^i} - (\rho + \kappa) \right]. \quad (3)$$

The felicity function  $f^i$  is an aggregator over current consumption and future utility. The parameters  $\psi^i$  and  $\gamma^i$ ,  $i = A, B$ , denote agents' elasticity of intertemporal substitution (EIS) and relative risk aversion (RRA).

Without loss of generality, we assume that type  $A$  agents are more risk tolerant,  $\gamma^A < \gamma^B$ . We view these agents' wealth as the equity capital of the "banks" in the economy (or more generally the financial sector). In order to focus on risk taking, we abstract from other aspects of financial intermediation and adopt a simplified view in which banks are set up to pool the economy's risk-bearing capital to earn a risk premium.<sup>4</sup> We therefore often refer to the type- $A$  agents as the banks and their wealth as the equity of the banking sector.

Let  $W_t^i$  denote the total wealth of type- $i$  agents at time  $t$ . We denote the wealth share of type- $A$  agents by  $\omega_t$ :

$$\omega_t = \frac{W_t^A}{W_t^A + W_t^B}. \quad (4)$$

We show below that  $\omega_t$  summarizes the state of the economy. To derive its dynamics, we assume that the wealth of agents who die is bestowed on the newly born on a per-capita basis. We can then write the law of motion of  $\omega_t$  as

$$d\omega_t = \kappa (\bar{\omega} - \omega_t) dt + \omega_t (1 - \omega_t) \left[ \mu_\omega (\omega_t) dt + \sigma_\omega (\omega_t) dB_t \right]. \quad (5)$$

The evolution of  $\omega_t$  has an exogenous component due to demographic turnover and an endogenous component due to differences in the rates of saving and the portfolio choices of the two agent types. The exogenous component prevents banks from dominating the economy, ensuring a non-degenerate stationary distribution for  $\omega_t$ .

Agents trade a claim on the aggregate endowment. The price of this claim is  $P_t$ , its

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<sup>4</sup>For example, in our setup it is not necessary to impose a restriction on equity issuance as in He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2013).



dividend yield is  $F(\omega_t) = D_t/P_t$ , and its return process is

$$dR_t = \frac{dP_t + D_t dt}{P_t} = \mu(\omega_t) dt + \sigma(\omega_t) dB_t. \quad (6)$$

Agents also trade instantaneous risk-free bonds that pay the endogenously-determined real interest rate  $r(\omega_t)$ .

### A. Deposits and reserves

The difference in risk aversion between agent types lead to the emergence of a credit market (see e.g. Longstaff and Wang 2012). In particular, optimal risk sharing implies that the risk-averse type-*B* agents lend to the risk-tolerant type-*A* agents using the instantaneous risk-free bonds. Continuing with our interpretation of type-*A* agents as banks, we think of these bonds as demand deposits.

The central friction in our model is that the central bank regulates deposit taking. In particular, banks must hold reserves of no less than a fixed proportion of their deposits. Reserves are issued only by the central bank, though they can be exchanged freely in a secondary market.

A reserve requirement can be motivated in several ways. For example, it provides a lever for regulating deposit creation, and deposit creation entails an externality in the presence of deposit insurance. Deposit insurance itself might be optimal if deposits are susceptible to destructive bank runs as in Diamond and Dybvig (1983). A reserve requirement is particularly well-suited to regulating the externalities associated with deposit creation because it takes advantage of a price mechanism. The central bank monitors the cost of lending and borrowing reserves in interbank markets and it responds to fluctuations in that cost by conducting open market operation.<sup>5</sup>

Without directly modeling the externalities associated with deposit creation, our approach is to take reserve requirements as given and study their implications for risk-taking. Formally, let  $w_{S,t}$  be the banks' portfolio weight in the risky endowment claim. Of this,  $\max\{w_{S,t} - 1, 0\}$  must be deposit-financed (borrowed). Let  $w_{M,t}$  be the banks' portfolio

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<sup>5</sup>Stein (2012) draws the analogy with the market for pollution permits. Reserves regulate the supply of deposits based on a tradeoff between the private value of monetary services against the public cost of a negative externality due to fire sales following crashes. In general, any negative externality associated with deposit creation introduces a role for reserves.

weight in reserves. The reserve requirement imposes the constraint

$$w_{M,t} \geq \max \left[ \lambda \sigma_t^2 (w_{S,t} - 1), 0 \right]. \quad (7)$$

Agents must hold reserves in proportion to their deposits, if any, and reserves cannot be held short. The parameter  $\lambda$  controls the reserve requirement,  $\lambda \sigma_t^2$ . Scaling by  $\sigma_t^2$  simplifies the resulting expressions, but is not essential.<sup>6</sup> When  $\lambda = 0$ , there is no reserve requirement and asset markets are complete. This case might correspond to a frictionless economy in which deposit-related externalities do not arise.

Let  $M_t$  denote the total number of reserves and  $\Pi_t$  the total real value of reserves in units of the endowment. It will be useful to define

$$G(\omega_t) = \frac{\Pi_t}{P_t} \quad (8)$$

as the wealth share of reserves. Furthermore, let  $\pi_t = \Pi_t/M_t$  denote the consumption value of each dollar of reserves. We take reserves to be the numeraire in the economy, so  $\pi_t$  is the inverse price level.<sup>7</sup> It follows that the realized rate of inflation is  $-d\pi_t/\pi_t$ . We assume that the central bank sets the path of reserves  $dM_t/M_t$  so that inflation is locally deterministic,

$$-\frac{d\pi_t}{\pi_t} = i(\omega_t) dt. \quad (9)$$

Locally deterministic inflation simplifies the exposition of the model and is arguably realistic. In the solution section, we show precisely how the central bank implements (9) by adjusting the exposure of reserves growth to the endowment shock. Since the central bank also controls the drift of reserves growth, it can still achieve its nominal rate target and so (9) does not restrict the effectiveness of monetary policy.

Next, we define the nominal interest rate

$$n(\omega_t) = r(\omega_t) + i(\omega_t). \quad (10)$$

We treat  $n(\omega_t)$  as the central bank's policy and solve for the path of reserves that implements it. We write this policy as a function of  $\omega_t$  since it summarizes the state of the economy.

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<sup>6</sup>In the absence of this scaling, the tightness of the reserves requirements varies inversely with the return variance of the endowment claim. This adds a degree of complication to the expressions which is inconsequential.

<sup>7</sup>In practice, reserves are fungible with currency which serves as numeraire. Since our focus is on risk taking, we abstract from introducing a transactions medium such as currency.

Agents have rational expectations so they know this function. In an extension, we also consider policy shocks, which may take the nominal rate away from its benchmark rule.

The central bank controls the supply of reserves via open market operations, sales and purchases of bonds for reserves at prevailing market prices. Let  $B_t$  be the central bank's total holdings of bonds (hence the private sector holds  $-B_t$  in aggregate). If we think of reserves as the central bank's liability, then the central bank's net worth is  $B_t - \Pi_t$ . This net worth is not directly affected by open market operations since they are conducted at prevailing market prices. However, the central bank earns a stream of "seignorage" profit on its portfolio, which is given by the difference between the interest income on its bond assets and the realized deflation on its reserve liabilities. We assume that the central bank pays out the seignorage it earns so its net worth remains at zero. Thus, seignorage is

$$B_t r(\omega_t) dt - \Pi_t \frac{d\pi_t}{\pi_t} = \Pi_t n(\omega_t) dt. \quad (11)$$

As we show below, no-arbitrage requires  $n(\omega_t) \geq 0$  so seignorage is never negative. To close the model, we assume it gets refunded to all agents in the economy in proportion to their wealth.<sup>8</sup>

## B. Optimization

We begin with the Hamilton-Jacobi-Bellman (HJB) equation of an agent in our economy. Let  $V^i(W_t^h, \omega_t)$  denote the value function of agent  $h$  of type  $i = A, B$ . Also let  $c_t^h$ ,  $w_{S,t}^h$ , and  $w_{M,t}^h$  be the agent's consumption-wealth ratio, endowment claim portfolio weight, and reserves portfolio weight. The HJB equation is

$$0 = \max_{c_t^h, w_{S,t}^h, w_{M,t}^h} f^i(c_t^h W_t^h, V^i(W_t^h, \omega_t)) dt + E[dV^i(W_t^h, \omega_t)] \quad (12)$$

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<sup>8</sup>This keeps the refund policy from changing the wealth distribution.

subject to the agent's wealth dynamics<sup>9</sup> and the reserve requirement

$$\frac{dW_t^h}{W_t^h} = \left( r(\omega_t) - c_t^h + w_{S,t}^h [\mu(\omega_t) - r(\omega_t)] + w_{M,t}^h \left[ \frac{d\pi_t}{\pi_t} - r(\omega_t) \right] + G(\omega_t) n(\omega_t) \right) dt + w_{S,t}^h \sigma(\omega_t) dB_t \quad (13)$$

$$w_{M,t}^h \geq 0 \quad (14)$$

$$w_{M,t}^h \geq \lambda \sigma^2(\omega_t) (w_{S,t}^h - 1). \quad (15)$$

The diffusive component of wealth depends only on the weight of the risky claim and not on the reserves holdings which are locally risk-free. In the drift term,  $G(\omega_t) n(\omega_t)$  represents the stream of seignorage refund payments.<sup>10</sup> By (9) and (10), the excess return on reserves,  $d\pi_t/\pi_t - r(\omega_t)$ , equals  $-n(\omega_t)$ , the negative of the nominal rate. Hence, reserves are costly when the nominal rate is positive. The reserve requirement consists of two parts: the shorting restriction that prevents agents from increasing the effective supply of reserves on their own, and the leverage constraint.

The homogeneity of the felicity function implies that the consumption and portfolio policies are independent of wealth, so we can write them as functions of agent type only. Define the aggregate type- $i$  consumption-wealth ratio as  $c^i(\omega_t) = \int_i c^h(\omega_t) \frac{W^h}{W^i} dh$  for  $i = A, B$ , and similarly for the type- $i$  portfolio policies  $w_S^i(\omega_t)$  and  $w_M^i(\omega_t)$ .

### C. Equilibrium conditions

In equilibrium, the markets for goods (i.e. consumption), the endowment claim, and reserves must clear. The bond (deposit) market clears by Walras' law. Since the public's net bond holdings are simply minus the value of reserves, aggregate wealth equals the value of the endowment claim,  $W_t^A + W_t^B = P_t$ . The three market-clearing conditions can therefore be written as

$$\omega_t c^A(\omega_t) + (1 - \omega_t) c^B(\omega_t) = F(\omega_t) \quad (16)$$

$$\omega_t w_S^A(\omega_t) + (1 - \omega_t) w_S^B(\omega_t) = 1 \quad (17)$$

$$\omega_t w_M^A(\omega_t) + (1 - \omega_t) w_M^B(\omega_t) = G(\omega_t). \quad (18)$$

<sup>9</sup>These are the wealth dynamics should the agent manage to cheat death over the next instant. The agent accounts for the possibility of death directly in the felicity function (3) (see Gârleanu and Panageas 2008).

<sup>10</sup>Recall from (11) that total seignorage is  $\Pi_t n(\omega_t)$  and that it gets refunded in proportion to wealth, so an agent with wealth  $W_t^h$  gets  $\Pi_t n(\omega_t) \frac{W_t^h}{P_t} = G(\omega_t) n(\omega_t) W_t^h$ .

All three are normalized by total wealth. The first equation gives the goods-market clearing condition, the second gives the market-clearing condition for the endowment claim, and the third gives the market-clearing condition for reserves.

## IV. Solution

In this section we derive the equations that characterize the equilibrium. These equations do not permit closed-form solutions. However, we are able to derive analytical expressions that highlight key mechanisms. In the next section, we provide a full analysis of the model's implications by applying numerical methods.

### A. The value function and the demand for leverage

For simplicity of notation, we drop agent, type, and time subscripts though it should be understood that they apply. Let  $\theta_\lambda V_W W \geq 0$  and  $\theta_0 V_W W \geq 0$  be the Lagrange multipliers on the reserves and non-negativity constraints. By Ito's lemma we can rewrite the HJB equation as the Lagrangian

$$\begin{aligned}
0 = & \max_{c, w_S, w_M} f(cW, V) + V_W W [r - c + w_S (\mu - r) - w_M n + Gn] & (19) \\
& + V_\omega \left[ \kappa (\bar{\omega} - \omega) + \omega (1 - \omega) \mu_\omega \right] + V_{W\omega} W \omega (1 - \omega) w_S \sigma_\omega \sigma + \frac{1}{2} V_{WW} W^2 (w_S \sigma)^2 \\
& + \frac{1}{2} V_{\omega\omega} \omega^2 (1 - \omega)^2 \sigma_\omega^2 + \theta_\lambda V_W W [w_M - \lambda \sigma^2 (w_S - 1)] + \theta_0 V_W W w_M.
\end{aligned}$$

The following proposition gives the form of the value function up to an unknown function of the wealth distribution  $J(\omega)$  together with the equation that characterizes it.

**Proposition 1.** *Each agent's value function has the form*

$$V(W, \omega) = \left( \frac{W^{1-\gamma}}{1-\gamma} \right) J(\omega)^{\frac{1-\gamma}{1-\psi}}. \quad (20)$$

*The unknown function  $J(\omega)$  gives the agent's consumption-wealth ratio,  $c^* = J$  and solves*

the second order ordinary differential equation

$$\begin{aligned} \rho + \kappa = & 1/\psi J + (1 - 1/\psi) (r + \lambda \sigma^2 \theta_\lambda + Gn) - 1/\psi \frac{J_\omega}{J} \left[ \kappa (\bar{\omega} - \omega) \right. \\ & \left. + \omega (1 - \omega) \mu_\omega \right] - \frac{1/\psi}{2} \left[ \left( \frac{\psi - \gamma}{1 - \psi} \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1 - \omega)^2 \sigma_\omega^2 \\ & + \frac{1}{2} \left( \frac{1 - 1/\psi}{\gamma} \right) \left[ \frac{\mu - r}{\sigma^2} - \lambda \theta_\lambda + \left( \frac{1 - \gamma}{1 - \psi} \right) \frac{J_\omega}{J} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right]^2 \sigma^2 \end{aligned} \quad (21)$$

if  $\gamma^B - \gamma^A \geq \lambda n$  with  $\theta_\lambda = n$  if the agent is of type A and  $\theta_\lambda = 0$  if the agent is of type B. If instead  $\gamma^B - \gamma^A < \lambda n$ ,  $J$  solves

$$\rho + \kappa = 1/\psi J + (1 - 1/\psi) \left( \mu - \frac{\gamma}{2} \sigma^2 \right) - 1/\psi \frac{J_\omega}{J} \left[ \kappa (\bar{\omega} - \omega) + \omega (1 - \omega) \mu_\omega \right]. \quad (22)$$

*Proof of Proposition 1.* The proof is contained in Appendix A.  $\square$

The function  $J$  is type-specific but not agent-specific since it does not depend on wealth. Instead, it depends solely on the wealth distribution  $\omega$ . As a result,  $\omega$  is a sufficient statistic for asset valuations and other equilibrium quantities.

Using the value functions, we can solve for agents' portfolio demands. Proposition 2 below provides the conditions under which banks take leverage (by issuing deposits), and characterizes their demand for the risky endowment claim as it depends on the central bank's nominal rate policy.

**Proposition 2.** *Banks take leverage/deposits ( $w_S^A > 1$ ) if and only if*

$$\gamma^B - \gamma^A > \lambda n. \quad (23)$$

*In this case, banks' portfolio holdings of the endowment claim are given by*

$$w_S^A = \frac{1}{\gamma^A} \left[ \frac{\mu - r}{\sigma^2} - \lambda n + \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right]. \quad (24)$$

*Proof of Proposition 2.* The proof is contained in Appendix A.  $\square$

Equation (24) has three parts. The first term,  $(\mu - r)/\sigma^2$ , is the standard “myopic” mean-variance tradeoff for the endowment claim. It shows that banks take more leverage when there is a higher risk premium per unit of risk. The third term, which depends on  $J_\omega^A$ , represents the intertemporal hedging component of banks' demand for the risky asset.

This component determines how much banks adjust their current risk taking to hedge future changes in investment opportunities. The investment opportunity set is stochastic because of variation in aggregate risk tolerance that is induced by changes in the relative wealth  $\omega$  of the risk-tolerant and risk-averse agents.

The second term in equation (24) gives the direct impact of the nominal rate on bank leverage, which we summarize in the following corollary.

**Corollary 1.** *All else equal, an increase in the nominal interest rate reduces bank leverage.*

For every dollar of deposit funding, banks must increase their reserves holdings by the reserve requirement. Since the excess return on reserves is the negative of the nominal rate, holding reserves is costly. An increase in the nominal interest rate raises the effective cost of deposits and results in less leverage.

Using (24), we can see that an increase in the nominal rate works like an increase in banks' effective risk aversion. This in turn raises the economy's aggregate risk aversion and hence also the risk premium.

Recall that reserves are locally risk-free. Why is their excess return negative in equilibrium? The reason is that reserves give banks the right to take leverage. We can think of the flow value of this right as a latent dividend stream that banks receive from holding reserves. The "dividend" is given by the Lagrange multiplier on banks' reserve requirement,  $\theta_\lambda^A$ , which in equilibrium equals the nominal rate  $n$ . Now, in equilibrium the (risk-adjusted) total real return on any asset must equal  $r$ . The capital gain on reserves, given by the change in the inverse price level (or minus the inflation rate),  $d\pi/\pi = -\iota dt$ , adjusts so that:

$$r = n - \iota. \tag{25}$$

This is Fisher's equation. Interpreted through the lens of asset pricing, it states that the real risk-free rate  $r$  equals the capital gain on reserves  $-\iota$  plus the latent dividend stream  $n$ .

Proposition 2 also shows that banks lever only if agents' risk aversions differ sufficiently to overcome the cost of leverage. The difference in risk aversions multiplied by the return variance,  $(\gamma^A - \gamma^B) \sigma^2$ , measures the risk premium earned by banks on their first dollar of leverage. This premium reflects the gains from risk sharing. For banks to take leverage, it must be greater than the cost of leverage which is given by the nominal rate  $n$  multiplied by the reserve requirement  $\lambda \sigma^2$ .

**Corollary 2.** *If  $\lambda n \geq \gamma^B - \gamma^A$  then  $w_S^A = w_S^B = 1$ .*

If the cost of leverage exceeds the difference in risk aversions then banks do not raise deposits and the two groups remain in “financial autarky”.

### *B. Reserves value and dynamics*

The following proposition gives the value of reserves and solves for the law of motion for  $M_t$ , the quantity of reserves required to support the central bank’s nominal interest rate rule.

**Proposition 3.** *The value of reserves as a fraction of aggregate wealth is given by*

$$G(\omega_t) = \omega_t \lambda \sigma_t^2 (w_{S,t}^A - 1). \quad (26)$$

*Under the central bank’s nominal rate rule  $n(\omega_t)$ , the quantity of reserves  $M_t$  must follow the law of motion*

$$\frac{dM_t}{M_t} = [n(\omega_t) - r(\omega_t)] dt + \frac{d\Pi_t}{\Pi_t} \quad (27)$$

$$= [n(\omega_t) - r(\omega_t)] dt + \frac{dG(\omega_t)}{G(\omega_t)} + \frac{dP_t}{P_t} + \frac{dG(\omega_t)}{G(\omega_t)} \frac{dP_t}{P_t}. \quad (28)$$

*Proof of Proposition 3.* The proof is contained in Appendix A. □

The dynamics of the quantity of reserves in equation (27) are given as a function of the bank’s policy  $n(\omega)$ , and two endogenous quantities, the total value of reserves  $\Pi(\omega)$ , and the real rate  $r(\omega)$ . The central bank adjusts the growth rate of reserves to achieve the target while responding to underlying shocks.

The growth rate of reserves is stochastic even though realized inflation is locally deterministic. To attain the nominal rate  $n(\omega)$ , the central bank must influence the rate of return on reserves, which depends on the state of the economy  $\omega$ . In particular, to maintain a stable nominal rate, the quantity of reserves must keep up with aggregate wealth  $P$  and demand for reserves  $G$ .

Equation (27) also implies the following corollary.

**Corollary 3.** *The nominal interest rate depends on the growth rate of reserves, not their level, which is not separately identified.*

This result follows directly from equation (27), which shows that  $n(\omega)$  is related to the growth of  $M$ , not the level. A specific value of  $M$  pins down the price level ( $\Pi = M\pi$ ), but



the nominal rate depends only on the growth rate of  $M$ . Equation (26) shows that it is the real value of reserves  $\Pi = GP$ , not the quantity  $M$ , that affects risk taking. The quantity of reserves is not separately identified from the price level since  $\Pi = M\pi$ . Thus, the model features neutrality with respect to the quantity but not the growth rate of reserves.

This mechanism contrasts with classic models of reserves where the quantity of reserves matters via sticky prices. For example, this is the case in the canonical formulation of the bank lending channel in Bernanke and Blinder (1988). Similarly, in the discussion of Kashyap and Stein (1994), and in the model of Stein (2012), the determining factor is the level of reserves made available by the central bank.

### *C. The Fed Funds rate and the external finance spread*

The Fed Funds market is a short-term (mostly overnight) uncollateralized lending market for banks in the US.<sup>11</sup> The rate that prevails in this market, the Fed Funds rate, has emerged as a key target for monetary policy. Fed Funds loans are not subject to reserve requirements as they are not deposits. In equilibrium, banks must be indifferent between raising a dollar of deposits versus borrowing Fed Funds. We can therefore define the real Fed Funds rate ( $FFr$ ) as the real deposit (or risk-free bond) rate inclusive of the cost of the reserve requirement:

$$FFr_t = r(\omega_t) + \lambda\sigma_t^2 n(\omega_t). \quad (29)$$

The term  $\lambda\sigma^2 n$  is then the spread between  $FFn$  and the rate on deposits/Treasurys, and it captures the cost of the reserves requirement. We interpret this spread as the external finance premium, since it is the difference between the value of a dollar inside the banking system versus outside. This term is used similarly in the literature (e.g. Bernanke and Gertler 1995) to refer to the gap between the cost of banks' marginal sources of funding and the rate on risk-free deposits or short-term TBills.

We highlight the importance of the external finance premium in the model by rewriting equation (24) for banks' optimal leverage/risky claim holdings,

$$w_S^A = \frac{1}{\gamma^A} \left[ \frac{\mu - FFr}{\sigma^2} + \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right]. \quad (30)$$

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<sup>11</sup>The Fed Funds market represents a substantial source of overnight funding for large US money-center banks. The other significant source of interbank uncollateralized dollar funding is the Eurodollar market. The prevailing rate in that market, LIBOR, typically tracks the Fed Funds rate very closely (Kuo, Skeie, and Vickery 2010).

Banks' constrained leverage choice can therefore be cast equivalently as an unconstrained optimal portfolio decision if the real interest rate is replaced by the cost of external financing, which exceeds the risk-free rate by the external finance premium.<sup>12</sup>

Equation (30) shows how changes in the nominal rate affect bank leverage by changing the external finance premium. When the central bank increases the nominal rate, the external finance premium widens, reducing banks' demand for leverage. The ultimate result is an increase in aggregate risk aversion and the price of risk.

Figure 1 plots the empirical relationship between the level of the Fed Funds rate (solid line, left axis) and the Fed Funds-TBill spread (dashed line, right axis) for the period July 1980 to May 2008. The beginning of this period corresponds roughly to the abolition of Regulation Q, which limited the rate banks could pay on deposits, while the end corresponds roughly to the beginning of the financial crisis, which brought about a spike of credit risk in the Fed Funds market. The figure plots 20-week moving averages of these series.

The figure displays a remarkably tight relationship between the two series over this roughly twenty-eight year period. Indeed, the correlation is 86%. Moreover, the Fed Funds-TBill spread closely tracks both the trend and the cycles in the Fed Funds rate over this period.

The average Fed Funds rate over this period is 6.25%, while the average Fed Funds-Tbill spread is 0.57%. The sensitivity of the Fed Funds-TBill spread to a change in the Fed Funds rate, estimated by OLS regression, is 0.14. In the model, this value corresponds to the reserve requirement,  $\lambda\sigma^2$ , which we set to be 0.10.

The tight relationship between the external finance premium and the interest rate in Figure 1 suggests a stable relationship between the interest rates and banks' funding costs. This relationship presents a challenge to models based on interest rate shock rather than levels.

## V. Results

To analyze further the impact of monetary policy on the economy, we choose values for the model parameters, specify a nominal rate policy, and solve for the resulting equilibrium.

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<sup>12</sup>The risk-free in the model is given equivalently by either the rate on deposits, or the rate of return on (instantaneous) government bonds. As noted above, we think of deposits as being protected by deposit insurance and hence equivalent to government bonds from investors viewpoint.

Since the model does not permit a closed-form solution, we solve it numerically. This requires solving the HJB equations of the two agent types simultaneously. We do this using Chebyshev collocation, which provides a global solution.

### A. Parameters

Table I displays our benchmark parameter values. We set the risk aversions of the two agents at 1.5 for type *A* and 15 for type *B* in order to generate a high demand for deposits and leverage. This allows monetary policy to have large effects on aggregate risk aversion and equilibrium asset prices.

Description	Parameter	Value
Risk aversion A	$\gamma^A$	1.5
Risk aversion B	$\gamma^B$	15
EIS	$\psi^A, \psi^B$	3.5
Reserve requirement	$\lambda\sigma_D^2$	0.1
Endowment growth	$\mu_D$	0.02
Endowment volatility	$\sigma_D$	0.02
Time preference	$\rho$	0.01
Death rate	$\kappa$	0.01
Type- <i>A</i> share of population	$\bar{\omega}$	0.10
Nominal Rate 1	$n_1$	0%
Nominal Rate 2	$n_2$	5%

Table I: **Parameter values.** This table lists the benchmark parameter values used to illustrate the results of the model.

We set the elasticity of intertemporal substitution (EIS) to 3.5 for all agents so that the two types differ only in risk aversion. An EIS value greater than one implies that an increase in effective risk aversion, for example generated by a rise in the nominal interest rate, results in a decrease in the equilibrium wealth-consumption ratio. Thus, as rates rise, prices fall.

We pick the reserve requirement parameter  $\lambda$  so that  $\lambda\sigma_D^2 = 0.1$ . Since return volatility is similar to fundamental volatility, this corresponds to a reserve requirement of about 10%, which is the reserve requirement for net transactions accounts in the US. We set the endowment growth rate and volatility to 2%, consistent with standard estimates for US aggregate consumption growth and volatility. We set agents' time preference parameter  $\rho$  and death rate  $\kappa$  both to 0.01, which leads to real interest rates near 2%, consistent with

the data. To stabilize banks' wealth share  $\omega$  at moderate levels, we set the population share of type- $A$  agents  $\bar{\omega}$  to 10%.

We compare equilibria across two nominal rate policies. In the first policy, the nominal rate is identically zero. In this case, holding reserves is costless, so the model is equivalent to a frictionless one with no reserve requirement. This case represents a useful frictionless benchmark. In the second policy, the nominal rate is set to 5%, making reserves costly and constraining leverage. While the model allows for much more complex policy rules, we restrict attention to these simple cases in order to convey the main intuition. We consider dynamic policies later in the paper.

### *B. Portfolio Choice*

Figure 2 shows the impact of higher nominal rates on the holdings of risky claims by banks (top panel) and depositors (bottom panel). The plots show portfolio weights across different values of the wealth distribution  $\omega$  under policy  $n_1 = 0\%$  (blue triangles) and policy  $n_2 = 5\%$  (red squares).

Bank leverage falls as the nominal rate rises at all values of the wealth distribution. The decrease is larger when banks' wealth is small relative to overall wealth (low  $\omega$ ). For the chosen parameters, when  $\omega$  is close to zero, banks' risky asset holdings decrease from around 10 times their net worth to less than 2 times. At moderate levels of  $\omega$  between 0.2 and 0.4 where the economy spends the most time, banks' holdings of risky assets decrease from between 2 and 4 times net worth under  $n_1$  to slightly above 1 under  $n_2$ , so that a near complete deleveraging takes place.

As the bottom panel of Figure 2 shows, depositor holdings of the risky asset offset the decrease in bank holdings. For instance, when  $\omega$  falls between 0.2 and 0.4, depositors hold 40% of their wealth in risky claims under  $n_1 = 0\%$ , rising to almost 100% under  $n_2 = 5\%$ . The changed allocation of risk in the direction of the more risk-averse depositors can be thought of as increasing the effective risk aversion of the representative investor.

The relationship between the portfolio weight and the wealth share  $\omega$  in Figure 2 is a result of market clearing. When  $\omega$  is close to either zero or one, a single type of agent dominates the economy, which reduces the opportunity for risk sharing. Agents of the remaining type must hold all their wealth in the endowment claim, whereas agents of the disappearing type can be satisfied with a vanishingly small amount of borrowing and lending. Thus, when  $\omega$  is

near zero, prices are set by depositors causing banks to demand very high leverage as long as the nominal rate is not too high. By contrast, when  $\omega$  is near one, banks set prices making risky claims unattractive to depositors unless a high nominal rate keeps the risk premium high.

We see that under  $n_2 = 5\%$ , reserves are sufficiently costly for banks to take almost no leverage. At even higher levels of  $n$ , the economy enters financial autarky (see Corollary 2): the credit market shuts down and both types of agents hold all their wealth in the risky endowment claim at all levels of  $\omega$ . This is why under  $n_2 = 5\%$  portfolio demand is relatively flat in  $\omega$  for both types of agents.

### *C. The price of risk and the risk premium*

The top panel of Figure 3 shows how the Sharpe ratio (top panel) and risk premium (bottom panel) of the endowment claim change with the interest rate policy. As noted above, the effective risk aversion in the economy is higher at the higher nominal rate, and this is indeed reflected in a higher Sharpe ratio. At moderate levels of  $\omega$  between 0.2 and 0.4, the price of risk goes up by a factor of between two and four in going from the low-rate policy  $n_1 = 0\%$  to the high-rate policy  $n_2 = 5\%$ . The effect is even stronger at higher levels of  $\omega$ , rising to an almost ten-fold increase near  $\omega = 1$ .

The upper value of the Sharpe ratio near 0.3 is due to the high risk aversion of depositors. When rates are high and depositors are required to hold almost 100% of their wealth in risky claims, the price of risk approaches  $\gamma^B \sigma_D$ , its value in an economy solely inhabited by the more risk averse agents.

The bottom panel of Figure 3 shows that the increase in the risk premium largely tracks the increase in the Sharpe ratio. At  $\omega$  between 0.2 and 0.4, the risk premium rises from 0.15–0.3% under  $n_1 = 0\%$  to near 0.6% under  $n_2 = 5\%$ . The small differences in the shapes of the risk premium and Sharpe ratio curves are due to changes in the volatility of the endowment claim induced by the two policies.

### *D. Volatility*

Figure 4 plots the volatility of returns. Although cash flow volatility is constant, return volatility is time varying. Moreover, it exceeds cash flow volatility in a hump-shaped pattern.

Under the low-rate policy  $n_1 = 0\%$ , return volatility peaks near  $\omega = 0.2$  at about 0.028, which is 40% higher than fundamental volatility.

The excess volatility of returns is a result of changes in discount rates. For a given nominal rate policy, the aggregate discount rate is determined by a weighted average of the risky-asset demands of the two agent types. The weights depend on  $\omega$ . At moderate values of  $\omega$ , banks take significant leverage and at the same time command enough wealth to affect prices. As a result, in this region a shock to the endowment has a large effect on banks' wealth share  $\omega$ . This makes aggregate risk tolerance and the discount rate volatile, which in turn makes realized prices volatile. By contrast, when either type of agent dominates the economy, returns do not change the risk tolerance of the representative investor by much and there is little variation in discount rates. Return volatility is then close to fundamental volatility.

Note that excess volatility is much lower under the high-rate policy  $n_2 = 5\%$ . This is because bank leverage is reduced so shocks do little to change the wealth distribution and by extension discount rates. Figure 4 thus shows that a low interest rate policy is associated with greater endogenous risk.

We note that return volatility is higher than fundamental volatility because discount rates are “counter-cyclical”. The presence of leverage implies that a positive endowment shock disproportionately raises the net worth of banks, which lowers effective aggregate risk aversion and the discount rate. As a result, endowment shocks and discount rate shocks reinforce each other, amplifying realized returns.

### *E. The real interest rate*

Figure 5 plots the equilibrium real interest rate under the two nominal interest rate policies. We see that the real rate is actually lower under the high nominal rate policy  $n_2 = 5\%$  than under  $n_1 = 0\%$ . This result is due to the impact of policy on aggregate risk aversion. When nominal rates are high, the allocation of risk is suboptimal, which raises the precautionary savings motive of the representative investor and pushes down the real interest rate.

The difference between the real rates under the two policies is greatest near  $\omega = 1$ . Recall that the same pattern holds for depositors' portfolio holdings in Figure 2. At high nominal rates, depositors retain a large amount of risk and so the real rate falls.

It may seem surprising that the increase in the nominal rate has opposing effects on the

risk premium and risk free rate. Yet, this is a direct consequence of the higher nominal rate increasing aggregate risk aversion. Higher risk aversion increases both risk prices and precautionary savings. The risk premium rises and the (real) interest rate falls.<sup>13</sup>

### *F. Valuations*

Figure 6 plots the wealth-consumption ratio under the two policies. Although a higher nominal rate has opposing effects on the risk premium and real risk-free rate, the net impact on the value of the endowment claim is unambiguous. For all values of  $\omega$ , the valuation ratio is higher under the low-rate policy  $n_1$ . The effect is strongest near the middle of the state-space where the value of the endowment claim is about 15% higher under  $n_1$ .

The sign of the net impact of nominal rates on valuations is a function of the EIS. When the EIS is greater than one, greater risk aversion reduces demand for assets causing valuations to fall. In this case the rise in the risk premium exceeds the fall in the interest rate. In contrast, when the EIS is less than one, the opposite occurs and valuations actually rise in risk aversion.

While the higher nominal rate uniformly decreases valuations, the size of the impact is non-monotonic in  $\omega$ . In particular, it is highest at intermediate values of  $\omega$ , when the wealth levels of banks and depositors are comparable. In this region, the deleveraging induced by high nominal rates has a large impact on the allocation of risk: it causes demand for risky assets and the supply of deposits to shrink substantially. In contrast, when  $\omega$  is near zero, a reduction in leverage has little effect on allocations since banks hold few assets. Similarly, when  $\omega$  is close to one, the supply of deposits is low regardless of the nominal rate. Thus, the effect of monetary policy on valuations is largest when aggregate risk sharing (measured either by aggregate leverage or aggregate deposits), is at its greatest extent.

### *G. Wealth distribution*

While the nominal rate has no effect on aggregate leverage when  $\omega$  equals zero or one, it still has an effect on the price of the endowment claim, as Figure 6 shows. This is due to the impact that the nominal rate has on the dynamics of the wealth distribution. At higher

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<sup>13</sup>The same result obtains in homogeneous economies in a comparative static with respect to risk aversion. Specifically, in a homogeneous economy with RRA  $\gamma$  and EIS  $\psi$ , we have  $\frac{\partial}{\partial \gamma}(\mu - r) = \sigma^2 > 0$  and  $\frac{\partial}{\partial \gamma}r = -\frac{1}{2}\sigma^2(1 + 1/\psi) < 0$ .

nominal rates, banks take less risk and their wealth tends to grow more slowly. As a result, the stationary distribution for their wealth share,  $\omega$  centers around a lower value. This is shown in Figure 7, which depicts this stationary distribution under the two nominal rate policies.

Since a higher nominal rate diminishes the expected future size of banks, it increases the expected aggregate risk aversion of the economy, and hence also discount rates. This dynamic effect on prices via expected future risk aversion is in addition to the local, direct effect of higher nominal rates on risk taking. Below, we further explore the dynamic asset price effects of changes in interest rates by looking at policy shocks and forward guidance.

## *H. Reserves*

Figure 8 plots the ratio of the value of reserves to total wealth ( $G$ ) under each policy. The wealth share of reserves is very small under the high nominal rate policy  $n_2 = 5\%$ , and for most values of  $\omega$  it is much greater under  $n_1 = 0\%$ . Since higher nominal rates make holding reserves more costly, banks hold less reserves (and take less leverage). Indeed, if the interest rate is high enough as to induce financial autarky (Corollary 2) reserves holdings fall to zero.

As the nominal rate decreases, banks take more leverage and hold a larger share of reserves on their balance sheets. In Figure 8, the increase in equilibrium reserves holdings in moving from policy  $n_2$  to a zero-interest rate policy is large, especially at intermediate values of  $\omega$ . Indeed, near  $\omega = 0.2$ , the wealth share of reserves  $G$  rises to almost 0.1. This occurs because under a zero nominal rate there is no cost to holding reserves. In fact, at that point there is no difference between holding reserves and holding bonds.

Figure 8 further shows that reserves holdings depend on the relative size of bank wealth  $\omega$ . The relationship is non-monotonic. Holding the nominal rate fixed, equilibrium reserves holdings at first increase in banks' wealth, and then start to decrease. This result shows that aggregate reserves can both increase and decrease independently of any change in the stance of monetary policy as measured by the nominal rate.

The relation between bank wealth and reserves reflects the aggregate leverage in the economy,  $\omega (w_S^A - 1)$ . When bank wealth  $\omega$  is small, aggregate leverage is small even though per dollar banks are highly levered ( $w_S^A - 1$  is high). As banks' wealth increases, their per-dollar leverage decreases but it does so less rapidly at first as the risk premium remains high. Aggregate leverage therefore increases. As bank wealth continues to rise, however, per-dollar



leverage starts to decline more rapidly. This occurs as risk prices decline due to the increased amount of risk-tolerant capital. Aggregate leverage decreases and it approaches zero as  $\omega$  approaches one. This pattern is responsible for the hum-shaped value of total reserves in Figure 8.

Figure 9 shows the growth rate in the quantity of reserves required to implement the low and high nominal rate policies,  $n_1$  and  $n_2$ . This rate is characterized in Proposition 3. Note that reserves growth is higher under  $n_2$  than  $n_1$ . The reason is that in order to obtain a higher nominal rate, the central bank must make the cost of holding reserves higher. It does so by issuing reserves at a higher rate, which reduces the rate at which existing reserves appreciate, creating higher inflation. The rise in inflation makes the higher nominal rate consistent with the real interest rate.

Looking across the range of the wealth share  $\omega$ , the growth rate of reserves is lowest between 0.2 and 0.5. This is also the region where the real value of reserves  $G$  is relatively flat in  $\omega$  (see Figure 8). In this case, from Proposition 3, the demand for reserves is stable, so money growth is approximately the sum of inflation and expected wealth growth, which is close to the growth in output. In this region, the central bank can maintain a steady nominal rate with steady reserves growth.

In contrast, money growth rises steeply at low or high levels of  $\omega$ . When  $\omega$  is low, the value of reserves is also small since aggregate leverage is small (banks are highly levered but they have little capital). At the same time, banks earn a high risk premium, so their wealth is expected to grow quickly. This means that all else equal, the aggregate value of reserves is expected to rise. To keep the capital gain on reserves from being too high, and hence the cost of holding reserves, the nominal rate, too low, the central bank must increase the growth rate of reserves. At the other end, if  $\omega$  is high, then it is also expected to fall, causing demand for reserves to rise as bank leverage increases. Again, to achieve the desired nominal rate the central bank must increase the growth rate of reserves to prevent them from appreciating too quickly. Note that at either extreme the required growth rate in reserves is unbounded since as  $\omega$  approaches zero or one, the aggregate value of reserves vanishes (see Figure 8).

Turning to the volatility of reserves growth, the picture is asymmetric. When  $\omega$  is low, bank leverage is high, so a positive endowment shock has a strong positive impact on the demand for reserves. To avert unexpected deflation, the quantity of reserves must be increased quickly. For the high rate policy  $n_2$ , demand for reserves rises more slowly, and the required response is less aggressive. When  $\omega$  is high, however, leverage is low and

endowment shocks have a smaller impact on the wealth distribution. Hence, the effect on the reserves demand is much smaller. In this case the central bank does not need to intervene as aggressively.

Overall, Figure 9 shows that in normal times (intermediate  $\omega$  values) the central bank can maintain its policy stance with steady reserves growth close to the sum of endowment growth and the target inflation rate. By contrast, in a downturn, when bank wealth is low, it must increase the rate of reserves growth and also intervene more aggressively when shocks arrive in order to maintain a stable nominal rate.

## VI. Dynamic policies

So far we have restricted attention to constant interest rate policies. We now analyze two settings in which dynamic policies play a central role. The first is one in which the central bank has already lowered the nominal rate to zero, and yet it wishes to further support asset prices. In the literature this situation is often referred to as hitting the “zero lower bound”. We show how in this situation the central bank can support asset prices by guiding down investors’ expectations of future nominal interest rates, i.e. “forward guidance”.

Our second setting examines the effect of an interest rate policy that responds to negative economic shocks by decreasing nominal rates. This type of policy is exemplified by “Greenspan put”, the notion that the Federal Reserve responds to negative shocks in asset markets by cutting interest rates.

### *A. The zero lower bound and forward guidance*

The monetary policy literature usually posits that the nominal rate cannot go below zero. This bound arises endogenously in the model in this paper. Mathematically speaking, the nominal rate must be nonnegative because it equals the Lagrange multiplier on the reserves constraint. More intuitively, the nominal rate reflects the tightness of the constraint on banks’ risk taking. When the nominal rate is zero, banks are satiated in their demand for risk, as shown in Proposition 2. In this case, their weight in the risky asset equals the optimal weight for an unconstrained investor. Banks have no desire to increase risk taking beyond this point.

If the central bank did try to decrease the nominal rate below zero, banks would borrow

deposits to invest in reserves, rather than in risky assets. Since reserves are riskless, this combination would represent an arbitrage. The resulting demand for reserves would then force the nominal rate back up to zero. This asymmetry between positive and negative nominal rates reflects the fact that the reserves requirement forces banks to hold a minimum amount of reserves, but does not prevent them from holding excess reserves.

Nevertheless, the central bank can still use monetary policy to influence asset prices by changing the course of expected future interest rates. This is illustrated in Figure 10. The top panel plots two nominal rate policies. Both policies set the nominal rate to zero when bank capital  $\omega$  is low, as in a financial crisis. Under policy  $n_{fg,2}$  (red squares), the nominal rate increases once bank capital recovers to a value of  $\omega = 0.25$ . In contrast, policy  $n_{fg,1}$  (blue triangles) delays the increase in the nominal rate until  $\omega = 0.3$ . Viewed from states where  $\omega < 0.25$ , it is meant to capture the idea of the central bank providing investors with guidance that rates will remain low for a longer period.

The bottom panel of Figure 10 plots the ratio of the prices of the endowment claim under the two policies,  $P_{fg,1}/P_{fg,2}$ . We focus on values of  $\omega$  less than 0.25, where the nominal rate is zero under both policies. The plot shows that even though the nominal rate is at its lower bound, the expectation that future rates will be lower under policy  $n_{fg,1}$  has a substantial impact on the current price of the endowment claim. For example, for  $\omega = 0.25$  the price of the endowment claim is around 4% higher under the forward guidance policy  $n_{fg,1}$ .

Guiding future nominal rates down increases prices by inducing a decrease in future discount rates. Investors expect that assets will be worth more in the future, and they are therefore willing to pay more for them today. Note that this effect is purely dynamic, it does not work by changing the cost of taking leverage today since this cost is already zero.

Finally, note that prices are higher under  $n_{fg,1}$  than under  $n_{fg,2}$  even when  $\omega$  is such that both policies have equally high nominal rates. The reason is that prices reflect the possibility that  $\omega$  will be low in the future.

## *B. Greenspan put*

As a second example of a state-contingent policy, we analyze the impact of a “Greenspan put”.<sup>14</sup> Within the context of our model, we interpret a Greenspan put as a policy that

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<sup>14</sup>The term dates to the late 1990s when critics faulted Federal Reserve chairman Alan Greenspan for “encouraging excessive risk-taking by creating what came to be called ‘the Greenspan put’, that is, the belief that the Fed would, if necessary, support the economy and therefore the stock market” (Blinder and Reis

reduces nominal interest rates in the event of a large enough sequence of negative shocks. Specifically, we consider the simple example of a constant benchmark and a Greenspan put alternative defined as

$$n_{gp,1}(\omega_t) = 0.04 \quad (31)$$

$$n_{gp,2}(\omega_t) = \min \left\{ 0.05, \frac{0.05}{0.3} \omega_t \right\}. \quad (32)$$

Under policy  $n_{gp,2}$ , the nominal rate rises from 0% at  $\omega = 0$  at a constant rate before reaching 5% at  $\omega = 0.3$  at which point it levels off. Thus, a sequence of negative shocks that pushes the bank capital share  $\omega$  below 0.3 triggers progressive rate decreases. The level of the constant benchmark  $n_{gp,1}$  is set so that the two policies have similar unconditional average nominal rates (integrated against the stationary distribution of  $\omega$ ).

The results are presented in Figure 11. The top left panel plots the two policies, while the top right panel displays the wealth-consumption ratio. The Greenspan put policy  $n_{gp,2}$  results in lower prices when  $\omega$  is high as it implements a higher nominal rate. However, when  $\omega$  approaches the cutoff 0.3 from above, the valuation under  $n_{gp,2}$  approaches that under the benchmark  $n_{gp,1}$ . This occurs because of the nearing prospect of lower nominal rates. As  $\omega$  falls below 0.3, the valuation under  $n_{gp,2}$  flattens out and even mildly increases, whereas it falls under  $n_{gp,1}$ . In this way, the central bank is supporting asset prices by cutting nominal rates. As  $\omega$  continues to fall however, there is little room for further rate cuts and valuations begin to fall steeply so that they are nearly equal under the two policies at  $\omega = 0$ . The Greenspan put policy therefore has the effect of stabilizing prices in a moderate downturn but it cannot forestall a severe price decline in a highly adverse scenario.

The bottom left panel of Figure 11 plots the risk premium. When  $\omega$  is high, the higher nominal rates of the Greenspan put policy result in a higher risk premium. As  $\omega$  declines towards 0.3, the stabilization effect of the policy results in reduced risk premia. Once rates actually start falling past 0.3, the risk premium drops precipitously as a result of the aggressive rate cutting. However, when  $\omega$  nears zero, prices are set to fall even more steeply than under the benchmark policy, so the risk premium under  $n_{gp,2}$  eventually falls below that under  $n_{gp,1}$ .

The bottom right panel of Figure 11 plots volatility. The pattern here is also driven by the behavior of prices. Under the Greenspan put policy  $n_{gp,2}$ , volatility is lower when  $\omega$  is high. This is due to the higher nominal rate in this region, which suppresses risk taking and stabilizes aggregate risk aversion. As  $\omega$  declines towards 0.3, it dips further as the prospect of

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2005).

intervention keeps prices from falling. Past 0.3, the “put” goes into the money and the rate cutting kicks in, causing volatility to fall even further. In this way, the Greenspan put policy is able to reduce volatility in moderate downturns. However, if  $\omega$  declines even further, the temporary support runs out and prices enter a steep descent. As a result, the Greenspan put results in higher volatility in severe downturns.

## VII. Policy shocks

In the baseline model the interest-rate policy gives the nominal rate as a function of the single state variable  $\omega$ . The only shock is the endowment growth shock and there is no independent monetary policy shock. Hence, in examining the impact of different nominal rates on equilibrium, we have so far compared across equilibria under different nominal rate rules. In this section we extend the model to incorporate an independent shock to the interest rate policy.

We model the monetary policy shock as exogenous. Under this shock the central bank “surprises the market” by raising or lowering nominal rates independently of the endowment growth shock. A positive shock has two effects. The first is direct: it increase banks’ cost of taking leverage. This is the same effect as in the baseline comparison across policies. The second, indirect effect is that a surprise rate change impacts prices and causes the wealth distribution to change. This change in the state variable produces second-round effects on prices that amplify the direct impact of the rate change.

### A. Model extension

We alter the policy rule with two objectives in mind. First, we want to allow for independent policy shocks. These shock the nominal rate away from a benchmark policy rule that agents know. As under the baseline model, this benchmark rule is a function of  $\omega$ . At the same time, we do not want the nominal rate to stray too far from the benchmark rule. This leads to a clearer interpretation of both the benchmark rule and the ensuing quantitative results.

To that end, let  $n_b(\omega_t) \in [\underline{n}, \bar{n}]$  be the benchmark policy rate and suppose the nominal rate  $n_t$  follows the process

$$dn_t = -\kappa_n \left[ n_t - n_b(\omega_t) \right] dt + (n_t - \underline{n}) (\bar{n} - n_t) \sigma_n dB_t^n, \quad (33)$$

where  $dB^n$  are the policy shocks, which we assume are independent of the endowment growth shocks  $dB$ . The nominal rate tends to drift towards the benchmark rate  $n_b(\omega_t)$  at the rate  $\kappa_n$ . Note that due to the structure of the diffusive component,  $n$  is bounded below by  $\underline{n}$  and above by  $\bar{n}$ .

Since shocks to  $n$  are persistent, the equilibrium now depends on two state variables, the wealth share  $\omega$  and nominal rate  $n$ . Moreover, the dynamics of  $\omega$  now depend on  $n$ , so that  $\mu_\omega = \mu_\omega(\omega, n)$  and  $\sigma_\omega = \sigma_\omega(\omega, n)$ . Hence, we rewrite all endogenous functions in terms of the two state variables. For example,  $F = F(\omega, n)$ . Furthermore, we maintain the same notation for the diffusions, but now the exposures (i.e.,  $\sigma$ ) are  $2 \times 1$  vectors whose first and second components correspond to the endowment shock  $dB$  and the policy shock  $dB^n$ . The rest of the model, including the reserve requirement, is unchanged.

We now state the form of the agent's value function and optimal portfolio choice under the extended model, leaving the full derivation to the Appendix.

**Proposition 4.** *The agents' value functions are given by*

$$V(W, \omega, n) = \left( \frac{W^{1-\gamma}}{1-\gamma} \right) J(\omega, n)^{\frac{1-\gamma}{1-\psi}}, \quad (34)$$

where  $J(\omega, n)$  represents the agents' optimal consumption-wealth ratio,  $c^* = J$ . Banks take leverage ( $w_S^A > 1$ ) when (and only when)

$$\begin{aligned} \lambda n < \gamma^B - \gamma^A \\ - \left[ \left( \frac{1-\gamma^A}{1-\psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1-\gamma^B}{1-\psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{\frac{F_n}{F} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right]. \end{aligned} \quad (35)$$

In this case, their holdings of the endowment claim are given by

$$\begin{aligned} w_S^A = \frac{1}{\gamma} \left\{ \frac{\mu - r}{\sigma' \sigma} - \lambda n \right. \\ \left. + \left( \frac{1-\gamma}{1-\psi} \right) \left[ \frac{J_\omega}{J} \omega (1-\omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n}{J} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\}. \end{aligned} \quad (36)$$

*Proof.* The proof is contained in Appendix B. □

The solution of the extended model generally follows that of the benchmark model. We note two differences. First, the portfolio demand (36) includes a hedging term for policy shocks. Second, the boundary of the region in which banks take leverage in  $n \times \omega$  space

(equation 35) depends on the difference in hedging demands between the two agent types. The reason is that, even though  $\omega$  becomes locally deterministic in the no-leverage region,  $n$  does not.

## B. Results

To illustrate the model with policy shocks, we set  $n_b = 0.03$ ,  $\underline{n} = 0.00$ ,  $\bar{n} = 0.06$ ,  $\kappa_n = 0.01$ , and  $\sigma_n = 5$ . A high persistence (low  $\kappa_n$ ) for policy shocks increases their impact on valuations, while the boundaries  $\underline{n}$  and  $\bar{n}$  keep the nominal rate from drifting very far away from the benchmark policy. Figure B.1 in Appendix B plots the joint stationary density of  $n$  and  $\omega$  under the model to give a sense of the distribution of these state variables.

Figure 12 shows the impact of a policy shock that raises the nominal interest rate from 1% to 4%, at different values of  $\omega$ . As in the benchmark case, the increase in the nominal rate leads to a higher risk premium, lower real interest rate, and lower valuation of the endowment claim. Since the shock is highly persistent, the effects are similar to the changes observed in the benchmark model across the high and low nominal rate policies.

Figure 12 further shows that policy shocks have a second-round effect on prices. This occurs because in changing prices the interest rate shock causes a change in the wealth distribution. This is a result of the differences in risk taking across agent types. When the nominal rate is 1%, banks employ high levels of leverage. As a result, the fall in the value of the wealth claim due to the nominal rate increase causes them to lose wealth disproportionately. The top left panel in the figure shows the fall in banks' wealth share  $\omega$ . The change in the wealth distribution is greatest when  $\omega$  is between one quarter and one third, since at this point banks' wealth share is significant and their leverage is high.

The top right panel shows that the wealth redistribution effect amplifies the first-round fall in prices. The dashed red line isolates the direct effect of the policy shock on the valuation ratio, calculated by holding  $\omega$  constant. The total effect, given by the solid red line, also incorporates the additional price impact of the policy shock due to the induced change in the wealth distribution. By reducing the size of bank balance sheets, higher rates reduce banks' capacity to bear risk, causing prices to fall further. Hence, the total effect is always greater than the direct effect. This amplification resembles the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), except it is driven by policy shocks.

The bottom panels of Figure 12 show that under these parameters the amplification

arises mainly through the real interest rate rather than the risk premium. Looking first at the risk premium, the indirect effect of the policy shock is small, as seen in the narrow gap between the direct and total effects, though the two go in the same direction. This effect is comparatively small because at these parameters leverage is quite low at the high nominal rate and hence the risk premium is nearly flat in  $\omega$ .

Turning to the real interest rate, the first-round effect of the policy shock is negative, as in the baseline model. However, because the shock shifts the wealth distribution towards the risk averse depositors who have a strong precautionary motive, the second-round effect actually raises the real rate. In this way, the policy shock dampens the fall in the real interest rate, leading to a greater fall in prices.

We note that policy shocks in our model have pronounced asymmetric effects. In particular, a rate hike leads to greater amplification than a rate drop. The reason is that at higher rates banks take less leverage, so the wealth distribution is not as significantly affected when prices change.

## VIII. Conclusion

Contemporary monetary policy is substantially concerned with the functioning of the financial system and with valuations in the markets for risky assets. Through their effects on financial institutions, central bank interventions drive not only the level of interest rates in the economy, but also the level of risk premia.

We present an asset pricing framework that enables us to study the relationship between monetary policy and risk premia in an environment with no nominal rigidities. When raising deposits to invest in risky assets is subject to a reserve-type requirement, the nominal interest rate represents the effective cost of leverage. Lower rates lead to more risk taking and lower though more volatile risk premia. Unexpected rate changes affect bank balance sheets and have amplified effects. A zero lower bound reflects satiation in risk taking and the central bank can support asset prices further through forward guidance.



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## Appendix A

*Proof of Proposition 1.* Conjecture that  $V$  has the form in (20). After substituting for  $V$  and  $f$  from (3), wealth drops out of the HJB equation (19):

$$\begin{aligned}
0 = & \max_{c, w_S, w_M} \left( \frac{1-\gamma}{1-1/\psi} \right) \left[ \left( \frac{c}{J^{1-\psi}} \right)^{1-1/\psi} - (\rho + \kappa) \right] \\
& + (1-\gamma) \left[ r - c + w_S (\mu - r) - \frac{\gamma}{2} (w_S \sigma)^2 - w_M n + Gn \right] \\
& + \left( \frac{1-\gamma}{1-\psi} \right) \left[ \frac{J_\omega}{J} \omega (1-\omega) \mu_\omega + (1-\gamma) \frac{J_\omega}{J} \omega (1-\omega) w_S \sigma_\omega \sigma \right] \\
& + \frac{1}{2} \left( \frac{1-\gamma}{1-\psi} \right) \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1-\omega)^2 \sigma_\omega^2 \\
& + (1-\gamma) \theta_\lambda [w_M - \lambda \sigma^2 (w_S - 1)] + (1-\gamma) \theta_0 w_M.
\end{aligned} \tag{A.1}$$

The FOC for consumption gives

$$c = J. \tag{A.2}$$

Substituting and rearranging,

$$\begin{aligned}
(\rho + \kappa) \left( \frac{1-\gamma}{1-1/\psi} \right) = & \max_{w_S, w_M} (1-\gamma) \left( \frac{1/\psi}{1-1/\psi} \right) J \\
& + (1-\gamma) \left[ r + w_S (\mu - r) - \frac{\gamma}{2} (w_S \sigma)^2 - w_M n + Gn \right] \\
& + \left( \frac{1-\gamma}{1-\psi} \right) \left[ \frac{J_\omega}{J} \omega (1-\omega) \mu_\omega + (1-\gamma) \frac{J_\omega}{J} \omega (1-\omega) w_S \sigma_\omega \sigma \right] \\
& + \frac{1}{2} \left( \frac{1-\gamma}{1-\psi} \right) \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1-\omega)^2 \sigma_\omega^2 \\
& + (1-\gamma) \theta_\lambda [w_M - \lambda \sigma^2 (w_S - 1)] + (1-\gamma) \theta_0 w_M.
\end{aligned} \tag{A.3}$$

Portfolio demand is characterized by

$$w_S = \frac{1}{\gamma} \left[ \frac{\mu - r}{\sigma^2} - \lambda \theta_\lambda + \left( \frac{1-\gamma}{1-\psi} \right) \frac{J_\omega}{J} \omega (1-\omega) \frac{\sigma_\omega}{\sigma} \right] \tag{A.4}$$

$$n = \theta_0^B + \theta_\lambda. \tag{A.5}$$

Let

$$\underline{w}_S = \frac{1}{\gamma} \left[ \frac{\mu - r}{\sigma^2} - \lambda n + \left( \frac{1-\gamma}{1-\psi} \right) \frac{J_\omega}{J} \omega (1-\omega) \frac{\sigma_\omega}{\sigma} \right] \tag{A.6}$$

$$\bar{w}_S = \frac{1}{\gamma} \left[ \frac{\mu - r}{\sigma^2} + \left( \frac{1-\gamma}{1-\psi} \right) \frac{J_\omega}{J} \omega (1-\omega) \frac{\sigma_\omega}{\sigma} \right]. \tag{A.7}$$

There are three possible cases:

$$w_S = \begin{cases} \bar{w}_S & \text{if } \bar{w}_S \leq 1 \\ 1 & \text{if } \underline{w}_S \leq 1 < \bar{w}_S \\ \underline{w}_S & \text{if } 1 < \underline{w}_S. \end{cases} \quad (\text{A.8})$$

The corresponding multipliers are

$$\{\theta_0, \theta_\lambda\} = \begin{cases} \{n, 0\} & \text{if } \bar{w}_S \leq 1 \\ \left\{ \frac{\gamma}{\lambda} (1 - \underline{w}_S), \frac{\gamma}{\lambda} (\bar{w}_S - 1) \right\} & \text{if } \underline{w}_S \leq 1 < \bar{w}_S \\ \{0, n\} & \text{if } 1 < \underline{w}_S. \end{cases} \quad (\text{A.9})$$

Substituting into the HJB equation and simplifying,

$$\begin{aligned} \rho &= 1/\psi J + (1 - 1/\psi) (r + \lambda \sigma^2 \theta_\lambda + Gn) & (\text{A.10}) \\ &- 1/\psi \left( \frac{J_\omega}{J} \omega (1 - \omega) \mu_\omega + \frac{1}{2} \left[ \left( \frac{\psi - \gamma}{1 - \psi} \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1 - \omega)^2 \sigma_\omega^2 \right) \\ &+ \frac{1}{2} \left( \frac{1 - 1/\psi}{\gamma} \right) \left[ \frac{\mu - r}{\sigma^2} - \lambda \theta_\lambda + \left( \frac{1 - \gamma}{1 - \psi} \right) \frac{J_\omega}{J} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right]^2 \sigma^2. \end{aligned}$$

The market-clearing equation (17) for the endowment claim implies that only one type of agents, if any, takes leverage, so the equilibrium must be in one of the three cases,

$$w_S^A > 1, \quad w_S^B < 1 \quad (\text{A.11})$$

$$w_S^A = 1, \quad w_S^B = 1 \quad (\text{A.12})$$

$$w_S^A < 1, \quad w_S^B > 1. \quad (\text{A.13})$$

Substituting,

$$\{w_S^A, w_S^B\} = \begin{cases} \{\underline{w}_S^A, \bar{w}_S^B\} & \text{if } \bar{w}_S^B \leq 1 < \underline{w}_S^A \\ \{1, 1\} & \text{if } \underline{w}_S^A, \bar{w}_S^B \leq 1 < \bar{w}_S^A, \underline{w}_S^B \\ \{\bar{w}_S^A, \underline{w}_S^B\} & \text{if } \bar{w}_S^A \leq 1 < \underline{w}_S^B. \end{cases} \quad (\text{A.14})$$

Call these three cases (i), (ii), and (iii).

Under case (i),

$$w_S^A = \frac{1}{\gamma^A} \left[ \frac{\mu - r}{\sigma^2} - \lambda n + \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} \omega (1 - \omega) \left( \frac{\sigma_\omega}{\sigma} \right) \right] \quad (\text{A.15})$$

$$w_S^B = \frac{1}{\gamma^B} \left[ \frac{\mu - r}{\sigma^2} + \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \omega (1 - \omega) \left( \frac{\sigma_\omega}{\sigma} \right) \right]. \quad (\text{A.16})$$

Note that

$$\frac{\sigma_\omega}{\sigma} = \frac{1}{1 - \omega} (w_S^A - 1). \quad (\text{A.17})$$

Stock-market clearing gives

$$1 = \omega w_S^A + (1 - \omega) w_S^B \quad (\text{A.18})$$

$$= \omega w_S^A + (1 - \omega) \frac{1}{\gamma^B} \left\{ \gamma^A w_S^A + \lambda n \right. \\ \left. - \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (w_S^A - 1) \right\}. \quad (\text{A.19})$$

This gives a linear equation for  $w_S^A$  in terms of exogenous and conjectured quantities. The solution is

$$w_S^A = \frac{1 - \frac{1}{\gamma^B} (1 - \omega) \lambda n - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (1 - \omega)}{\omega + (1 - \omega) \frac{\gamma^A}{\gamma^B} - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (1 - \omega)}. \quad (\text{A.20})$$

We need to verify  $w_S^A > 1$ , which gives

$$\lambda n < \gamma^B - \gamma^A. \quad (\text{A.21})$$

From here we can get  $(\mu - r) / \sigma^2$ , and  $\sigma_\omega / \sigma$ . This also gives  $\sigma$  and hence  $\sigma_\omega$ , and as a result,  $\mu - r$ . To get the drift of  $\omega$ , apply Ito's Lemma to (4) and use  $W^A + W^B = P$  to obtain

$$\frac{d\omega}{\omega(1 - \omega)} = \left( \frac{dW^A}{W^A} - \frac{dW^B}{W^B} \right) - \left( \frac{dW^A}{W^A} - \frac{dW^B}{W^B} \right) \left( \frac{dP}{P} \right). \quad (\text{A.22})$$

Substituting for the evolution of aggregate type- $A$  and type- $B$  wealth gives (4) and

$$\mu_\omega = (w_S^A - w_S^B) (\mu - r) + \lambda \sigma^2 (w_S^B - 1) n - (J^A - J^B) - \sigma_\omega \sigma. \quad (\text{A.23})$$

This can be plugged into the dynamics of returns to get  $\mu$ :

$$dR = \frac{dD/F}{D/F} + F dt \quad (\text{A.24})$$

$$= \frac{dD}{D} - \frac{dF}{F} - \left( \frac{dD}{D} \right) \left( \frac{dF}{F} \right) + \left( \frac{dF}{F} \right)^2 + F dt \quad (\text{A.25})$$

$$\mu = \mu_D + F - \frac{F_\omega}{F} \omega (1 - \omega) (\mu_\omega + \sigma_\omega \sigma_D) \\ + \left[ \left( \frac{F_\omega}{F} \right)^2 - \frac{1}{2} \frac{F_{\omega\omega}}{F} \right] \omega^2 (1 - \omega)^2 \sigma_\omega^2 \quad (\text{A.26})$$

$$\sigma = \sigma_D - \frac{F_\omega}{F} \omega (1 - \omega) \sigma_\omega. \quad (\text{A.27})$$

From here, get  $r$  using  $r = \mu - (\mu - r)$ . Finally, plug the constraints  $\theta_\lambda^A = n$  and  $\theta_\lambda^B = 0$  into the HJB equations to verify the conjectures for  $J^A$  and  $J^B$ . To obtain the value of reserves,

use the reserves-market clearing equation (18),

$$\{w_M^A, w_M^B\} = \left\{ \frac{G}{\omega}, 0 \right\}. \quad (\text{A.28})$$

The binding leverage constraint pins down the value of reserves:

$$G = \omega \lambda \sigma^2 (w_S^A - 1). \quad (\text{A.29})$$

Under case (ii),

$$\{w_S^A, w_S^B\} = \{1, 1\} \quad (\text{A.30})$$

$$\{w_M^A, w_M^B\} = \{0, 0\}. \quad (\text{A.31})$$

The stock market clears and  $G = 0$ . From here, we get  $\sigma_\omega = 0$  and so  $\sigma = \sigma_D$ . Next, use

$$\mu_\omega = -(J^A - J^B). \quad (\text{A.32})$$

in the dynamics of returns (A.26) and (A.27) to get  $\mu$  and  $\sigma$ . Substituting into the HJB equations and simplifying,

$$\rho = 1/\psi J + (1 - 1/\psi) \left( \mu - \frac{\gamma}{2} \sigma^2 \right) - 1/\psi \frac{J_\omega}{J} [\kappa (\bar{\omega} - \omega) + \omega (1 - \omega) \mu_\omega]. \quad (\text{A.33})$$

This case requires

$$\lambda n > |\gamma^A - \gamma^B|. \quad (\text{A.34})$$

The real interest rate lies inside a range between a lending and a borrowing rate.

Case (iii) is analogous to Case (i) with the roles reversed. It requires  $\lambda n < \gamma^A - \gamma^B$ , which is ruled out by assumption. This completes the proof.  $\square$

*Proof of Proposition 2.* Banks take leverage under case (i) in the proof of Proposition 1 above. This case From (A.21), requires  $\lambda n < \gamma^B - \gamma^A$ . Banks' portfolio demand is then given by (A.15).  $\square$

*Proof of Proposition 3.* Equation (26) follows from the fact that reserves are costly and therefore the reserve requirement binds, see (A.29). To obtain (27), apply Ito's Lemma to  $\Pi_t = M_t \pi_t$  and use the fact that inflation  $-d\pi_t/\pi_t = \iota(\omega_t) dt = [n(\omega_t) - r(\omega_t)] dt$  is locally deterministic (equations 9 and 10). Finally, apply Ito's Lemma to  $\Pi_t = G_t P_t$  to obtain (28).  $\square$



## Appendix B

This Appendix contains the derivations for the model with policy shocks. Denote the dynamics of  $\omega$  by

$$d\omega = \kappa(\bar{\omega} - \omega) dt + \omega(1 - \omega) \left[ \mu_\omega(\omega, n) dt + \sigma_\omega(\omega, n)' dB \right]. \quad (\text{B.1})$$

Write the return process

$$dR = \frac{dP + Ddt}{P} \quad (\text{B.2})$$

$$= \mu(\omega, n) dt + \sigma(\omega, n)' dB, \quad (\text{B.3})$$

the instantaneous real risk-free rate as  $r = r(\omega, n)$ , and the dividend yield as  $F = F(\omega, n)$ . Applying Ito's Lemma gives

$$\begin{aligned} \mu &= \mu_D + F - \frac{F_\omega}{F} [\kappa(\bar{\omega} - \omega) + \omega(1 - \omega)(\mu_\omega + \sigma'_\omega \sigma_D)] - \frac{F_n}{F} \kappa_n (n - n_b) \\ &+ \left[ \left( \frac{F_\omega}{F} \right)^2 - \frac{1}{2} \frac{F_{\omega\omega}}{F} \right] \omega^2 (1 - \omega)^2 \sigma'_\omega \sigma_\omega + \left[ \left( \frac{F_n}{F} \right)^2 - \frac{1}{2} \frac{F_{nn}}{F} \right] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2 \\ &+ 2 \left[ \left( \frac{F_\omega}{F} \right) \left( \frac{F_n}{F} \right) - \frac{1}{2} \frac{F_{\omega n}}{F} \right] \omega (1 - \omega) (n - \underline{n}) (\bar{n} - n) \sigma_{\omega,2} \sigma_n \end{aligned} \quad (\text{B.4})$$

$$\sigma = \begin{bmatrix} \sigma_D \\ 0 \end{bmatrix} - \frac{F_\omega}{F} \omega (1 - \omega) \sigma_\omega - \frac{F_n}{F} (n - \underline{n}) (\bar{n} - n) \begin{bmatrix} 0 \\ \sigma_n \end{bmatrix}. \quad (\text{B.5})$$

The reserve requirement is

$$w_M \geq \max \left[ \lambda \sigma' \sigma (w_S - 1), 0 \right]. \quad (\text{B.6})$$

The wealth dynamics are as in the benchmark case.

*Proof of Proposition 4.* Dropping agent subscripts and applying LaGrange multipliers  $\theta_\lambda V_W W$  and  $\theta_0 V_W W$  on the leverage and non-negativity constraints, the HJB equation is

$$\begin{aligned} 0 &= \max_{c, w_S, w_M} f(cW, V) + V_W W [r - c + w_S(\mu - r) - w_M n + Gn] \quad (\text{B.7}) \\ &+ V_\omega [\kappa(\bar{\omega} - \omega) + \omega(1 - \omega)\mu_\omega] + V_n \kappa_n (n - n_b) \\ &+ V_{W\omega} W \omega (1 - \omega) w_S \sigma'_\omega \sigma + V_{Wn} W w_S (n - \underline{n}) (\bar{n} - n) \sigma_2 \sigma_n + \frac{1}{2} V_{WW} W^2 w_S^2 \sigma' \sigma \\ &+ \frac{1}{2} V_{\omega\omega} \omega^2 (1 - \omega)^2 \sigma'_\omega \sigma_\omega + V_{\omega n} \omega (1 - \omega) (n - \underline{n}) (\bar{n} - n) \sigma_n \sigma_{\omega,2} \\ &+ \frac{1}{2} V_{nn} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2 + \theta_\lambda V_W W [w_M - \lambda \sigma' \sigma (w_S - 1)] + \theta_0 V_W W w_M. \end{aligned}$$

Conjecture that the value function has the form

$$V(W, \omega, n) = \left( \frac{W^{1-\gamma}}{1-\gamma} \right) J(\omega, n)^{\frac{1-\gamma}{1-\psi}}. \quad (\text{B.8})$$

Then wealth drops out of the HJB equation:

$$\begin{aligned} 0 = & \max_{c, w_S, w_M} \frac{1}{1-1/\psi} [c^{1-1/\psi} J^{1/\psi} - (\rho + \kappa)] + r - c + w_S (\mu - r) - \frac{\gamma}{2} w_S^2 \sigma' \sigma \quad (\text{B.9}) \\ & - w_M n + Gn - \frac{1/\psi}{1-1/\psi} \left[ \frac{J_\omega}{J} [\kappa (\bar{w} - \omega) + \omega (1 - \omega) \mu_\omega] + \frac{J_n}{J} \kappa_n (n - n_b) \right. \\ & \left. + (1 - \gamma) \frac{J_\omega}{J} \omega (1 - \omega) w_S \sigma'_\omega \sigma + (1 - \gamma) \frac{J_n}{J} w_S (n - \underline{n}) (\bar{n} - n) \sigma_2 \sigma_n \right] \\ & - \frac{1}{2} \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1 - \omega)^2 \sigma'_\omega \sigma_\omega \\ & - \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right) \left( \frac{J_n}{J} \right) + \frac{J_{\omega n}}{J} \right] \omega (1 - \omega) (n - \underline{n}) (\bar{n} - n) \sigma_n \sigma_\omega \\ & - \frac{1}{2} \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_n}{J} \right)^2 + \frac{J_{nn}}{J} \right] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2 \\ & + \theta_\lambda [w_M - \lambda \sigma' \sigma (w_S - 1)] + \theta_0 w_M. \end{aligned}$$

The FOC for consumption gives

$$c = J. \quad (\text{B.10})$$

Substituting and rearranging,

$$\begin{aligned} 0 = & \max_{w_S, w_M} \frac{1/\psi J - (\rho + \kappa)}{1-1/\psi} + r + w_S (\mu - r) - \frac{\gamma}{2} w_S^2 \sigma' \sigma - w_M n + Gn \quad (\text{B.11}) \\ & - \frac{1/\psi}{1-1/\psi} \left[ \frac{J_\omega}{J} [\kappa (\bar{w} - \omega) + \omega (1 - \omega) \mu_\omega] + \frac{J_n}{J} \kappa_n (n - n_b) \right. \\ & \left. + (1 - \gamma) \frac{J_\omega}{J} \omega (1 - \omega) w_S \sigma'_\omega \sigma + (1 - \gamma) \frac{J_n}{J} w_S (n - \underline{n}) (\bar{n} - n) \sigma_2 \sigma_n \right] \\ & - \frac{1}{2} \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right)^2 + \frac{J_{\omega\omega}}{J} \right] \omega^2 (1 - \omega)^2 \sigma'_\omega \sigma_\omega \\ & - \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_\omega}{J} \right) \left( \frac{J_n}{J} \right) + \frac{J_{\omega n}}{J} \right] \omega (1 - \omega) (n - \underline{n}) (\bar{n} - n) \sigma_n \sigma_\omega \\ & - \frac{1}{2} \frac{1/\psi}{1-1/\psi} \left[ \left( \frac{1-\gamma}{1-\psi} - 1 \right) \left( \frac{J_n}{J} \right)^2 + \frac{J_{nn}}{J} \right] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2 \\ & + \theta_\lambda [w_M - \lambda \sigma' \sigma (w_S - 1)] + \theta_0 w_M. \end{aligned}$$

Portfolio demand is characterized by

$$w_S = \frac{1}{\gamma} \left\{ \frac{\mu - r}{\sigma' \sigma} - \lambda \theta_\lambda \right. \quad (\text{B.12})$$

$$\left. + \left( \frac{1 - \gamma}{1 - \psi} \right) \left[ \frac{J_\omega}{J} \omega (1 - \omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n}{J} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\}$$

$$n = \theta_0^B + \theta_\lambda. \quad (\text{B.13})$$

Let

$$\underline{w}_S = \frac{1}{\gamma} \left\{ \frac{\mu - r}{\sigma' \sigma} - \lambda n \right. \quad (\text{B.14})$$

$$\left. + \left( \frac{1 - \gamma}{1 - \psi} \right) \left[ \frac{J_\omega}{J} \omega (1 - \omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n}{J} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\}$$

$$\bar{w}_S = \frac{1}{\gamma} \left\{ \frac{\mu - r}{\sigma' \sigma} \right. \quad (\text{B.15})$$

$$\left. + \left( \frac{1 - \gamma}{1 - \psi} \right) \left[ \frac{J_\omega}{J} \omega (1 - \omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n}{J} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\}.$$

There are three possible cases:

$$w_S = \begin{cases} \bar{w}_S & \text{if } \bar{w}_S \leq 1 \\ 1 & \text{if } \underline{w}_S \leq 1 < \bar{w}_S \\ \underline{w}_S & \text{if } 1 < \underline{w}_S. \end{cases} \quad (\text{B.16})$$

The corresponding multipliers are

$$\{\theta_0, \theta_\lambda\} = \begin{cases} \{n, 0\} & \text{if } \bar{w}_S \leq 1 \\ \left\{ \frac{\gamma}{\lambda} (1 - \underline{w}_S), \frac{\gamma}{\lambda} (\bar{w}_S - 1) \right\} & \text{if } \underline{w}_S \leq 1 < \bar{w}_S \\ \{0, n\} & \text{if } 1 < \underline{w}_S. \end{cases} \quad (\text{B.17})$$

Substituting into the HJB equation and simplifying,

$$\rho + \kappa = 1/\psi J + (1 - 1/\psi) \left( r + \lambda \sigma' \sigma \theta_\lambda + G n + \frac{\gamma}{2} w_S^2 \sigma' \sigma \right) \quad (\text{B.18})$$

$$- 1/\psi \left[ \frac{J_\omega}{J} \omega (1 - \omega) [\kappa (\bar{w} - \omega) + \omega (1 - \omega) \mu_\omega] + \frac{J_n}{J} \kappa_n (n - n_b) \right]$$

$$- \frac{1/\psi}{2} \left[ \frac{J_{\omega\omega}}{J} \omega^2 (1 - \omega)^2 \sigma'_\omega \sigma_\omega + 2 \frac{J_{\omega n}}{J} \omega (1 - \omega) (n - \underline{n}) (\bar{n} - n) \sigma_n \sigma_{\omega,2} \right.$$

$$\left. + \frac{J_{nn}}{J} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2 \right] - \frac{1/\psi}{2} \left( \frac{\psi - \gamma}{1 - \psi} \right) \left[ \left( \frac{J_\omega}{J} \right) \omega (1 - \omega) \sigma_\omega \right.$$

$$\left. + \left( \frac{J_n}{J} \right) (n - \underline{n}) (\bar{n} - n) \left[ \begin{array}{c} 0 \\ \sigma_n \end{array} \right] \right]' \left[ \left( \frac{J_\omega}{J} \right) \omega (1 - \omega) \sigma_\omega + \left( \frac{J_n}{J} \right) (\bar{n} - n) \left[ \begin{array}{c} 0 \\ \sigma_n \end{array} \right] \right].$$

The markets for goods, stocks, and reserves must clear (the bond market clears by Walras' law):

$$\omega c^A + (1 - \omega) c^B = F \quad (\text{B.19})$$

$$\omega w_S^A + (1 - \omega) w_S^B = 1 \quad (\text{B.20})$$

$$\omega w_M^A + (1 - \omega) w_M^B = G. \quad (\text{B.21})$$

Market-clearing implies that only one type of agents, at most, takes leverage, so there are three possible cases in equilibrium:

$$w_S^A > 1, \quad w_S^B < 1 \quad (\text{B.22})$$

$$w_S^A = 1, \quad w_S^B = 1 \quad (\text{B.23})$$

$$w_S^A < 1, \quad w_S^B > 1. \quad (\text{B.24})$$

Call these three cases (i), (ii), and (iii). Under case (i),

$$w_S^A = \frac{1}{\gamma^A} \left\{ \frac{\mu - r}{\sigma' \sigma} - \lambda n + \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \left[ \frac{J_\omega^A}{J^A} \omega (1 - \omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n^A}{J^A} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\} \quad (\text{B.25})$$

$$w_S^B = \frac{1}{\gamma^B} \left\{ \frac{\mu - r}{\sigma' \sigma} + \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \left[ \frac{J_\omega^B}{J^B} \omega (1 - \omega) \left( \frac{\sigma'_\omega \sigma}{\sigma' \sigma} \right) + \frac{J_n^B}{J^B} (n - \underline{n}) (\bar{n} - n) \left( \frac{\sigma_2 \sigma_n}{\sigma' \sigma} \right) \right] \right\}. \quad (\text{B.26})$$

Note that

$$\begin{aligned} \frac{\sigma'_\omega \sigma}{\sigma' \sigma} &= \frac{1}{1 - \omega} (w_S^A - 1) \quad (\text{B.27}) \\ \frac{\sigma_2 \sigma_n}{\sigma' \sigma} &= \left[ 1 + \frac{F_\omega}{F} \omega (w_S^A - 1) \right] \frac{-\frac{F_n}{F} (n - \underline{n}) (\bar{n} - n) \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}. \end{aligned}$$

Stock-market clearing gives

$$1 = \omega w_S^A + (1 - \omega) w_S^B \quad (\text{B.28})$$

$$\begin{aligned} &= \omega w_S^A + (1 - \omega) \frac{1}{\gamma^B} \left\{ \gamma^A w_S^A + \lambda n \right. \\ &\quad - \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (w_S^A - 1) - \left[ 1 + \frac{F_\omega}{F} \omega (w_S^A - 1) \right] \\ &\quad \left. \cdot \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{-\frac{F_n}{F} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right] \right\}. \quad (\text{B.29}) \end{aligned}$$

This gives a linear equation for  $w_S^A$  in terms of exogenous and conjectured quantities. The

solution is

$$w_S^A = \frac{1 - \frac{1}{\gamma^B} (1 - \omega) \lambda n - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (1 - \omega) - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{\frac{F_n}{F} [(1 - \omega) - \frac{F_\omega}{F} \omega (1 - \omega)] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right]}{\omega + (1 - \omega) \frac{\gamma^A}{\gamma^B} - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_\omega^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_\omega^B}{J^B} \right] \omega (1 - \omega) - \frac{1}{\gamma^B} \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{\frac{F_n}{F} [-\frac{F_\omega}{F} \omega (1 - \omega)] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right]} \quad (\text{B.30})$$

We need to verify  $w_S^A > 1$ :

$$\lambda n < \gamma^B - \gamma^A \quad (\text{B.31})$$

$$- \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{\frac{F_n}{F} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right].$$

From here we can get  $(\mu - r) / (\sigma' \sigma)$ ,  $(\sigma'_\omega \sigma) / (\sigma' \sigma)$ , and  $(\sigma_2 \sigma_n) / (\sigma' \sigma)$ . This also gives  $\sigma$  and hence  $\sigma_\omega$ , and as a result,  $\mu - r$ .

Next, calculate  $w_S^B$  (verify  $w_S^B < 1$ ) and calculate

$$\mu_\omega = (w_S^A - w_S^B) (\mu - r) + \lambda \sigma' \sigma (w_S^B - 1) n - (J^A - J^B) - \sigma'_\omega \sigma. \quad (\text{B.32})$$

This can be plugged into the dynamics of returns to get  $\mu$ , which also gives  $r$ . Finally, plug the constraints  $\theta_\lambda^B = n$  and  $\theta_\lambda^A = 0$  into the HJB equations to verify the conjectures for  $J^A$  and  $J^B$ .

Money-market clearing gives

$$\{w_M^A, w_M^B\} = \left\{ 0, \frac{G}{1 - \omega} \right\}. \quad (\text{B.33})$$

The binding leverage constraint pins down the value of reserves:

$$G = (1 - \omega) \lambda \sigma' \sigma (w_S^B - 1). \quad (\text{B.34})$$

Under case (ii),

$$\{w_S^A, w_S^B\} = \{1, 1\} \quad (\text{B.35})$$

$$\{w_M^A, w_M^B\} = \{0, 0\}. \quad (\text{B.36})$$

The stock market clears and  $G = 0$ . From here, we get  $\sigma_\omega = 0$  and so  $\sigma = \sigma_D$ . Next, use  $\mu_\omega = -(J^A - J^B)$  in the dynamics of returns to get  $\mu$  and  $\sigma$ . Substituting into the HJB

equations and simplifying,

$$\begin{aligned}
\rho + \kappa &= 1/\psi J + (1 - 1/\psi) \left[ \mu + \left( \frac{1 - \gamma}{1 - \psi} \right) \frac{J_n}{J} (n - \underline{n}) (\bar{n} - n) \sigma_n \sigma_2 - \frac{\gamma}{2} \sigma' \sigma \right] \quad (\text{B.37}) \\
&\quad - 1/\psi \left[ \frac{J_\omega}{J} [\kappa (\bar{\omega} - \omega) + \omega (1 - \omega) \mu_\omega] + \frac{J_n}{J} \kappa_n (n - n_b) \right] \\
&\quad - \frac{1/\psi}{2} \left[ \left( \frac{\psi - \gamma}{1 - \psi} \right) \left( \frac{J_n}{J} \right)^2 + \frac{J_{nn}}{J} \right] (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2.
\end{aligned}$$

This case requires

$$\begin{aligned}
\lambda n &> |\gamma^A - \gamma^B| \\
&\quad + \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left( \frac{\frac{F_n}{F} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right) \quad (\text{B.38})
\end{aligned}$$

The real interest rate lies inside a range between a lending and a borrowing rate.

Case (iii) is analogous to Case (i) with the roles reversed. It requires

$$\begin{aligned}
\lambda n &< \gamma^A - \gamma^B \quad (\text{B.39}) \\
&\quad + \left[ \left( \frac{1 - \gamma^A}{1 - \psi^A} \right) \frac{J_n^A}{J^A} - \left( \frac{1 - \gamma^B}{1 - \psi^B} \right) \frac{J_n^B}{J^B} \right] \left[ \frac{\frac{F_n}{F} (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left( \frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\bar{n} - n)^2 \sigma_n^2} \right].
\end{aligned}$$

This completes the proof.  $\square$

We solve the model using Chebyshev collocation with complete polynomials up to order  $N$  in  $\omega$  and  $n$  with  $N = 30$ .

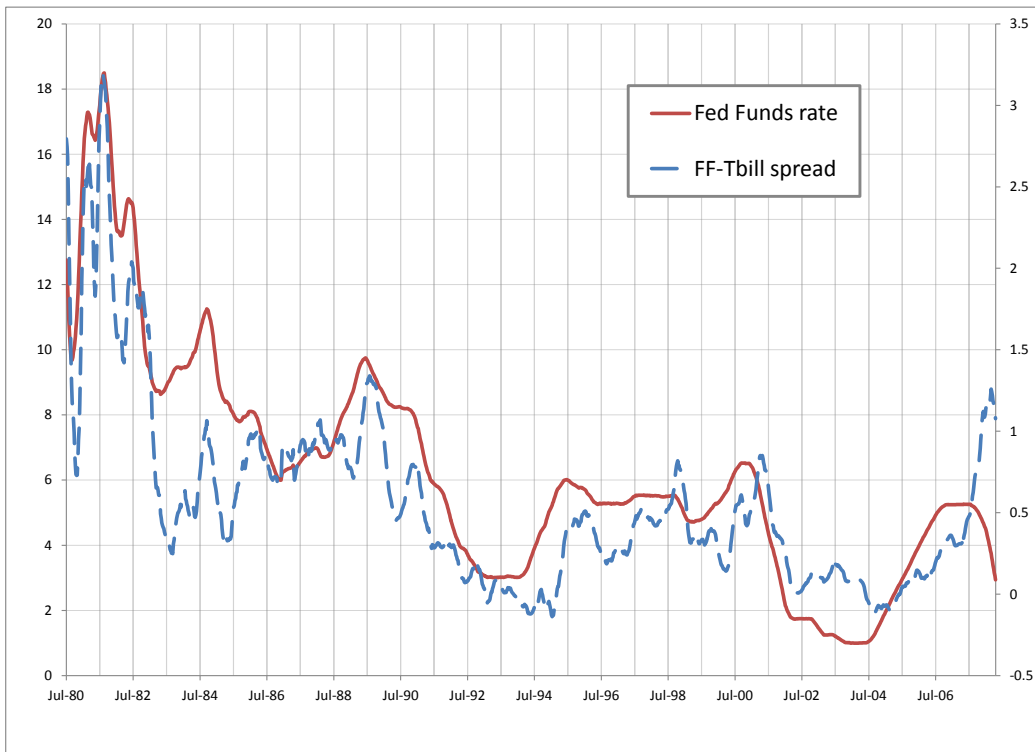


Figure 1: **Fed Funds-TBill spread vs. Fed Funds rate.** The figure plots the 20-week moving averages of the Fed Funds rate (solid red line) and the difference between the Fed Funds rate and the 1-month Treasury bill rate (dashed blue line). The sample is 7/25/1980 to 5/9/2008.

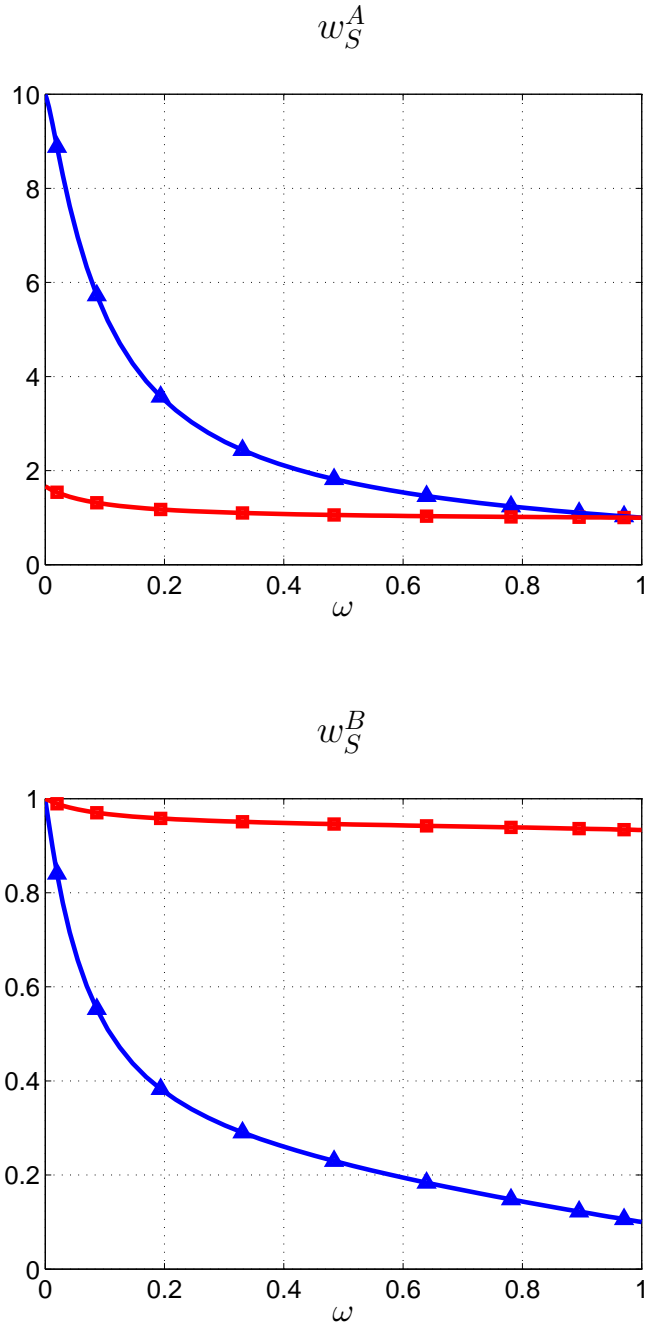


Figure 2: **Risk taking.** The figure plots the risky claim portfolio weight for agent A (top panel) and agent B (bottom panel) under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest-rate policies.



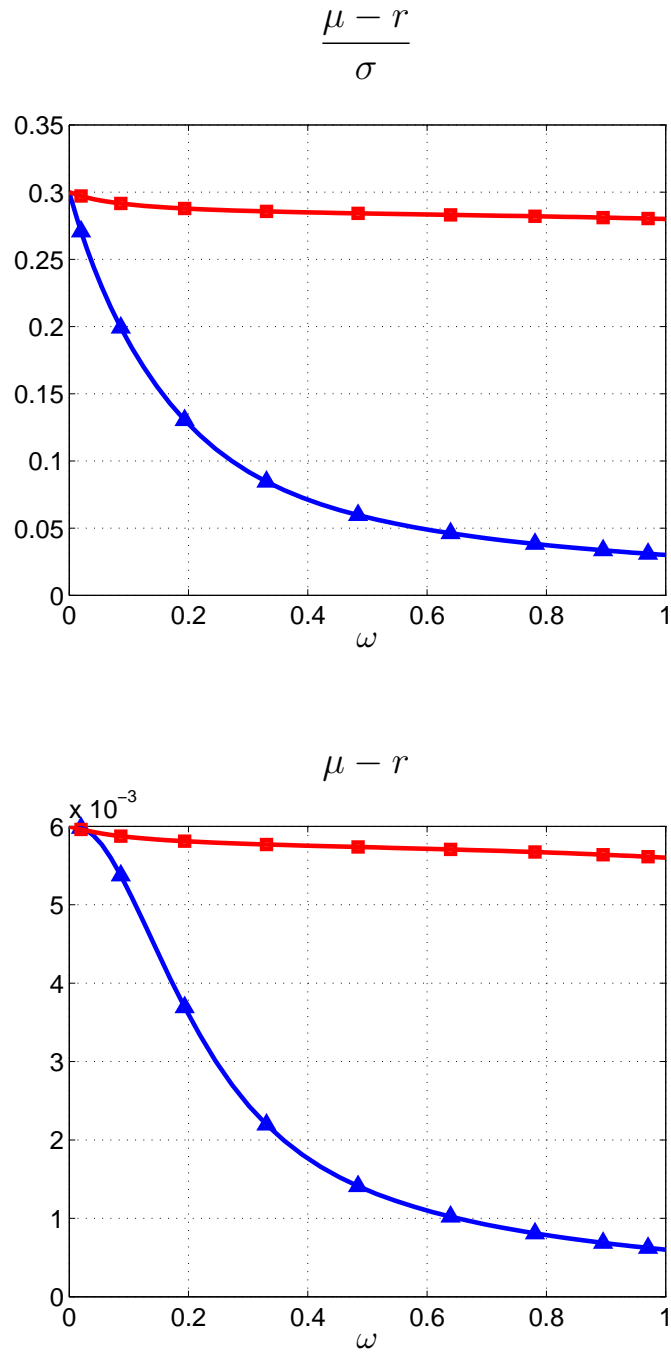


Figure 3: **The price of risk and the risk premium.** The figure plots the Sharpe ratio (top panel) and risk premium (bottom panel) of the endowment claim under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest rate policies.

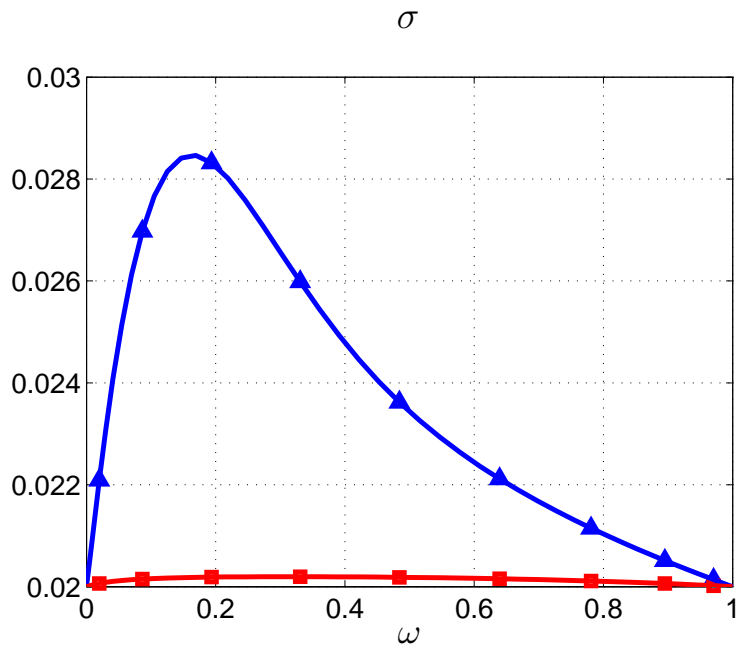


Figure 4: **Volatility.** The figure plots the volatility of the endowment claim under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest rate policies.

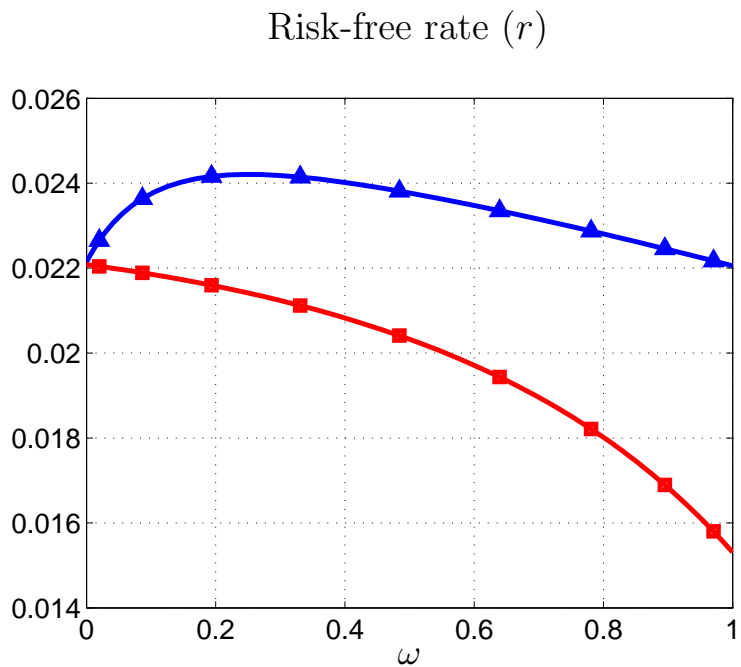


Figure 5: **Risk-free rate.** The figure plots the risk-free rate under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest rate policies.

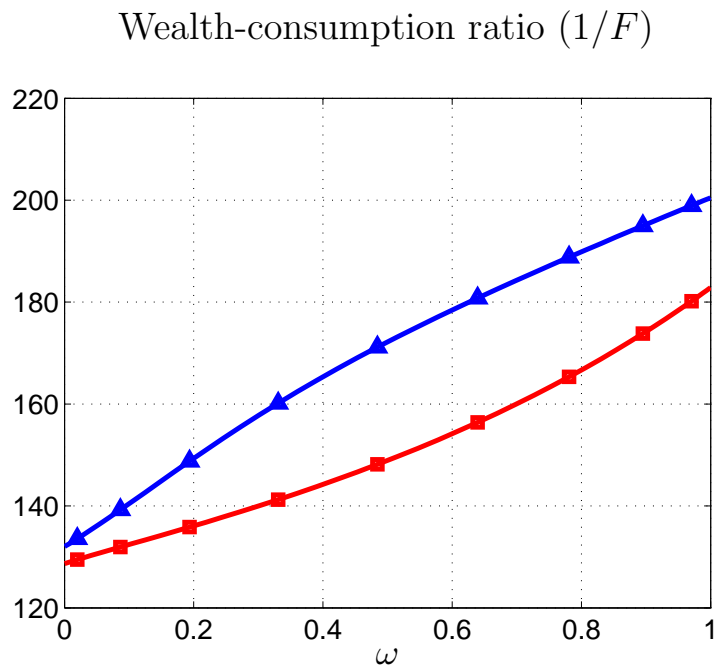


Figure 6: **Valuations.** The figure plots the wealth-consumption ratio under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest rate policies.

### Stationary Density of $\omega$

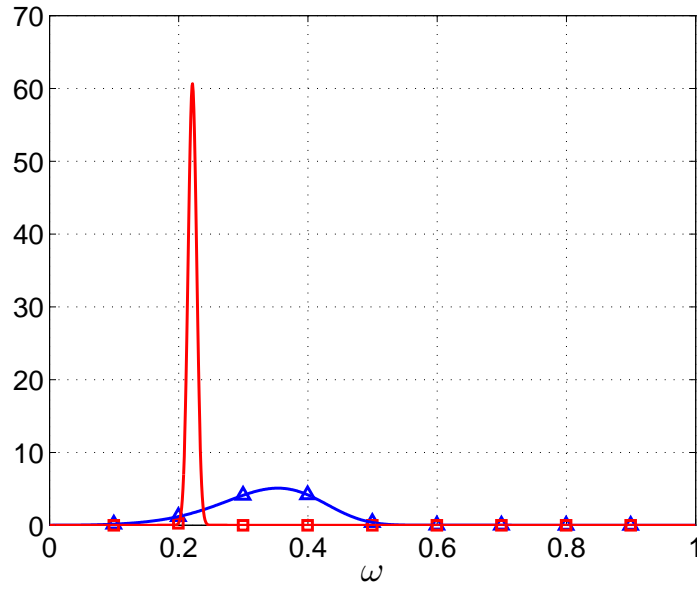


Figure 7: **The Stationary Density of Banks' Wealth Share ( $\omega$ ).** The figure plots the stationary density of  $\omega$ , the share of wealth owned by banks, under the  $\mathbf{n}_1 = 0\%$  (blue triangles) and  $\mathbf{n}_2 = 5\%$  (red squares) interest rate policies.

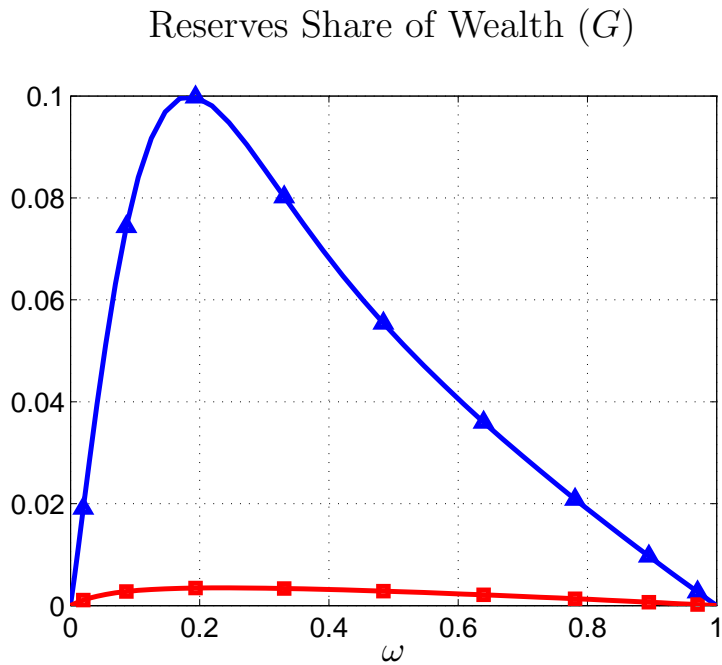
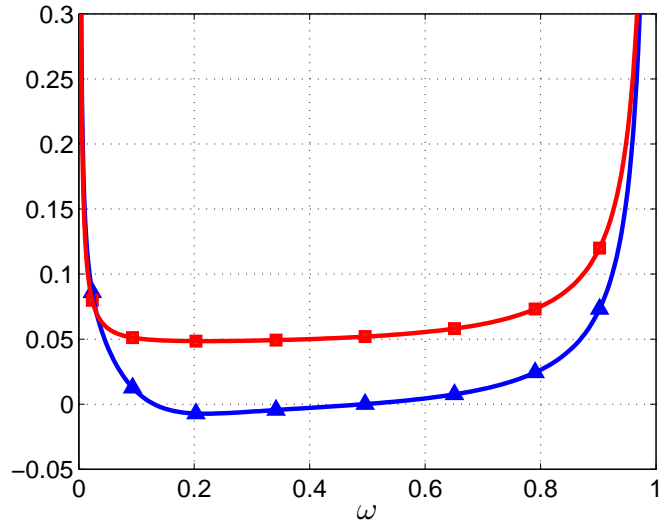


Figure 8: **Ratio of real reserves to wealth.** The figure plots the ratio of the real value of reserves to wealth  $G$  under the  $n_1 = 0\%$  (blue triangles) and  $n_2 = 5\%$  (red squares) interest rate policies.

### Reserves growth rate drift



### Reserves growth rate loading

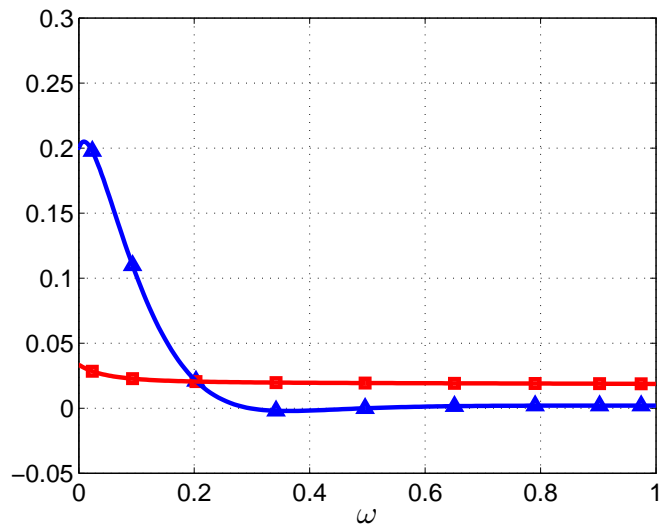
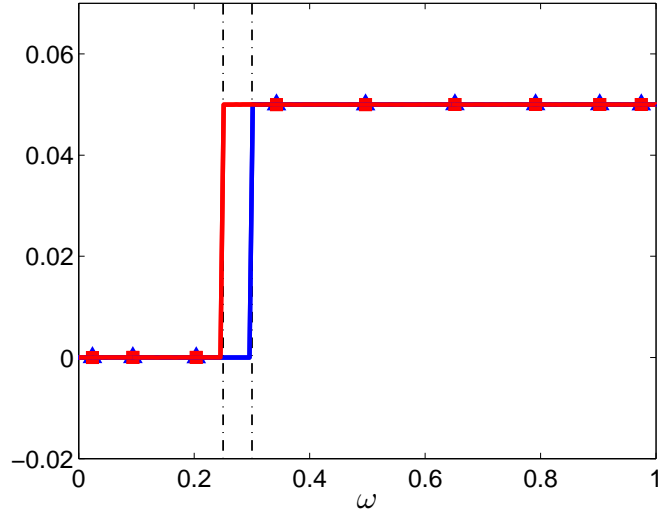


Figure 9: **Quantity of reserves dynamics.** The figure plots the drift and diffusion of the growth rate in the quantity of reserves  $M$  under the  $\mathbf{n}_1 = 0\%$  (blue triangles) and  $\mathbf{n}_2 = 5\%$  (red squares) interest rate policies.

### Nominal rate policies



### Ratio of Prices ( $P_{fg,1} / P_{fg,2}$ )

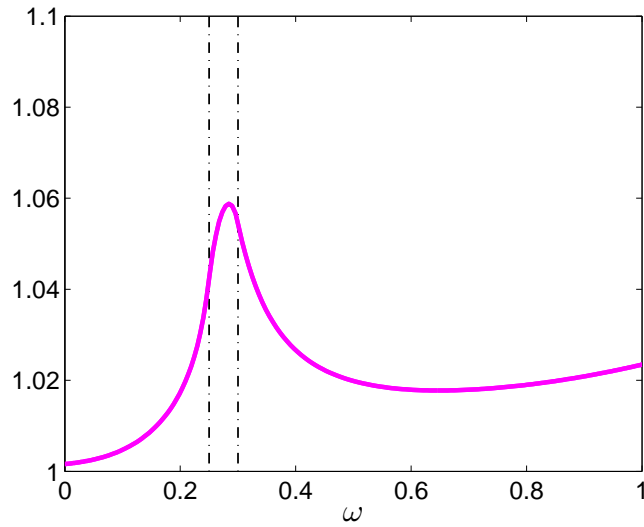


Figure 10: **Impact of forward guidance on prices.** The figure plots the impact of forward guidance on asset prices. The top panel plots the two nominal rate policies  $n_{fg,1}$  (blue triangles) and  $n_{fg,2}$  (red squares). The bottom panel plots the ratio of the price of the endowment claim for  $n_{fg,1}$  relative to  $n_{fg,2}$  ( $P_{fg,1} / P_{fg,2}$ ).



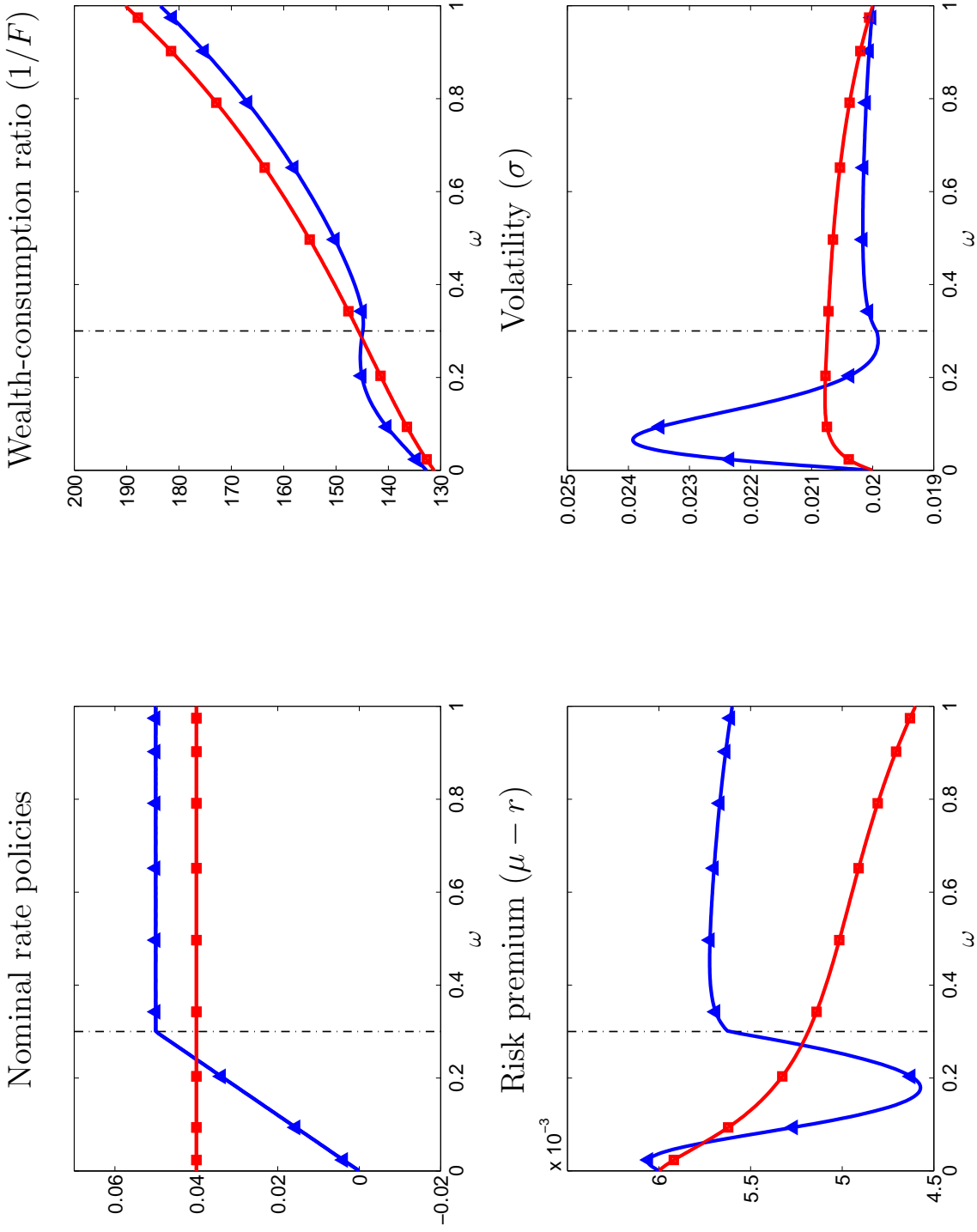


Figure 11: **Impact of Greenspan put policy on prices and volatility.** The figure plots the impact of a Greenspan put policy on prices, risk premia, and volatility. The top left panel plots the two nominal rate policies  $\eta_{gp,1}$  (blue triangles) and  $\eta_{gp,2}$  (red squares). The top right panel plots the wealth-consumption ratio  $1/F$ , the bottom left panel plots the risk premium  $\mu - r$ , and the bottom right panel plots volatility  $\sigma$ .

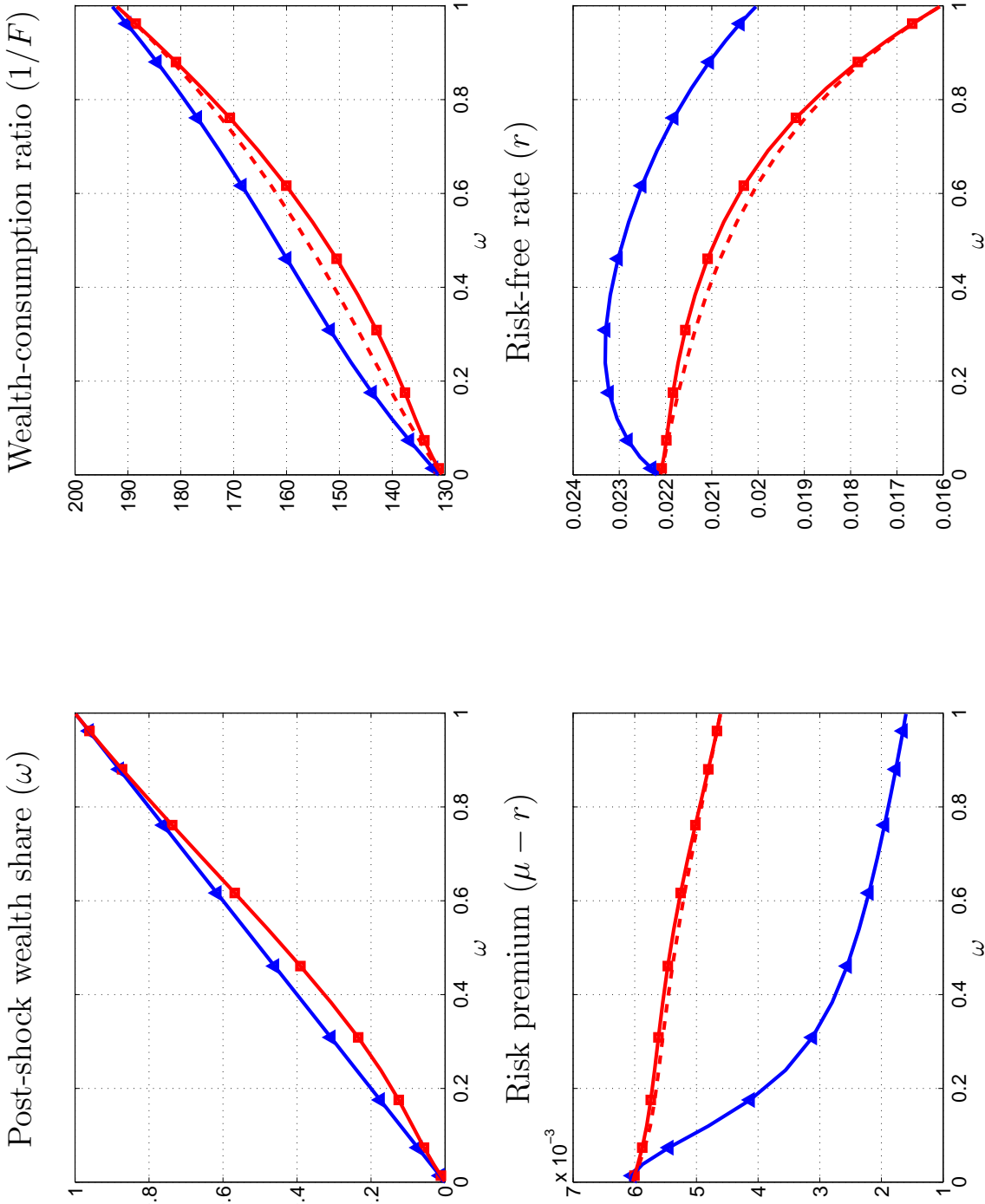


Figure 12: **Results from the model with policy shocks.** The figure plots the response of the wealth share  $\omega$ , the wealth-consumption ratio  $1/F$ , the risk premium  $\mu - r$ , and the real rate  $r$  to a policy shock that raises the nominal interest rate  $n$  from 1% to 4%. **Blue** triangles represent initial values, **red** squares represent after-shock values. Dashed red lines correspond to the “direct effect” of the policy shock, which excludes the impact on the wealth share  $\omega$ .

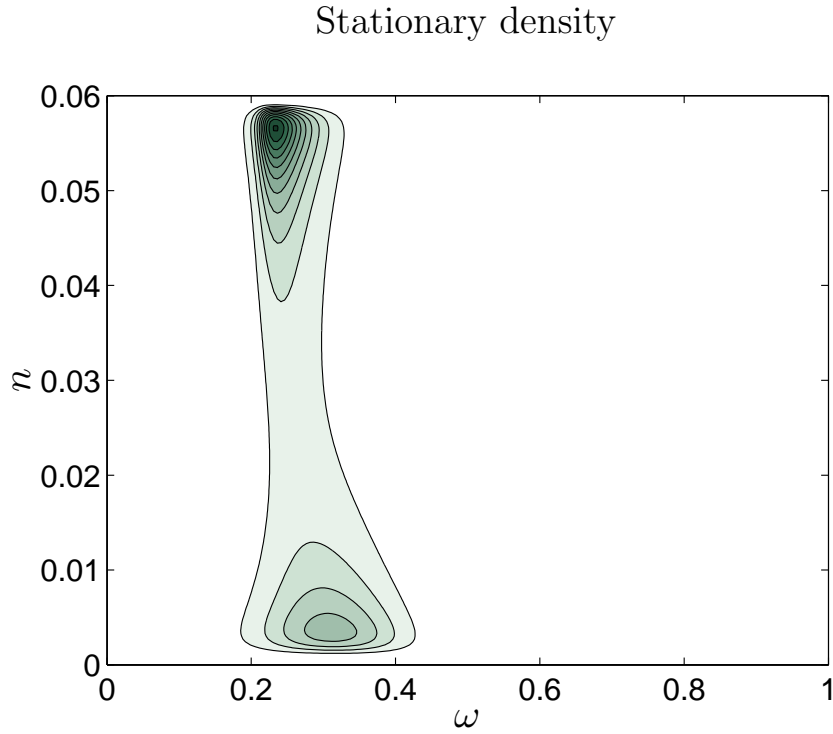


Figure B.1: **Stationary density: model with policy shocks.** Stationary density of the state variables  $\omega$  and  $n$  in the extended model with policy shocks. We set  $\underline{n} = 0.00$ ,  $\bar{n} = 0.06$ ,  $\kappa_n = 0.01$ , and  $\sigma_n = 5$ . The density is obtained by solving the forward Kolmogorov equation of the system. The contour lines are at increments of 50.