Female Feticide: Some Results on Parental Choices of Conceptions and Abortions

Raaj K. Sah
University of Chicago

Javier A. Birchenall
University of California at Santa Barbara

September 18, 2013

Abstract

For the first time in human history, in the last few decades, parents can nearly ascertain the gender of a fetus, and can potentially undertake gender-specific abortions. Contemporaneously, the newborns' gender ratio, in some countries, shows more sons than in the previous era. We present many intuitive, and previously unavailable, qualitative results on the parents' choices concerning conceptions and abortions. By aggregating individuals' behaviors, we also obtain results which concur with, and potentially explain, the existing empirical findings. We obtain many of our results partly because we use integers to represent abortions and conceptions.

Keywords: Gender detection technologies; Abortions, Sex Ratios at Birth.

1 Introduction

Much progress has been made during the post-War period in the technology for the *in vivo* testing of fetuses for a number of diseases and abnormalities. This technology, which includes sex determination by obstetric ultrasound and amniocentesis, is now widely available almost throughout the world. As often happens with many innovations, technologies produce their own chain of events. For the first time in history, parents can know with near certainty the gender of a fetus. Technology has expanded the choice set of parents. For the first time in human history, parents can potentially undertake gender-specific abortions.

Contemporaneous with the availability of gender-detection technology, declines in the number of girls (relative to boys) have been observed in some Asian countries; e.g., in China, India and South Korea. Analogous declines have been observed in some Asian migrants living in Western countries. An important empirical yardstick is the *sex ratio at birth* (SRB), which is the number of boys born per 100 of girls. We will be using this yardstick in several parts of the paper.[1] SRBs have typically been in the range of 103 to 107 in the era before gender-biased abortions. As the next section shows, much larger SRBs have been observed subsequently. In that section, we also briefly discuss some patterns of how SRBs differ across subpopulations of parents, such as those with different numbers and/or different gender-compositions (the number of girls versus boys) of pre-existing children. As will be seen later, theoretical predictions of such patterns is among the contributions of this paper.

The primary objective of this paper is to present qualitative results on the parents' choices of conceptions, and of abortions of female fetuses. Almost all other choices are assumed to be made optimality, and are kept in the background. We do this through the use of indirect utility functions, which have other advantages as well, briefly discussed

[1] One reason for the importance of SRBs is that it is difficult for researchers to collect direct data on the use of gender-detection technology for gender-biased abortions. Detection of the gender of a fetus is legally prohibited in all of the countries mentioned above, and it is done in illegal markets. Subject to such limitations, field surveys on reproductive histories indicate widespread incidence of gender-biased abortions; see, e.g., Khana (1997), Ganatra, Siddhi and Rao (2001), and Junhong (2001).

later. We present results on how the choices of a set of parents are influenced by their preferences, and by their pecuniary considerations. We also present results on how the parents' choices are influenced by the number and the gender-composition of the children that they already have. When these choices of individuals are aggregated across parents, we obtain qualitative predictions concerning patterns of SRBs for different subpopulations of parents. These predicted patterns are consistent with the observed ones, alluded to in the previous paragraph, and discussed briefly in the next section. To our knowledge, this paper is the first to predict such patterns.

Our analysis highlights parental choices (of abortions of female fetuses) as a source of SRBs that are abnormal, in that they are larger than those in the era before gender detection. Other sources, not involving parental choices, have been studied in the literature, such as biological and genetic predispositions for boys or for girls.[2] As we show in the paper, our analysis is fully consistent with these other sources of gender imbalance.

We construct the simplest possible models to analyze the choices of our concern. This is because if qualitative results are not available in such models, they are unlikely to be available in more general ones. On the other hand, results from simple models may potentially give some insights on some of the trade-offs that might arise within more general models. An example of this simplicity is that we keep our analysis of the parents' choices quite bare, leaving out many details that are potentially associated with these choices. We note some of these details in the concluding section; i.e., in Section 7.

Another example of the simplicity mentioned earlier is our modeling of the dynamics of choices. A set of parents first make the choice to conceive or not to conceive. If they choose to conceive then, after gender detection, they potentially make another choice concerning abortion. Regardless of what the choice is concerning the abortion just mentioned, the preceding sequence of choices (that of conception, and then that of potential abortion) is followed by an analogous sequence, and so on. All of these choices are dynamically linked. In this paper, we analyze two different abstractions which, as we shall see, complement

[2] As populations differ in their biological and genetic predisposition for boys or girls, biological factors have been studied in the literature though with less relative success; see, e.g., Lin and Luoh (2008), Oster (2005), and Oster and Chen (2008).

each other. In the first abstraction, there are two stages of choices, which is the minimum number of stages required for the problem at hand. The number of conceptions is chosen in the first stage, and the number of abortions in the second stage. In the second abstraction, there are multiple stages, as described earlier in this paragraph. In each stage, the parents make the choice concerning one conception, potentially followed by the choice of one abortion. As it should be, in making the choice at any stage, parents treat the outcomes of the preceding stages as parametrically specified. In this model, we analyze the parents' choice of the last conception, and that of the abortion following this conception.

We use integers to describe the number of conceptions, and the number of abortions. Both of these are so in reality. In economics, better tractability is generally achieved by treating intrinsically integer or discrete variables as continuous ones. In contrast, our analysis is considerably facilitated by the use of integer representations. If, instead, continuous representations were to be used, then many of our results will be less general and less usable. This is because, as we illustrate in the paper, we will then need additional assumptions with little or no economic meaning.

The analysis presented in this paper is strictly positive. We do not deal with any of the many important normative issues. We do not deal with the society-level general-equilibrium consequences of gender imbalances. We also do not deal with the profound moral and ethical issues raised by gender-biased abortions. We briefly remark on these issues in the concluding section.

Related literature. A long analytical tradition in economics, beginning with Becker (1960), has fruitfully employed single-stage deterministic models to analyze fertility-related choices. Such models are not aplicable to the questions studied in this paper. Recall from our earlier discussion that a minimum of two stages of choices are needed, and it is essential that gender outcomes of pregnancies be stochastic.

An important literature has studied how fertility choices are influenced by parental preferences for the genders of their children, in the absence of gender biased abortions. Leung (1991), for instance, considers a sequential model of fertility in which parents choose the probability of conception for an additional child, based on the existing number and gender-composition of children. Our main observation on this literature is that the availability of gender detection has added newer choices without restricting the previously-available choices.[3]

The economic literature on parental choices, incorporating selective abortions, is quite limited, and the present paper complements this literature. Kim (2005) uses a dynamic stochastic model of parental choices in which parents choose conception probabilities. He focuses on the effects of access to gender detection technologies on the parents' choice of the number of sons and daughters, and on the aggregate sex ratio at birth. For example, Kim (2005) finds that a reduction in the cost of gender detection increases aggregate SRBs due to more sex-selective abortions. He, however, does not focus on how the number of existing children or their gender composition influence parental decisions or disaggregated sex ratios at birth. These disaggregated patterns are one of our focus of analysis.

In addition, Kim's (2005) paper uses a direct utility function. The use of a direct utility function and of continuous choices for the probability of conception yields many advantages to this model; e.g., one can directly specify variables such as the prices of children, parents' income, and the reliability of contraception. An analytical disadvantage of this modeling approach is that the effects of a parameter on the outcomes typically depend in complicated ways on the cross-partial derivatives and on other aspects of the direct utility function, and that these aspects of the direct utility function do not always have intuitive signs or other properties. In Kim (2005), for instance, it is not possible to study how the gender composition of existing children influences parental choices without making assumptions on the third-order derivative of the utility function.[4] No such assumptions are needed here. Our methodology uses an indirect utility function which does not require

[3]See Ben-Porath and Welch (1976, 1980) and Bloom and Grenier (1983) for early economic work on the role of parental preferences in the absence of gender selective abortions. Bloom and Grenier (1983) recognize that gender selection will affect parental choices, but abstract from it in their analysis of parental behavior. See also Cleland et al. (1983) and Goody (1981) for cross-country demographic analyses, and the former for an analysis of self-reported parental attitudes. See Leung (1988) for an empirical analysis of the role of parental preferences on fertility in the absence of gender selective abortions.

[4]Leung (1994) and Davies and Zhang (1997) consider stochastic and deterministic models of sex selection but their focus is on comparing human capital and fertility in a model in which parents can choose the gender of their children and in a model in which gender selection is not possible. In these papers, the effect of gender selection typically depends on the second and third derivatives of the direct utility function and on specific values of the many different elasticities of the direct utility function.

an explicit identification of all the behavioral or pecuniary factors that might motivate a propensity for sons.[5] Taking all of this into account, we believe that our methodological approach and that of Kim (2005) are complementary rather than competing.

Organization of the paper. Section 2 gives a brief background on the current estimates of the prevalence of sex selective abortions. Section 3 describes a model of the parents' choices. It also presents some comparative statics results that are employed repeatedly in the paper. Section 4 examines the parents' choices concerning selective abortions. Section 5 examines their choices concerning conceptions. Section 6 provides a sequential analysis. Section 7 notes some extensions of the preceding analysis and some concluding remarks. Proofs of the propositions presented are available in the Appendix, except when the proof follows readily from the material preceding it in the text.

2 A brief discussion of some sex ratios at birth (SRBs)

The limited purpose of this section is to motivate the present paper, including some specific analytical results that we present. A full review of the literature, or even a partial one, on SRBs is unnecessary here because our analysis does not depend on any of the details; e.g., on measurement-related issues concerning SRBs, or on their precise magnitudes.

We highlight China, India and South Korea in this section. At the end of this section, we remark briefly on SRBs of some Asian migrant groups in Western countries. We focus on the following aspects of SRBs: (i) the aggregate country-level SRBs, (ii) the SRBs at different birth orders, and (iii) the SRBs for different gender compositions. *Birth order* of a particular birth is the number of births given by a woman, including the one under consideration. For example, the SRB at birth order one includes only the women who are giving birth to the their first child. SRB at birth order two includes only the women

[5] Das Gupta et al. (2003, p. 154), discussing the roots of son preference in Asia, noted that "in India the main cause is, it is argued, the need to pay dowries for daughters. In the context of China it has been suggested that stringent fertility regulation is responsible for heightened discrimination against daughters. In South Korea, son preference is attributed more to patriarchal family systems and low female autonomy. In South Korea and China son preference is sometimes also attributed to Confucian values." Ebenstein (2011) provides an empirically-oriented study of the role of the one-child policy in promoting selective abortions in China.

who are giving birth to their second child, and so on. It is generally accepted that, in the era before gender detection, SRBs declined slightly with birth order or remained constant (Zeng et al., 1993). The idea of gender composition is straightforward; given a particular number of previous births, how many boys and how many girls did a woman gave birth to.

The following broad picture emerges. Roughly since 1980s, the aggregate SRBs have been noticeably larger than the historical normal.[6] SRBs have been larger at higher birth orders than at lower birth orders. Given any particular number of previous total births, the SRBs have been larger if there were a larger number of girls previously born. In the paragraphs below, we present some brief data that illustrates the preceding conclusions; additional data will unlikely alter the very broad conclusions that are of interest to the present paper.

Table 1. SRBs at Different Birth Orders in South Korea.

		SRB at birth order			
Period	Aggregate SRBs	1	2	3	4+
1970-1974	108.4	109.6	108.1	108.1	107.5
1990-1994	114.6	106.6	114.1	192.7	215.2
2000-2004	109.2	105.6	106.9	137.8	152.1
2005-2009	106.9	105.0	106.0	119.1	122.3

Source: Korean Statistical Information Service, http://www.kosis.kr/index.html

Table 1 displays the SRBs in South Korea at first, second, third, and fourth (and larger) birth orders in different time-periods. During the 1970s, SRBs were roughly similar

[6] Johansson and Nygren (1991), Zeng et al. (1993), Park and Cho (1995), Gu and Roy (1995), Lavely (2001), Poston et al. (2003), Arnold et al. (2002), Retherford and Roy (2003), Jayaraj and Subramanian (2004) are among the many studies that provide detailed demographic accounts of SRBs in China, Taiwan, South Korea, Hong Kong, and India. Chung and Das Gupta (2007) empirically studied the evolution of the SRBs in South Korea and its decline since the 2000s. It is presently unclear whether the decline is significant at higher birth orders, or with larger proportions of females previously born. Even if there are declines, the current SRBs are likely to be significantly larger than the historical normal range for particular subpopulations of parents.

at all birth orders. The subsequent SRBs are larger than before and the magnitude of the increase in the SRB is greater at higher birth orders. Similar patterns have been observed in China and India. Given our limited purpose, we have highlighted here only the South Korean data, which is often considered more reliable than, say, that from China or India.[7]

Next consider SRBs for different gender compositions, given a particular birth order. Table 2 provides a glimpse of SRBs for the third birth order, for China, India, and South Korea. In the first column, MM means that the previous two births yielded two sons. MF or FM means that they yielded one son and one daughter, and FF means that they yielded two daughters. The general pattern is quite clear. The third birth following two daughters has a larger SRB than that following one daughter and one son, which in turn is larger than that following two sons. Analogous patterns have been observed at first and second birth orders, and for the SRBs at the last birth (that is, for the birth just before the completed fertility).

Table 2. SRBs at Third Birth Order, by the Gender Composition of Previous Births.

Preceding	South Korea		India		China
sex sequence	1974	1991	1992	2002	2000
MM	105.4	105.3	104.4	109.7	106.0
MF,FM	112.1	101.6	110.7	112.5	124.0
FF	111.2	136.3	119.8	131.8	200.3

Notes. For South Korea: Park and Cho (1995, Table 7) based on small samples from household surveys. For India: Jha et al. (2011, Tables 1 and 2) for the average between 1991 and 1993, and 2001 and 2003, respectively, for nationally representative Census data. For China: Wei (2005, Table 6.5), for families with sons only, equal number of sons and daughters, and daughters only, for nationally representative Census data.

We conclude this section with a pattern that to us appears quite striking. Historically,

[7] In China in 1990, SRBs for birth order three and four and above were 125 and 132, respectively; see Zeng et al. (1993, pp. 283-284). Similar patterns are observed in India; see Das Gupta and Bhat (1997) and Retherford and Roy (2003).

there has been no imbalance in SRB in Western countries. Data has now begun to show an increase in SRB among children of Chinese, Indian, and South Korean immigrants in Canada (e.g., Almond et al., 2010, Ray et al., 2012), the UK (e.g., Dubuc and Coleman, 2007, and Dubuc, 2009), and the US (e.g., Almond and Edlund, 2008, Abrevaya, 2009). As in Tables 1 and 2, the recent increase in the SRB in these subpopulations is more pronounced at higher birth orders and for families with fewer or no previous sons.

3 A two-stage model

This section presents a basic, bare-bones model of parents' choices. We analyze this model and then expand and modify it later in the paper. Our narrow objective is to study female foeticide and to obtain qualitative results on the resulting parental behavior concerning conceptions and abortions. Thus, we do not deal with abortions that are largely unrelated to the gender of the fetus. These include abortions motivated by the total number of offsprings, accidental pregnancy, and the termination of pregnancy for medical reasons.

Given our objective of studying female foeticide, we abstract from the parents who abort male fetuses. Put differently, we consider those parents who may potentially abort female fetuses; whether they actually do it or not will depend, as we shall later see, on the external parameters that they face and on their preferences. One simple formal way to state this aspect of the parents under consideration here is that their utility cost of aborting male fetuses is sufficiently large, such that an explicit analysis of this choice is not necessary. We abstract from these categories of abortions because these are not the most central to the study of female foeticide. It is not because these phenomena are not important in themselves. For analogous reasons, we abstract from multiple births, infant mortality, and errors in gender detection.

For brevity, we use the following short-hands in the rest of the paper. An *abortion* means the abortion of a female fetus. An *unaborted female fetus* becomes a *daughter*. A *male fetus* becomes a *son*; we therefore use the preceding two phrases interchangeably.

Consider a set of parents who undertake n conceptions. n is a finite nonnegative integer. $C(n)$ is the cost of conceptions including the cost of gender detection. We do not make any assumptions about $C(n)$.

For a given n, the parents ascertain the gender outcomes of their conceptions. That is, they observe the number of sons, denoted by the random variable M (where $n \geq M \geq 0$), and the number of female fetuses, $n - M$. The gender outcome of each conception is assumed to be stochastically independent. Hence, the probability that n conceptions yield M sons is the binomial density $b(M, n, q) = \binom{n}{M} q^M (1-q)^{n-M}$. Here q is the exogenous probability that a conception yields a son, and $0 < q < 1$. The value of q may differ across time or parents, including for biological reasons. Using our framework, it is possible to examine the effects of such differences in q on the parents' conception and abortion behavior.

Having ascertained M, the parents choose the number of abortions they wish to undertake. We denote this number by the integer variable a. The cost and inconvenience of an abortion is represented by the parameter ℓ. Feasibility requires that $0 \leq a \leq n - M$. That is, the number of abortions cannot be negative and that it cannot exceed the number of available female fetuses. Let F denote the number of daughters that the parents choose to have. This number is connected to the number of abortions through the identity $F \equiv n - M - a$. Hence, the number of daughters that the parents have is determined arithmetically from their choice of the number of abortions and vice-versa. Accordingly, the feasibility requirement noted earlier can be restated as $0 \leq F \leq n - M$. That is, the number of daughters cannot be negative and it cannot exceed the number of female fetuses.

For any given number of sons, M, and daughters, F, the parents *indirect* utility is $u(M, F)$. This utility (or expected utility) is indirect in that, for a given M and F, this is the parents' utility which has been optimized over all of their other choices (such as those about work, leisure, consumption, and so on), subject to various constraints that they face. To reflect the physical reality, $u(M, F)$ is defined only for nonnegative values of M and F.

Consider a particular set of parents who undertake n conceptions, who have M sons out of these, and who undertake a abortions. Their net utility, after taking into account the cost of abortions, is

$$U(n, M, a) \equiv u(M, n - M - a) - \ell a. \tag{1}$$

The parents choose the number of abortions, a, subject to the feasibility requirement $0 \leq a \leq n - M$. Thus, their *maximized post-abortion utility* is:[8]

$$v(n, M) \equiv \max_{a \geq 0} U(n, M, a). \tag{2}$$

As described earlier, $b(M, n, q)$ is the binomial density of M sons out of n conceptions. Hence, using (2), the expected benefit from n conceptions is

$$V(n, q) \equiv \sum_{M=0}^{n} b(M, n, q) v(n, M). \tag{3}$$

Next, taking into account the cost of conceptions, the parents' *expected valuation of n conceptions* is

$$w(n, q) \equiv -C(n) + V(n, q). \tag{4}$$

The parents maximize this valuation with respect to n, and they subsequently choose the number of abortions after they have ascertained the value of M.

Some assumptions concerning the parents. Define $u_F(M, F) \equiv u(M, F + 1) - u(M, F)$ as the marginal utility of an additional daughter if the parents have M sons and F daughters. Define $u_{FF}(M, F) \equiv u_F(M, F + 1) - u_F(M, F)$. These are the discrete counterparts of the first and second derivatives, respectively. In the rest of the paper,

[8] Besides $a \geq 0$, the other feasibility requirement in (2) is $a \leq n - M$. The parents will have a negative number of daughters if the latter is not met. However, we do not need to impose this requirement explicitly because, as noted earlier, $u(M, F)$ is defined only for $M \geq 0$ and $F \geq 0$.

letter subscripts denote such derivatives. The properties of $u(M,F)$ that we employ are[9]

$$u_{FF}(M,F) < u_{MF}(M,F) < 0. \tag{5}$$

There are three economic assumptions in the above expression. The inequality $u_{FF}(M,F) < 0$ says that the marginal utility of an additional daughter is smaller if the parents have one more daughter. The inequality $u_{MF}(M,F) < 0$ says that daughters and sons are substitutes. The inequality $u_{FF}(M,F) < u_{MF}(M,F)$ says that marginal utility of an additional daughter is larger if the parents have one more son instead of one more daughter.

One of the objectives of this paper is to compare the behaviors of different types of parents. Since our analysis is based on an indirect utility function, we can do this quite parsimoniously. In particular, it is not necessary to separately examine the effects of the parents' pure preferences on their behavior, versus those of their pecuniary considerations. An example of pure preferences is that, even if all of the pecuniary costs and benefits of an additional son and of an additional daughter are identical for two sets of parents, one set might prefer an additional son over an additional daughter with a greater intensity than the other set. An example of pecuniary considerations is that, with all else being the same, the expected future dowry to be received for a son, compared to that to be paid for a daughter, may be larger for one set of parents than for another.

Thus, instead of separately studying such different kinds of forces which may affect the parents' behavior, we use a parameter λ to denote their propensity for sons. This parameter can represent, depending on the context, one or another aspect of pure preferences or of pecuniary considerations. A larger value of λ implies a greater propensity for sons. We then compare the behaviors of sets of parents with different values of λ.[10] We

[9] The economic content of our results does not change if one or both inequalities in (5) is weak. We use strict inequalities because this considerably simplifies the derivation and presentation of the results. This simplification is especially helpful given that we are dealing with discrete variables, and not with continuous ones. The same observations apply to the inequalities in (6) below.

[10] For simplicity, λ is a continuous scalar. If λ were a vector of parameters then, with appropriate reformulations, the effects of each of the parameters can be studied using the analysis of the kind that we present. If we wish to understand the joint effect of two or more parameters together, then these would be aggregations of the separate effects of each of these parameters.

assume that

$$\frac{\partial}{\partial\lambda}u_M(M,F,\lambda) > -\frac{\partial}{\partial\lambda}u_F(M,F,\lambda) > 0. \tag{6}$$

That is: (i) a greater propensity for sons raises the marginal utility of a son, (ii) a greater propensity for sons lowers the marginal utility of a daughter, and (iii) the magnitude of the former increase is larger than that of the latter decrease.

4 The parents' abortion behavior

In this section we analyze the parents' choices of the number of abortions, or equivalently, that of the number of daughters. Our qualitative analysis yields some patterns of these choices. It also yields results on how these choices are affected by the external summary parameter λ. The number of conceptions n is taken as given here; the choice of n is endogenized in the next section. One way to look at the present section is this. A valuable and well-understood method of stochastic dynamic analysis is to begin with the last stage of decision, and then to proceed to the earlier stages. Here, the last stage of the decision is (2), which we analyze in this section. We then proceed, in the next section, to the first stage where the parents maximize (4) by choosing the number of conceptions.

The proposition below, concerning the optimal number of abortions, is proven in the Appendix.

Proposition 1 *The utility of any given set of parents is maximized either at a unique number of abortions, or at two neighboring numbers.* ■

This proposition says that up to two values of a maximize the parents' utility in (2), for a given (n, M). If so, the parents will be indifferent between these two maximizing values of a. For situations of such indifference, we follow the convention that the parents choose the smaller of the two maximizing values of a. Any other convention in this regard does not affect our qualitative results. We refer to the smaller number just mentioned as the *optimal* number of abortions, and denote it as $a(n, M)$. That is,

$$a(n,M) \equiv \text{ The smallest } a \text{ that maximizes } U(n,M,a). \tag{7}$$

Naturally, $a(n, M)$ is defined only for $M = 0$ to n. Hence, for our analysis to be nontrivial, we assume that $n \geq 1$. That is, the parents undertake at least one conception.

The optimal choices of conceptions and abortions will generally depend on parameters such as λ and ℓ. For brevity in the rest of this paper, we will suppress one or more of these parameters (as arguments of functions) if their explicit mention is not needed in the context at hand. For example, we will often write $a(n, M, \lambda, \ell)$ as $a(n, M)$ and $w(n, \lambda, \ell)$ as $w(n, \lambda)$.

Some effects of the number of sons on the parents' abortion behavior. Here the question is: With a given number of conceptions, how does the number of abortions chosen by a set of parents change if they have more or fewer sons? Recall our notation that, with n conceptions, the parents undertake $a(n, M)$ abortions if they have M sons, and $a(n, M + 1)$ abortions if they have $M + 1$ sons. We show in the Appendix that:

$$a(n, M + 1) = a(n, M), \text{ or } a(n, M) - 1. \tag{8}$$

We present this result and some of its immediate consequences as:

Proposition 2 *(a) An additional son either reduces the number of abortions by one or leaves it unchanged.*

(b) Suppose that the parents do not abort any female fetus when they have a given number of sons. They will not abort any female fetus when they have more sons.

(c) Suppose that the parents abort at least one female fetus when they have a given number of sons. They will abort at least one female fetus when they have fewer sons. ■

It is easier to qualitatively interpret these results, and also those presented later, if the cost of abortions is suppressed in the explanation. No such suppression is needed in the actual results. An understanding of many of our results is also facilitated if we use the language of an *initial* regime and a *changed* regime. For example, in the initial regime in (8) the parents have M sons and $n - M$ female fetuses. In the changed regime, they have $M + 1$ sons and $n - M - 1$ female fetuses. Thus, (8) states that the number of abortions in the changed regime does not increase, and that it does not decrease by more than one, compared to that in the initial regime. We use similar artifacts of language to interpret many other results in this paper.

To understand the result (8), suppose hypothetically that the number of abortions in the changed regime is the same as that in the initial regime. Then, in the changed regime, the parents will have one more son and one fewer daughter. Hence, an additional daughter will become more desirable than before.[11] Accordingly, an additional abortion will become less desirable. Thus, the number of abortions will not exceed that in the initial regime.

Consider now a different thought experiment. Suppose that the number of abortions in the changed regime is one fewer than that in the initial regime. Then the parents now will have one more son and the same number of daughters. Since daughters and sons are substitutes, this will make an additional daughter less desirable. Hence an additional abortion will become more desirable. Thus, the number of abortions will not decrease by more than one.

Restatement of the results in terms of the number of daughters. Our formal analysis employs the number of abortions as the parents' choice variable. Each of this paper's results can easily be restated, without any change in its economic implications, if we were to instead employ the number of daughters as the parents' choice variable. This is because the latter choice will get arithmetically translated into the number of abortions. We briefly illustrate this in the present subsection. We then continue with the formulation in which the number of abortions is the parents choice variable.

Let $F(n, M)$ denote the number of daughters that the parents choose to have with n conceptions and M sons. Then, recalling the identity connecting the number of abortions and that of daughters, $F(n, M) \equiv n - M - a(n, M)$. Hence, from (8), $F(n, M+1) = F(n, M)$ or $F(n, M) - 1$. Thus, for example, we can restate Proposition 2(a) as:

[11] Throughout the paper, a daughter becoming *more desirable* means that the marginal utility of an additional daughter has increased. A daughter becoming *less desirable* means the opposite. An abortion becoming more, or less, desirable means that the marginal utility of an additional abortion has respectively increased or decreased. A larger marginal utility of an additional daughter naturally lowers the marginal utility of an additional abortion.

Proposition 3 *An additional son either reduces the number of daughters by one, or leaves it unchanged.* ■

Some effects of the propensity for sons, and of the cost of abortions, on the parents' abortion behavior. Let λ' and λ'' denote two values of the propensity for sons, λ, such that $\lambda'' > \lambda'$. Let ℓ' and ℓ'' denote two values of the cost of abortions ℓ, such that $\ell'' < \ell'$. Then, as derived in the Appendix

$$a(n, M, \lambda'') \geq a(n, M, \lambda'), \text{ and} \tag{9}$$

$$a(n, M, \ell'') \geq a(n, M, \ell'). \tag{10}$$

These results yield:

Proposition 4 *(a) Consider two sets of parents with different propensities for sons. The parents with a greater propensity will undertake more or the same number of abortions.*

(b) If the cost of abortions is smaller, then the parents will undertake more or the same number of abortions. ■

These results are quite intuitive. For example, a larger propensity for sons makes an additional daughter less desirable. Hence, it cannot decrease the number of abortions for any given number of male and female fetuses.

Some effects of the number of conceptions on the parents' abortion behavior. We now compare the parents' abortion behavior when they undertake $(n+1)$ versus n conceptions. This additional $(n+1)$th conception can yield a male or a female fetus. The parents being considered in this paper have gender biases. Hence, we would expect their abortion behavior to be meaningfully different under the two gender outcomes of the additional conception. Surprisingly, this is largely not true.

Recalling our notation, the parents undertake $a(n, M)$ abortions with n conceptions and M sons. Thus, if the $(n+1)$th conception yields a female fetus, then $a(n+1, M)$ is their number of abortions. If it yields a male fetus, then $a(n+1, M+1)$ is their number of abortions. As shown in the Appendix:

$$a(n+1, M) = a(n, M) + 1 \text{ if } a(n+1, M) \geq 1 \tag{11}$$

$$= a(n, M) = 0 \text{ otherwise.} \tag{12}$$

$$a(n+1, M+1) = a(n, M) \text{ or } a(n, M) + 1 \text{ if } a(n+1, M) \geq 1 \tag{13}$$

$$= a(n, M) = 0 \text{ otherwise.} \tag{14}$$

We now present and interpret some implications of these results.

Proposition 5 *(a) An additional conception, regardless of its gender outcome, increases the number of abortions by one or leaves it unchanged.*

(b) Suppose that an additional conception yields a female fetus, and the parents undertake at least one abortion. Then the number of abortions with the additional conception will be one larger than without.

(c) Suppose that an additional conception yields a female fetus, and the parents do not undertake any abortion. Then they will not undertake any abortion if: (i) the additional conception had instead yielded a male fetus, or (ii) they had not undertaken the additional conception. ■

The most transparent intuition is that of part (c) of the above proposition. Here the parents undertake zero total abortions if the additional conception yields a female fetus. Treat this as the initial regime. Now, suppose hypothetically that the additional conception had, instead of a female fetus, yielded a male fetus resulting in a son. Then, an additional daughter will become more desirable. Hence, these parents will also undertake zero total abortions under this outcome. Now consider a different thought experiment. Suppose that these parents had not undertaken the additional conception at all. Then, with zero total abortions, they will have the same number of sons and one fewer daughter than those in the initial regime. An additional daughter will then become more desirable than in the initial regime. Hence, the parents will undertake zero total abortions in this changed regime as well.

Other parts of Proposition 5 can likewise be understood by constructing thought experiments of the kind described above, and in the explanation of result (8). For brevity, therefore, we omit the details of the interpretations of parts (a) and (b) of Proposition 5. Instead, we interpret a remark made at the beginning of this subsection. We noted there that the change in parents' abortion behavior with an additional conception does not markedly depend on its gender outcome. This is summarized by Proposition 5(a). Whether the additional conception yields a female or a male fetus, the change in the number of abortions has the same direction (that this number does not decrease) and the same maximum magnitude (that it increases by at most one).[12]

It is easy to understand why this is so. Whether an additional conception yields a male or a female fetus, it does not make an additional daughter more desirable. Hence, regardless of its gender outcomes, an additional conception does not induce fewer abortions. Next, under either gender outcome, the extent to which an additional conception increases the desirability of an extra abortion is limited. Hence, regardless of its gender outcome, an additional conception does not induce more than one extra abortion.

5 The parents' conception behavior

In this section, we first analyze some basic aspects of the parents' choice of conceptions, with the objective of adding to our understanding of this choice. We then examine how the number of conceptions undertaken by the parents is affected by their propensity for sons, and by the cost of abortions.

Recall from (4) that $w(n)$ is the parents' valuation of n conceptions. This valuation takes into account each of the states-of-the-world that they will face, in terms of the number of sons yielded by the n conceptions. It also takes into account the number of abortions that they will choose in each of these states. A given set of parents therefore

[12] It can be seen from (11) and (12) that the change in the optimal a differs under the two gender outcomes, only if the parents undertake at least one abortion under the outcome that the additional conception yields a female fetus. That is, only if $a(n+1, M) \geq 1$. In this case, if the $(n+1)$th conception yields a female fetus, then the optimal a increases by one as shown by (11). Otherwise, it increases by zero or one, as shown in (13).

choose n by maximizing their $w(n)$. If more than one value of n maximizes $w(n)$, then we use the convention that the parents choose the largest among these values. That is,

$$n(\lambda, \ell) \equiv \text{ The largest } n \text{ that maximizes } w(n, \lambda, \ell). \tag{15}$$

The parents' choice of the number of conceptions will naturally be based on their marginal valuation of a conception. The marginal valuation of $n+1$ conceptions versus n conceptions is $w_n(n) \equiv w(n+1) - w(n)$. In the Appendix, we obtain the following expression for this marginal valuation.

$$\begin{aligned} w_n(n) = {} & -C_n(n) + q \sum_{M=0}^{n} b(M,n,q) \left[v(n+1, M+1) - v(n,M) \right] \\ & + (1-q) \sum_{M=0}^{n} b(M,n,q) \left[v(n+1, M) - v(n,M) \right]. \end{aligned} \tag{16}$$

This expression has a clear economic interpretation. Consider the state-of-the-world in which there are M sons out of n conceptions. The probability of this state is $b(M, n, q)$. Recall from (2) that, under this state, the parents' maximized post-abortion utility is $v(n, M)$. Now consider the outcome in which the $(n+1)$th conception results in a male fetus. The probability of this outcome is q. Correspondingly, the maximized post-abortion utility changes from $v(n, M)$ to $v(n+1, M+1)$. This yields the terms in the first summation in (16). Next, consider the outcome in which the $(n+1)$th conception results in a female fetus. The probability of this outcome is $1-q$. Correspondingly, the maximized post-abortion utility changes from $v(n, M)$ to $v(n+1, M)$. This yields the terms in the second summation in (16).

Some effects of the propensity for sons, and of the cost of conceptions, on

conception behavior. As derived in the Appendix:[13]

$$n(\lambda'') \geq n(\lambda') \text{ for } \lambda'' > \lambda', \text{ and} \tag{17}$$

$$n(\ell'') \geq n(\ell') \text{ for } \ell'' < \ell'. \tag{18}$$

We state these results as:

Proposition 6 *(a) The parents with a higher propensity for sons will undertake a larger or the same number of conceptions.*

(b) If the cost of abortions declines, then the parents will undertake a larger or the same number of conceptions. ■

The intuition of these results can be understood as follows. Take the above result concerning the propensity for sons. Solely for illustration, consider the extreme case of a set of parents who abort all female fetuses regardless of how many male fetuses they have. They can further increase the chances of having more sons by undertaking an additional conception. In particular, with $n+1$ conceptions, they will have one more son (with probability q) than those under each of the gender outcomes of n conceptions. The number of conceptions they choose will, of course, be influenced by the cost of conceptions and abortions. Compare this set of parents with another set which exhibits the same extreme abortion behavior as described above, but who have an even greater propensity for sons. The latter set of parents will not undertake fewer conceptions than the former set.

Our results in (17) and (18) are much more comprehensive than that for the extreme abortion behavior used for the illustration in the previous paragraph. Our results include every possible abortion behavior. For example, they include the opposite extreme in which one of the two sets of parents being compared do not undertake any abortions

[13] A sufficient condition for (17) is that $q \geq 0.5$. This condition is supported by the facts briefly summarized in Section 2. A sufficient condition for (18) is that the parents undertake at least one abortion under at least one of the gender outcomes of their chosen number of conceptions. This makes economic sense. The cost of abortions will not affect the conception behavior if the parents do not undertake any abortion under any of the circumstances that they face.

under any of the circumstances that they face. It further allows the two sets of parents being compared to have partly or entirely different abortion behaviors.

Some additional remarks. Before concluding this section, we briefly note another result which is quite intuitive. Suppose that the marginal cost of conception, $C_n(n)$, declines due to technological and other reasons. Then the number of conceptions will increase or will remain unchanged. This can be seen from (16). A reduction in $C_n(n)$ implies that the marginal valuation of a conception, $w_n(n)$, increases. Accordingly, the number of conceptions chosen by a set of parents will not decrease.

Finally, note that we obtain the results in Proposition 6 even though the parents' valuation $w(n)$ does not necessarily possess any concavity properties. This has largely been possible because we represent conceptions and abortions as discrete variables, and not as continuous ones. Additional observations on some of the advantages of our discrete representations are presented later in the paper.

6 A multi-stage framework of conceptions and abortions

In this section, we present some analysis of a framework in which the time-sequence of parents' choices is more comprehensive than that in the basic (two-stage) model in Section 3. One aim of this section is to show, with greater concreteness than before, how parents' parameters influence their conception and abortion behaviors. This analysis yields many results which provide newer insights and also complement the results presented earlier. Another aim of this section is to present some economy-wide results. Recall that we briefly outlined in Section 2 some empirical patterns of SRBs. In this section, we present analytical results which not only concur with the empirically observed patterns, but also help understand them.

Consider a framework in which a given set of parents faces $t = 1, \ldots, T$ stages of choices. At any given stage t, the parents decide whether or not to undertake *one* conception. Denote this choice by the binary integer variable $n^t = 0$ or 1, where $n^t = 1$

represents a conception. We would expect this choice to be potentially influenced by the number of sons and daughters that these parents have inherited from their previous stages of choice. For brevity, we refer to these numbers as the inherited *stock of children*. We denote this stock for the choice at stage t as (M^t, F^t).

Let c^t denote the cost of conception at stage t. If the parents conceive at stage t (that is, if $n^t = 1$) then, before reaching to the next stage $t+1$, they ascertain the gender outcome of the current conception. If this conception yields a female fetus, they decide whether or not to abort it. Let the binary integer variable $a^t = 0$ or 1 denote this choice at stage t, where $a^t = 1$ represents an abortion. The parents begin with no stock of children. That is, $M^1 = F^1 = 0$.

Thus, the conception choice, and the subsequent abortion choice, that the parents make at stage t determine the stock of children at the beginning of next stage, that is, the stage $t+1$. The choice $n^t = 0$ yields $(M^{t+1}, F^{t+1}) = (M^t, F^t)$. The choice $n^t = 1$ yields $(M^{t+1}, F^{t+1}) = (M^t + 1, F^t)$, with probability q. With probability $1-q$, the same choice (that is, $n^t = 1$), yields $(M^{t+1}, F^{t+1}) = (M^t, F^t + 1)$ or (M^t, F^t), depending on whether the choice a^t is 0 or 1.

A central concept in demography is that of *completed fertility*. It is the number of sons and daughters that a set of parents have, after they have concluded all of their fertility-related activities. We analyze here the stage of parental choices which immediately precedes the completed fertility. The last stage of choices is stage T. The choices made in stage T lead immediately to the completed fertility. Recalling our symbols, (M^{T+1}, F^{T+1}) is the stock of children with completed fertility.

For brevity, let $\bar{u} \equiv u(M^{T+1}, F^{T+1}, \lambda)$ denote the utility from completed fertility, for the parents whose propensity for sons is λ. Various outcomes concerning $\bar{u}$ can be determined easily from the dynamics presented in an earlier paragraph, which described the transition from the stage T, in which the stock of children was (M^T, F^T), to the stage $T+1$. Specifically, the choice $n^T = 0$ yields $\bar{u} = u(M^T, F^T, \lambda)$. The choice $n^T = 1$ yields $\bar{u} = u(M^T + 1, F^T, \lambda)$, with probability q. With probability $(1-q)$, the value of $\bar{u}$ under the choice $n^T = 1$ is $u(M^T, F^T + 1, \lambda)$ or $u(M^T, F^T, \lambda)$, depending on whether the choice of a^T is 0 or 1. Expression (19) below summarizes these stage T choices; namely, that of n^T and a^T. We suppress the superscript T in (19) and later, because our present analysis focuses on the choices in stage T.[14]

$$\max_{n\in\{0,1\}} \left\{ -cn + qu(M+n, F, \lambda) + (1-q) \left[\max_{a\in\{0,n\}} u(M, F+n-a, \lambda) - \ell a \right] \right\}. \tag{19}$$

Let $n(M, F, \lambda)$ and $a(M, F, \lambda)$ denote the optimal number of conceptions and abortions in (19).[15] We do not explicitly discuss the parameter ℓ in this section, though it is fully included in the derivations of our results. We do this for brevity. Also, the analysis of the effects of ℓ on the parental behavior is formally analogous to that of λ, which we present here.[16]

There are only three economically relevant groups of parents: those who (i) do not conceive, (ii) conceive and, if this conception yields a female fetus, do not undertake an abortion, and (iii) undertake an abortion in the preceding circumstance. For brevity, we refer to the second and the third groups of parents respectively as those *who do not abort* and those *who abort*. This is because abortion without conception is ruled out. Further, an abortion here means that the one conception that was undertaken yielded a female fetus which was aborted.[17]

[14] We have already noted the importance of completed fertility in demography. The following are among the additional reasons why we abstract from the choices before the last stage, that is, those in the stages $t = 1$ to $T - 1$. Multi-stage stochastic dynamic systems have been studied extensively in economics. Leaving aside exceptions, this valuable literature suggests that analytical results are generally not obtainable for all of the many stages of decisions without strong assumptions. Accordingly, researchers have used numerical simulation methods, based on specific functional forms (such as log-linear and Cobb-Douglas) and on plausible ranges of values for the exogenous parameters (see, for example, Stokey, Lucas, and Prescott (1989)). We abstract from these methods in this paper because, in the present context, these represent a distinct and separate research agenda, and we have not undertaken it. Even otherwise, this research agenda will likely require one or more separate papers.

[15] We follow the convention that the parents conceive if they are indifferent between this and not conceiving, and that they do not abort if they are indifferent between this and aborting. That is, $a(M, F, \lambda)$ is the smallest value of a, and $n(M, F, \lambda)$ is the largest value of n, which maximize the optimand in (19). This is analogous to the conventions used earlier in (7) and (15).

[16] Note that the sequential framework, leading to (19), is structurally different from the setup in (7) and (15) studied in the previous sections. We emphasize this difference because, to avoid cluttering, some of the symbols used in this section are the same as those used earlier. Among these symbols are a, n, u, M, F. The contents of these symbols are different in the present framework. For example, M and F are choice variables in the model in Section 3, whereas they are exogenous parameters in (19).

[17] To avoid unnecessary details, we consider only those values of the parameters, including of (M, F), for which each of the three groups just mentioned contains parents who are heterogeneous in their propensity

Some effects of the parents' propensities for sons on their behaviors. These effects are summarized crisply and nearly-completely by Figure 1 below.[18] An immediate implication of this figure is that: *The parents who conceive have a greater propensity for sons than those who do not. The parents who abort have a greater propensity for sons than those who do not.*

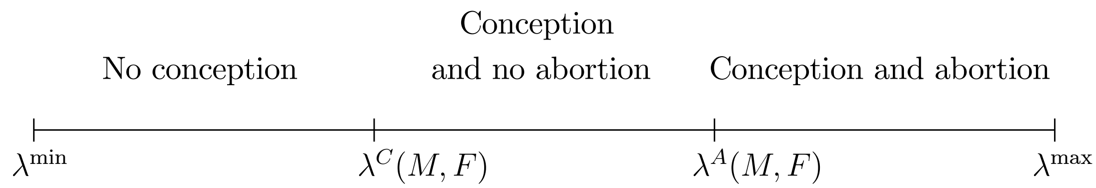

Figure 1: Parents with different conception and abortion behaviors, arranged according to their propensity for sons, λ.

Another way to look at Figure 1 is as follows. Consider parents with any given stock of children, (M, F). The parents with propensity parameters below $\lambda^C(M, F)$ do not conceive, and the others do. Among the parents who conceive, those with propensity parameters up to $\lambda^A(M, F)$ do not abort, and the others do. We can therefore refer to λ^C and λ^A respectively as the *conception threshold* and the *abortion threshold*. A formal statement of these results, proven in the Appendix, is

Proposition 7 *For any given (M, F), there exists λ^C and λ^A, such that*

$$\lambda^{\min} \le \lambda < \lambda^C \iff n(\lambda) = 0. \tag{20}$$

$$\lambda^C \le \lambda \le \lambda^A \iff n(\lambda) = 1, \text{ and } a(\lambda) = 0. \tag{21}$$

$$\lambda^A < \lambda \le \lambda^{\max} \iff n(\lambda) = 1, \text{ and } a(\lambda) = 1. \quad \blacksquare \tag{22}$$

parameter (that is, each group contains parents with more than one value of λ), and these within-group parents together are a positive fraction of all the parents in the economy. We assume that there are fixed lower and upper bounds of the distribution of λ on the economy. We denote these bounds respectively by $\lambda^{\min}$ and $\lambda^{\max}$.

[18] For the proofs of results (in the rest of this section) on the choice of conceptions, we have assumed that $q \ge 0.5$, and $u_{MM} \le 0$. We have earlier discussed, in Section 2, the empirical support for $q \ge 0.5$. The assumption $u_{MM} \le 0$ implies that the marginal utility of an additional son is not larger if a set of parents have more sons.

The above result on the parents who abort versus those who do not is straightforward. The result concerning conceptions can be understood as follows. If a set of parents conceive, they have an additional son with probability q and a female fetus with probability $1-q$. Under the latter outcome, the parents may choose to have or not have an abortion. Regardless of the abortion choice, conceiving will be more attractive to the parents with higher propensity for sons, primarily because their desirability of an additional son is greater. Therefore, for parents with propensity greater than some level, which is the conception threshold λ^C, conceiving will dominate not conceiving.

Some effects of the inherited stock of children on the parents' behaviors. As mentioned earlier, we would expect the parents' choices (concerning conception and abortion) to be potentially affected by the stock of children that they have inherited from their previous stages of choices. To analyze these effects, we compare parents with the same propensity for sons (that is, with the same λ), but with different stocks of children. Let (M', F') denote a stock of children which is *different* from (M, F), in that it differs in the number of sons, the number of daughters, or both. We show in the Appendix that

$$n(M', F', \lambda) \leq n(M, F, \lambda), \text{ and} \tag{23}$$

$$a(M', F', \lambda) \geq a(M, F, \lambda), \text{ if} \tag{24}$$

$$M' \geq M \text{ and } F' \geq F. \tag{25}$$

Some results on sex ratios at birth (SRBs). Recall from Section 2 that the SRB is a metric for a group of parents. Consider parents with the inherited stock of children (M, F). Let $S(M, F)$ denote their SRB. Then, $S(M, F)$ is the number of sons born to these parents, divided by the number of daughters born to them. Let $G(\lambda)$ denote the cumulative density of the distribution of λ within the group of parents under consideration. Then,

$$S(M, F) \equiv \frac{q \displaystyle\int n(M, F, \lambda) dG(\lambda)}{(1-q) \displaystyle\int \{n(M, F, \lambda) - a(M, F, \lambda)\} dG(\lambda)}. \tag{26}$$

The numerator in (26) is the probability q of conceiving a son, multiplied by the fraction of parents who undertake a conception. The denominator is the probability $(1-q)$ of conceiving a female fetus, multiplied by the fraction of unaborted females in the population. For instance, if $a(M, F, \lambda) = 0$ for all values of λ, then, (26) yields $S(M, F) = q/(1-q)$, for all (M, F). That is, in the absence of selective abortions, the SRB should largely be invariant to the parents' stocks of children. Also, by definition, *selective abortions of females must necessarily lead to a larger SRB, compared to the case of no selective abortions.*

The proposition presented below is derived in the Appendix. This result requires virtually no assumption concerning how the propensity parameter, λ, is distributed in the economy.[19]

Proposition 8 *For any $(M', F') \neq (M, F)$ with $M' \geq M$ and $F' \geq F$,*

$$S(M', F') \geq S(M, F). \qquad \blacksquare \tag{27}$$

The above proposition implies that: *The SRB is larger at higher birth orders, caused by more sons and the same number of daughters, by more daughters and the same number of sons, or by both.* This pattern is unambiguously supported by Table 1.

7 Some remarks and extensions

Our analysis has abstracted from many issues. We have dealt with abortion choices concerning female fetuses (which requires gender detection), and not with other abortion choices. The latter include abortions of male fetuses (which also requires gender detection), abortions for achieving particular gender composition of one's offspring (which may or may not require gender detection), and abortions undertaken without gender detection, whether for medical or other reasons. We have abstracted from the stochastic nature of conceptions, errors in gender detection, multiple births, miscarriages, infant mortality,

[19] To avoid unnecessary details, we have assumed in the proof of (27) that the integral of the density of λ on any closed interval of positive length is positive.

biological limits to the number of conceptions (e.g., because of menopause), biological and genetic predispositions for producing more male or more female fetuses, and methods to influence the gender of the fetus without abortion.[20]

Our paper does not deal with the society-level issues potentially induced by gender imbalances. Among them are: their effects on future roles and treatments of women and men, family formation, and other aspects of the economy. A related aspect is that the population growth is slowing down, and may become negative, in many parts of the world, including China. A society-level research topic that this potentially suggests is the combined effects of declining populations, and of gender imbalances. Finally, selective abortions raise numerous and profound moral and ethical issues not dealt with in the present paper.[21]

References

[1] Abeykoon, A.T.P.L., 1995, "Sex Preference in South Asia: Sri Lanka an Outlier," Asia-Pacific Population Journal 10(3), 5-16.

[2] Ahn, Namkee, 1995, "Measuring the Value of Children by Sex and Age Using a Dynamic Programming Model," Review of Economic Studies 62(3), 361-379.

[3] Arnold, Fred, Sunita Kishor, and T.K. Roy, 2002, "Sex-selective Abortions in India," Population and Development Review 28(4), 759-785.

[4] Becker, Gary S., 1960, "An Economic Analysis of Fertility," in Demographic and Economic Change in Developed Countries, Universities-National Bureau Conference Series No. 10, Princeton University Press, Princeton.

[20] Aside from elaborate folk practices (e.g., Khanna, 1997), several techniques are now available for peri-conceptual, pre-conceptual and post-conceptual sex selection with different degrees of accuracy (see the President's Council on Bioethics, 2003).

[21] The President's Council on Bioethics (2003) and Harris (1992) present analyses that deal with gender-specific abortions and other forms of sex-selection. They also examine more general ethical dilemmas that accompany advances in medical technology. We have abstracted entirely from welfare analyses in this paper because the current state of welfare economics has not fully resolved the treatment that should be given to unborn generations especially when their potential choices are affected by parental decisions.

[5] Bélanger, Danièle , K.T.H. Oanh, L. Jianye, L.T. Thuy, and P.V. Thanh, 2003, "Are Sex Ratios at Birth Increasing in Vietnam?" Population 58(2), 231-250.

[6] Ben-Porath, Yoram and Finis Welch, 1976, "Do Sex Preferences Really Matter?" Quarterly Journal of Economics 90, 285–307.

[7] Ben-Porath, Yoram and Finis Welch, 1980, "On Sex Preferences and Family Size," Research in Population Economics 2, 387–399.

[8] Blackorby, Charles, Daniel Primont and Robert R. Russell, 1978, Duality, Separability and Functional Structure: Theory and Economic Applications, North-Holland, New York.

[9] Bloom, David E. and Gilles Grenier, 1983, "The Economics of Sex Preference and Sex Selection," in N. G. Bennett, ed., Sex Selection of Children, Academic Press, New York.

[10] Chung, Woojin and Monica Das Gupta, 2007, "Why Is Son Preference Declining In South Korea ? The Role Of Development And Public Policy, And The Implications For China And India," Population and Development Review 33(4), 757-783.

[11] Cleland, John, Jane Verall, and Martin Vaessen, 1983, "Preferences for the Sex of Children and their Influence on Reproductive Behavior," World Fertility Survey Comparative Studies No. 27, International Statistical Institute, Voorburg, Netherlands.

[12] Coale, Ansley J., 1984, Rapid Population Change in China 1952–1982, National Academy Press, Washington, D.C.

[13] Das Gupta, Monica and P.N. Mary Bhat, 1997, "Fertility Decline and Increased Manifestation of Sex Bias in India," Population Studies 51(3), 307-315.

[14] Das Gupta, Monica, Jiang Zhenghua, Li Bohua, Xie Zhenming, Woojin Chung, Bae Hwa-Ok, 2003 , "Why is Son Preference so Persistent in East and South Asia? A Cross-Country Study of China, India, and the Republic of Korea," Journal of Development Studies 40(2), 153-187.

[15] Davies, James B. and Junsen Zhang, 1997, "The Effects of Gender Control on Fertility and Children's Consumption," Journal of Population Economics 10(1), 67-85.

[16] Diewert, W. E., 1982, "Duality Approaches to Microeconomic Theory," in K. J. Arrow and M. D. Intrilligator, eds., Handbook of Mathematical Economics, Vol. II, North-Holland, Amsterdam.

[17] Efrat, Z, Akinfenwa, O.O., and Nicolaides, K.H., 1999, "First-trimester Determination of Fetal Gender by Ultrasound," Ultrasound in Obstetrics and Gynecology 13(5), 305-307.

[18] Feller, William, 1968, An Introduction to Probability Theory and its Applications, Vol. I, John Wiley, New York.

[19] Freed, Ruth S. and Stanley A. Freed, 1989, "Beliefs and Practices Resulting in Female Deaths and Fewer Females than Males in India," Population and Environment 10, 144–161.

[20] Ganatra, Bela, Hirve Siddhi, and V.N. Rao, 2001, "Sex-selective Abortion: Evidence from a Community-based Study in Western India," Asia-Pacific Population Journal 16(2), 109-124.

[21] Goody, J. R., J.R. Duly, I. Beeson, and G. Harrison, 1981. "Implicit Sex Preferences: A Comparative Study," Journal of Biosocial Science 13(4), 455–466.

[22] Gu, B Baochang and Krishna Roy, 1995, "Sex Ratio at Birth in China, with Reference to Other Areas in East Asia: What We Know," Asia-Pacific Population Journal 10(3), 17-42.

[23] Harris, John, 1992, Wonderwoman and Superman: The Ethics of Human Biotechnology, Oxford University Press, New York.

[24] Heckman James, and Robert Willis, 1975, "Estimation of a Stochastic Model of Reproduction: An Econometric Approach," in N. Terleckyj, ed., Household Production and Consumption, Columbia University Press, New York.

[25] Hull, Terence H., 1990, "Recent Trends in Sex Ratios at Birth in China," Population and Development Review 16(1), 63–83.

[26] James, William H., 2004, "Further Evidence that Mammalian Sex Ratios at Birth are Partially Controlled by Parental Hormone Levels around the Time of Conception," Human Reproduction 19(6), 1250-1256.

[27] Jayaraj, Dhairiyarayar, and Sreenivasan Subramanian, 2004, "Women's Wellbeing and the Sex Ratio at Birth: Some Suggestive Evidence from India," Journal of Development Studies 40(5), 91-119.

[28] Jha, Prabhat, Rajesh Kumar, Priya Vasa, Neeraj Dhingra, Deva Thiruchelvam, and Rahim, Moineddin, 2006, "Low Male-to-Female Sex Ratio of Children Born in India: National Survey of 1.1 Million Households," The Lancet 367(9506), 211-218.

[29] Johansson, Sten and Ola Nygren, 1991, "The Missing Girls of China: A New Demographic Account," Population and Development Review 17(1), 35–51.

[30] Junhong, Chu, 2001, "Prenatal Sex Determination and Sex-selective Abortion in Rural Central China," Population and Development Review 27(2), 259-281.

[31] Khanna, Sunil, K., 1997, "Traditions and Reproductive Technology in an Urbanizing North Indian Village," Social Science and Medicine 44(2), 171-180.

[32] Kim, Jinyoung, 2005, "Sex Selection and Fertility in a Dynamic Model of Conception and Abortion," Journal of Population Economics 18(1), 41-67.

[33] Kim, Jinyoung and Hidehiko Ichimura, 2006, "Effects of Selective and Unselective Abortions on the Sex Ratio at Birth," Mimeo, Department of Economics, SUNY Buffalo.

[34] Lavely, William, 2001, "First Impressions of the 2000 Census of China," Population and Development Review 27(4), 755-769.

[35] Leung, Siu Fai, 1988, "On Tests for Sex Preferences," Journal of Population Economics 1(2), 95-114.

[36] Leung, Siu Fai, 1991, "A Stochastic Dynamic Analysis of Parental Sex Preferences and Fertility," Quarterly Journal of Economics 106(4), 1063-1088.

[37] Leung, Siu Fai, 1994, "Will Sex Selection Reduce Fertility?" Journal of Population Economics 7(4), 379-392.

[38] Lin, Ming-Jen and Ming-Ching Luoh, 2008, "Can Hepatitis B Mothers Account for the Number of Missing Women? Evidence from Three Million Newborns in Taiwan," American Economic Review, forthcoming.

[39] Martin, John F., 1995, "Hormonal and Behavioral Determinants of the Secondary Sex Ratio," Social Biology 42(3-4), 226-238.

[40] Oster, Emily, 2005, "Hepatitis B and the Case of Missing Women," Journal of Political Economy 113(6), 1163-1216.

[41] Oster, Emily and Gang Chen, 2008, "Hepatitis B Does Not Explain Male-Biased Sex Ratios in China," NBER Working Paper 13971.

[42] Park, Chai Bin, and Nam-Hoon Cho, (1995) "Consequences of Son Preference in a Low-Fertility Society: Imbalance of the Sex Ratio at Birth in Korea," Population and Development Review 21(1), 59-84.

[43] Pickens, Gary T. and Marc E. Jacobson, 1986, "Demographic Effects of Sex Preselection: A Formal Model," International Journal of Environmental Studies 28, 21–31.

[44] Poston, Dudley L., Julie Juan Wu, and Kim Han Gon, 2003, "Patterns and Variation in the Sex Ratio at Birth in the Republic of Korea," Development and Society 32(1), 47-60.

[45] President's Council on Bioethics, 2003, Beyond Therapy: Biotechnology and the Pursuit of Happiness, Washington DC.

[46] Retherford, Robert D. and T.K. Roy, 2003, "Factors Affecting Sex-selective Abortion in India and in 17 Major States," National Family Heath Survey Reports 21, Institutional Institute for Population Science, Mumbai.

[47] Wongboonsin, Kua, and Vipan Prachuabmoh, 1995, "Sex Preference for Children in Thailand and Some Other South-East Asian Countries," Asia-Pacific Population Journal 10(3), 43-62.

[48] Zeng, Yi, Tu Ping, Gu Baochang, Xu Yi, Li Bohua, and Li Yongpiing, 1993 "Causes and Implications of the Recent Increase in the Reported Sex Ratio at Birth in China," Population and Development Review 19(2), 283-302.

8 Appendix A

Proof of Proposition 1. From (1),

$$U_a(n, M, a) = -u_F(M, n - M - a - 1) - \ell, \text{ and} \tag{A1}$$

$$U_{aa}(n, M, a) = u_{FF}(M, n - M - a - 2) < 0, \tag{A2}$$

where the inequality in (A2) is from (5).

Let a^* be the smallest a for which $U_a(n, M, a) \leq 0$. If no such a^* exists, set $a^* = n - M$. If it does, then by definition,

$$U_a(n, M, a) > 0 \text{ for all } a \leq a^* - 1, \text{ and} \tag{A3}$$

$$U_a(n, M, a) \leq 0 \text{ for } a = a^*. \tag{A4}$$

We show that $a(n, M) = a^*$. For this, we first show the validity of $a^* = n - M$ if no a^* satisfying (A4) exists.

Consider the only two possibilities concerning M that: $M = n$, or $M \leq n - 1$. Under the first possibility, that $M = n$, the feasible number of abortions is zero. This yields $a(n, M) = a^* = n - M = 0$. Under the second possibility, that $M \leq n - 1$, the feasible number of abortions is at least one. The non-existence of a^* satisfying (A4) means that $U_a(n, M, a) > 0$ for all $a = 0$ to $n - M - 1$. That is, $U(n, M, a)$ increases with a, for all feasible values of a. Hence, the parents will choose the largest feasible value of a, which is $n - M$. Thus, $a(n, M) = a^* = n - M$, under both of the possibilities considered in this paragraph.

In the rest of the proof, an a^* satisfying (A4) exists. Recalling $U_{aa} < 0$ from (A2), expression (A4) implies that

$$U_a(n, M, a) < 0 \text{ for all } a \geq a^* + 1. \tag{A5}$$

There are only two possibilities in (A4): $U_a(n, M, a^*) < 0$, and $U_a(n, M, a^*) = 0$. Under the first possibility, that $U_a(n, M, a^*) < 0$, (A3) to (A5) imply that a^* is the unique optimum. That is, $a(n, M) = a^*$. The second possibility in (A4) is that $U_a(n, M, a^*) = 0$. Then, (A3) to (A5) imply that a^* and $a^* + 1$ are the only two maximizing values of a. As (7) states, if more than one value of a maximizes $U(n, M, a)$, then the parents choose the smallest of these values. Hence, $a(n, M) = a^*$ is the optimal value, and $a^*(n, M) + 1$ is the other maximizing value. Note that (A3) does not apply if $a^* = 0$ but this does not alter the results presented in this paragraph. We thus have shown that $a(n, M) = a^*$ under both possibilities considered in this paragraph.

Recalling the results presented earlier, we therefore have shown that $a(n, M) = a^*$, regardless of whether or not a value of a^* satisfying (A4) exists. We have also shown the following, which directly yields the desired proposition. If an a^* satisfying (A4) does not exists, then $U(n, M, a)$ is maximized by a unique value of a. If an a^* satisfying (A4) exists, then $U(n, M, a)$ is maximized either by a unique value of a, or by at most two neighboring integer values of a. ■

Proof of expression (8). Let $a^* \equiv a(n, M)$. We prove (A6) and (A7) below, which immediately yield (8).

$$a(n, M + 1) \leq a^* \text{ and} \tag{A6}$$

$$a(n, M + 1) \geq a^* - 1. \tag{A7}$$

We first prove (A6). A feasibility requirement is that $a(n, M + 1) \leq n - (M + 1) = n - M - 1$. Hence if $a^* = n - M$ or $n - M - 1$, then $a(n, M + 1) \leq a^*$, yielding (A6). Hence $a^* \leq n - M - 2$ in the rest of the proof of (A6).

From (A1), $U_a(n, M + 1, a) - U_a(n, M, a) = -u_F(M + 1, n - M - a - 2) + u_F(M, n - M - a - 1) = -\{u_F(M + 1, n - M - a - 2) - u_F(M, n - M - a - 2)\} + \{u_F(M, n - M - a - 1) - u_F(M, n - M - a - 2)\} = -u_{MF}(M, n - M - a - 2) + u_{FF}(M, n - M - a - 2)$. The last part of the preceding expression is negative, because of the first inequality in (5).

Hence,

$$U_a(n, M+1, a) < U_a(n, M, a). \tag{A8}$$

(A4) and (A5) imply $U_a(n, M, a) \leq 0$ for all $a \geq a^*$. Hence, (A8) yields $U_a(n, M+1, a) < 0$ for all $a \geq a^*$. Accordingly, $a(n, M+1)$ cannot exceed a^*. This yields (A6).

We now prove (A7). A feasibility requirement is that $a(n, M+1) \geq 0$. This yields (A7) if $a^* = 0$ or 1. Hence, $a^* \geq 2$ in the rest of the proof of (A7). From (A1), $U_a(n, M+1, a-1) - U_a(n, M, a) = -u_F(M+1, n-M-a-1) + u_F(M, n-M-a-1) = -u_{MF}(M, n-M-a-1)$. The last part of the preceding expression is positive, because of the second inequality in (5). Hence, $U_a(n, M+1, a-1) > U_a(n, M, a)$. This and (A3) yield

$$U_a(n, M+1, a-1) > 0 \text{ for all } a \leq a^* - 1. \tag{A9}$$

Defining $a' \equiv a - 1$, (A9) becomes $U_a(n, M+1, a') > 0$ for all $a' \leq a^* - 2$. Accordingly, $a(n, M+1)$ cannot be smaller than $a^* - 1$. This yields (A7). ■

Proof of expressions (9) and (10). Let $a^* \equiv a(n, M, \lambda')$. From the second inequality in (6), $-u_F(M, F, \lambda'') > -u_F(M, F, \lambda')$. In turn, from (A1), $U_a(n, M, a, \lambda'') > U_a(n, M, a, \lambda')$ for all a. This and (A3) yield $U_a(n, M, a, \lambda'') > 0$ for all $a \leq a^* - 1$. Hence, $a(n, M, \lambda'') \geq a^*$, yielding (9). An analogous reasoning yields (10). ■

Proof of Expressions (11) to (14). From (A1), $U_a(n+1, M, a+1) = -u_F(M, (n+1) - M - (a+1) - 1) - \ell = -u_F(M, n-M-a-1) - \ell$. Hence, from (A1)

$$U_a(n+1, M, a+1) = U_a(n, M, a). \tag{A10}$$

Define $a^{**} = a(n+1, M)$. Then, analogous to (A3) to (A5),

$$U_a(n+1, M, a') > 0 \text{ for all } a' \leq a^{**} - 1, \tag{A11}$$

$$U_a(n+1, M, a') \leq 0 \text{ for } a' = a^{**}, \text{ and} \tag{A12}$$

$$U_a(n+1, M, a') < 0 \text{ for } a' > a^{**} + 1. \tag{A13}$$

Using the definition $a \equiv a' - 1$, (A11) to (A13) respectively become $U_a(n+1, M, a+1) > 0$ for all $a \leq a^{**} - 2$, $U_a(n+1, M, a+1) \leq 0$ for $a = a^{**} - 1$, and $U_a(n+1, M, a+1) < 0$ for all $a \geq a^{**}$. The preceding and (A10) yield

$$U_a(n, M, a) > 0 \text{ for all } a \leq a^{**} - 2, \tag{A14}$$

$$U_a(n, M, a) \leq 0 \text{ for } a = a^{**} - 1, \text{ and} \tag{A15}$$

$$U_a(n, M, a) < 0 \text{ for } a > a^{**}. \tag{A16}$$

We now prove (11). A given in (11) is that $a^{**} \geq 1$. Hence, a value of a^{**} exists for which (A15) holds. (If $a^{**} = 1$, then (A14) does not apply, but this does not alter any conclusion in this paragraph.) A straightforward comparison of the set of expressions (A14) to (A16) to the set (A3) to (A5) shows that $a(n, M) = a^{**} - 1$. This yields (11), recalling the definition that $a^{**} \equiv a(n+1, M)$.

We next prove (12). A given in (12) is that $a^{**} = 0$. This implies that $U_a(n+1, M, a') \leq 0$ for all $a' \geq 0$. Using the definition $a \equiv a' - 1$, the preceding expression implies that $U_a(n+1, M, a+1) \leq 0$ for all $a \geq -1$. This and (A10) yield

$$U_a(n, M, a) \leq 0 \text{ for all } a \geq -1. \tag{A17}$$

This expression is not economically useful, because of the feasibility requirement that $a \geq 0$. However, recall from (A2) that $U_{aa} < 0$. Hence, (A17) yields $U_a(n, M, a) \leq 0$ for all $a \geq 0$. Accordingly, $a(n, M) = 0$. This yields (12).

We now prove (14) and then (13). From (8),

$$a(n+1, M+1) = a(n+1, M), \text{ or } a(n+1, M) - 1. \tag{A18}$$

If $a(n+1, M) = 0$, then (A18), and the feasibility requirement that $a(n+1, M+1) \geq 0$, yield $a(n+1, M+1) = 0$. This and (12) yield (14). If $a(n+1, M) \geq 1$, then (11) and (A18) yield (13). ■

Proof of expression (16). The relationship $b(M, n+1, q) = qb(M-1, n, q) + (1 -$

$q)b(M,n,q)$ is based on the Pascal's identity. This and (3) yield

$$V(n+1) = q\sum_{M=0}^{n+1} b(M-1,n,q)v(n+1,M) + (1-q)\sum_{M=0}^{n+1} b(M,n,q)v(n+1,M). \quad \text{(A19)}$$

The terms in the first summation in (A19) are $\{b(-1,n,q)v(n+1,0), b(0,n,q)v(n+1,1), ..., b(n,n,q)v(n+1,n+1)\}$, and the first of these terms is zero because $b(-1,n,q) = 0$. Hence, this summation can be written as $\sum_{M=0}^{n} b(M,n,q)v(n+1,M+1)$. In the second summation in (A19), the last term is zero because $b(n+1,n,q) = 0$. Hence, (A19) yields $V(n+1) = q\sum_{M=0}^{n} b(M,n,q)v(n+1,M+1) + (1-q)\sum_{M=0}^{n} b(M,n,q)v(n+1,M)$. Substract from this $\{q+(1-q)\}V(n)$ where $V(n)$ is given by (3). This yields

$$\begin{aligned} V_n(n) = {} & q\sum_{M=0}^{n} b(M,n,q)[v(n+1,M+1) - v(n,M)] \\ & + (1-q)\sum_{M=0}^{n} b(M,n,q)[v(n+1,M) - v(n,M)]. \end{aligned} \quad \text{(A20)}$$

Next, from (4),

$$w_n(n) = -C_n(n) + V_n(n). \quad \text{(A21)}$$

(A20) and (A21) yield (16). ■

Lemma 1 *Suppose that f is a continuous real-valued function on an interval in the real line x. If f is a strictly increasing function at all but finitely many points, f is a strictly increasing function at all points in x.*

Proof of Lemma 1. Consider a closed domain of f. Since f is strictly increasing at all but finitely many points in this domain, the set of points at which f is potentially not strictly increasing is a finite discrete set. Let $x = c$ be an element of this set. Then there is an open interval about c such that f is strictly increasing at all points other than c in that interval. Choose a sufficiently small $\varepsilon > 0$ such that $c-\varepsilon$ and $c+\varepsilon$ are in this open interval. Define $g(x) \equiv f(x) - f(x-\delta)$ for $c-\varepsilon < x-\delta < x < c$. Since f is continuous, g is continuous. $g(x) > 0$ because f is strictly increasing for the relevant values of x. Taking the limit as x approaches c from the left, $g(c) = \lim_{x \to c} g(x)$. The continuity of $g(x)$ implies that $g(c)$ exists. $g(x) > 0$ yields $g(c) > 0$. Similarly, let $h(x) \equiv f(x + \delta) - f(x)$ for $c < x < x + \delta < c + \varepsilon$. Then, $h(x)$ is continuous and $h(x) > 0$. Taking the limit as x approaches c from the right, $h(c) = \lim_{x \to c} h(x)$. The continuity of $h(x)$ implies that $h(c)$ exists. $h(x) > 0$ yields $h(c) > 0$. Combining this with $g(c) > 0$, obtained earlier, yields the desired result that $f(c - \delta) < f(c) < f(c + \delta)$. ■

Proof of Expression (17). The proof is in six parts.

Part (a): We can write (A20) as

$$V_n(n, \lambda) = \sum_{M=0}^{n} b(M, n, q) X(n, M, \lambda), \text{ where} \tag{A22}$$

$$X(n, M, \lambda) \equiv q\{v(n + 1, M + 1) - v(n, M)\} + (1 - q)\{v(n + 1, M) - v(n, M)\}. \tag{A23}$$

Part (b): From (1), U is continuous in λ for any given value of a. From (2), v is continuous in λ because it is the maximum of a finite set of continuous functions. Hence, from (A22) and (A23), $V_n(n, \lambda)$ is continuous in λ. Accordingly, from (A21), $w_n(n)$ is continuous in λ.

Part (c): From (1), (2), (7), and (A23),

$$X(n, M, \lambda) \equiv Y(n, M, \lambda) - \ell Z(n, M, \lambda), \text{ where} \tag{A24}$$

$$\begin{aligned} Y(n, M, \lambda) \equiv\; & q\{u(M + 1, n - M - a(n + 1, M + 1, \lambda), \lambda) - u(M, n - M - a(n, M, \lambda), \lambda\} \\ & + (1 - q)\{u(M, n - M - a(n + 1, M, \lambda) + 1, \lambda) \\ & - u(M, n - M - a(n, M, \lambda), \lambda)\}, \text{ and} \end{aligned} \tag{A25}$$

$$Z(n, M, \lambda) \equiv q\{a(n{+}1, M{+}1, \lambda){-}a(n, M, \lambda)\}{+}(1{-}q)\{a(n{+}1, M, \lambda){-}a(n, M, \lambda)\}. \tag{A26}$$

For brevity, define the set $A(n, M, \lambda)$ which contains all of the a's which appear in (A22).

Thus, from (A22) to (A26),

$$A(n,M,\lambda) \equiv \{a(n+1,M+1,\lambda), a(n+1,M,\lambda), a(n,M,\lambda); \text{ for } M = 0 \text{ to } n\}. \quad \text{(A27)}$$

Part (d): From (9), either $a(n,M,\lambda)$ does not change with λ, or it increases by a positive integer value. Separately, by definition, $a(n,M)$ can only take an integer value from 0 to $n-M$. Hence, there can at most be a finite number of discrete values of λ at which the value of $a(n,M,\lambda)$ can change. The same conclusion applies to $a(n+1,M+1,\lambda)$ and to $a(n+1,M,\lambda)$, and hence to the set $A(n,M,\lambda)$ in (A27). Let α denote the finite discrete set containing the values of λ at which any element of $A(n,M,\lambda)$ changes.

Part (e): We now calculate $\partial V_n(n)/\partial\lambda$, excluding at $\lambda \in \alpha$. This exclusion means that a change in λ does not affect any of the a's in (A25) and (A26). Also, this exclusion means that a change in λ does not affect Z, defined in (A26). Let $a^* \equiv a(n,M)$. Consider the following three possibilities for a given M:

(i) Suppose that $a(n+1,M)=0$. Then, (12) and (14) yield $a(n+1,M+1)=a(n+1,M)=a^*=0$. This and (A25) yield $Y = q\{u(M+1,n-M-a^*)-u(M,n-M-a^*)\} + (1-q)\{u(M,n-M-a^*+1)-u(M,n-M-a^*)\} = qu_M(M,n-M)+(1-q)u_F(M,n-M)$. This yields $\partial Y/\partial\lambda = (1-q)\{\partial u_M/\partial\lambda + \partial u_F/\partial\lambda\} + (2q-1)\partial u_M/\partial\lambda$. Assuming $q \geq 0.5$ for the rest of this proof, the preceding expression for $\partial Y/\partial\lambda$, and (6) yield $\partial Y/\partial\lambda > 0$.

(ii) Suppose that $a(n+1,M) \geq 1$ and, in (13), $a(n+1,M+1)=a^*$. Also recall from (11) that $a(n+1,M)=a^*+1$. Hence, from (A25), $Y = q\{u(M+1,n-M-a^*)-u(M,n-M-a^*)\}+(1-q)\{u(M,n-M-(a^*+1)+1)-u(M,n-M-a^*)\} = qu_M(M,n-M-a^*)$. This and (6) yield $\partial Y/\partial\lambda > 0$.

(iii) Suppose that $a(n+1,M) \geq 1$, and in (13), $a(n+1,M+1)=a^*+1$. Once again, $a(n+1,M)=a^*+1$ from (11). Hence (A25) yields $Y = q\{u(M+1,n-M-(a^*+1))-u(M,n-M-a^*)\}+(1-q)\{u(M,n-M-(a^*+1)+1) = u(M,n-M-a^*)\} = q\{u(M+1,n-M-a^*-1)-u(M,n-M-a^*-1)\} - q\{u(M,n-M-a^*)-u(M,n-M-a^*-1)\} = q[u_M(M,n-M-a^*-1) - u_F(M,n-M-a^*-1)]$. This yields $\partial Y/\partial\lambda = q(\partial u_M/\partial\lambda - \partial u_F/\partial\lambda)$. This and (6) yield $\partial Y/\partial\lambda > 0$.

The above three are the only possibilities. Under each, $\partial Y/\partial\lambda > 0$ for any given

M. This, combined with (A22) to (A26), yield $\partial V_n(n)/\partial\lambda > 0$. This and (A21) yield $\partial w_n(n)/\partial\lambda > 0$.

Part (f): From Part (b) above, $w_n(n)$ is a continuous function of λ. From Part (e), $\partial w_n(n)/\partial\lambda > 0$ except at $\lambda \in \alpha$, and α is a finite discrete set. Hence, Lemma 1 yields that

$$w_n(n) \text{ is strictly increasing in } \lambda, \text{ for all } \lambda. \tag{A28}$$

Let $n' \equiv n(\lambda')$ denote the optimal n at $\lambda = \lambda'$. From (15), $w_n(n, \lambda') \geq 0$ for all $n \leq n' - 1$. Hence, (A28) and $\lambda'' > \lambda'$ yield $w_n(n, \lambda'') > 0$ for all $n \leq n' - 1$. This yields (17). ■

Proof of Expression (18). This proof is identical to that of (17) above except for that corresponding to Part (e) of the latter. Hence for brevity, we outline only the derivations relating to the difference just noted. Analogous to Part (e) of the proof of (17), there are three steps here. (i) Temporarily disregard those values of ℓ at which the values of any of the a's in (A25) and (A26) changes. (ii) Assess $\partial Y/\partial\ell$ for the remaining values of ℓ. (iii) Then find sufficient conditions needed, if any, for $\partial V_n(n)/\partial\ell < 0$ for these remaining values of ℓ.

For step (ii) above, we use (11) to (14), and (A26), for each value of M. From these, $\partial Y/\partial\ell = 0$ if $a(n+1, M) = 0$. Hence, the sufficient condition $q \geq 0.5$, which was needed in the proof of (17) is not relevant for the present proof. Further, if $a(n+1, M) \geq 1$, then $\partial Y/\partial\ell$ is $-(1-q)$ or -1 depending on whether $a(n+1, M+1)$ is $a(n, M)$ or $a(n, M)+1$ in (18). But $\partial Y/\partial\ell < 0$, regardless. Hence, from (A22), a sufficient condition for $\partial V_n(n)/\partial\ell < 0$ is that $a(n+1, M) \geq 1$ for at least one value of M. This completes step (iii) described in the first paragraph of the present proof. ■

Proof of Proposition 7. For later convenience, rewrite (19) as

$$\max_{n\in\{0,1\}} \omega(n,M,F,\lambda), \text{ where} \tag{A29}$$

$$\omega(n,M,F,\lambda) \equiv -cn + qu(M+n,F,\lambda) + (1-q)\Psi(M,F,\lambda), \tag{A30}$$

$$\Psi(M,F,\lambda) \equiv \max_{a\in\{0,n\}} \psi(M,F,a,\lambda), \text{ and} \tag{A31}$$

$$\psi(M,F,a,\lambda) \equiv u(M,F+n-a,\lambda) - \ell a. \tag{A32}$$

For later use, define

$$L^0(M,F) \equiv \{\lambda | n^* = 0\}, \tag{A33}$$

$$L^{10}(M,F) \equiv \{\lambda | n^* = 1, a^* = 0\}, \text{ and} \tag{A34}$$

$$L^{11}(M,F) \equiv \{\lambda | n^* = 1, a^* = 1\}. \tag{A35}$$

These are three sets of λ's corresponding to the parents who: (i) do not conceive, (ii) do not abort, and (iii) abort. Throughout the rest of this Appendix, we use our assumption that each of these three groups contains more than one value of λ. Note that the first superscript of L is the value of n^*, and the second superscript is that of a^*.

Let $n^* \equiv n(M,F,\lambda)$, and $a^* \equiv a(M,F,\lambda)$. We first show that there exists a value of λ, which we denote as $\lambda^A(M,F)$, such that

$$a^* = 0 \text{ iff } \lambda \leq \lambda^A(M,F), \text{ and} \tag{A36}$$

$$a^* = 1 \text{ iff } \lambda > \lambda^A(M,F). \tag{A37}$$

To show this, we examine (A31), assuming that $n = 1$. Then, (A32) yields $\psi(M,F,0,\lambda) = u(M,F+1,\lambda) = u(M,F,\lambda) + u_F(M,F,\lambda)$, and $\psi(M,F,1,\lambda) = u(M,F,\lambda) - \ell$. Define

$u'(M,F,\lambda) \equiv \psi(M,F,1,\lambda) - \psi(M,F,0,\lambda)$. Then,

$$u'(M,F,\lambda) = -u_F(M,F,\lambda) - \ell. \tag{A38}$$

$$a^* = 0 \text{ iff } u'(M,F,\lambda) \leq 0, \text{ and} \tag{A39}$$

$$a^* = 1 \text{ iff } u'(M,F,\lambda) > 0. \tag{A40}$$

(A31) yields $\Psi(M,F,\lambda) = \max\{\psi(M,F,0,\lambda), \psi(M,F,1,\lambda)\}$. This and (A32) yield

$$\Psi(M,F,\lambda) = u(M,F,\lambda) + \max\{u_F(M,F,\lambda), -\ell\}. \tag{A41}$$

Expression (6) and (A38) yield

$$u' \text{ is continuous and strictly increasing in } \lambda. \tag{A42}$$

There are parents who abort, and also those who do not. This, (A39), and (A40) imply that there is a value of λ for which $u'(M,F,\lambda) \leq 0$, and a value of λ for which $u'(M,F,\lambda) > 0$. This, (A42), and the intermediate value theorem, yield that $\lambda^A(M,F,\lambda)$ exists such that

$$u'(M,F,\lambda^A(M,F)) = 0. \tag{A43}$$

(A42) and (A43) imply that $u'(M,F,\lambda) \leq 0$ for $\lambda \leq \lambda^A(M,F)$, and $u'(M,F,\lambda) > 0$ for $\lambda > \lambda^A(M,F)$. These two conclusions, combined with (A39) and (A40), yield the desired expressions (A36) and (A37).

We now show that there exists a value of λ, which we denote as $\lambda^C(M,F)$, such that

$$n^* = 0 \text{ iff } \lambda < \lambda^C(M,F), \text{ and} \tag{A44}$$

$$n^* = 1 \text{ iff } \lambda \geq \lambda^C(M,F). \tag{A45}$$

To show this, we examine (A29). If $n = 0$, then (A31) and (A32) yield $\Psi(M,F,\lambda) = u(M,F,\lambda)$. Hence, (A30) yields $\omega(0,M,F,\lambda) = u(M,F,\lambda)$. If $n = 1$, then (A30) and (A41) yield $\omega(1,M,F,\lambda) = -c + u(M,F,\lambda) + qu_M(M,F,\lambda) + (1-q)\max\{u_F(M,F,\lambda), -\ell\}$.

Define $\omega'(M,F,\lambda) \equiv \omega(1,M,F,\lambda) - \omega(0,M,F,\lambda)$. Then,

$$\omega'(M,F,\lambda) = \max\{\omega^0(M,F,\lambda), \omega^1(M,F,\lambda)\}, \text{ where} \tag{A46}$$

$$\omega^0(M,F,\lambda) \equiv -c + qu_M(M,F,\lambda) + (1-q)u_F(M,F,\lambda), \text{ and} \tag{A47}$$

$$\omega^1(M,F,\lambda) \equiv -c + qu_M(M,F,\lambda) - (1-q)\ell. \tag{A48}$$

The definition of ω' yields

$$n^* = 0 \text{ iff } \omega'(M,F,\lambda) < 0 \text{ and} \tag{A49}$$

$$n^* = 1 \text{ iff } \omega'(M,F,\lambda) \geq 0. \tag{A50}$$

We now show that

$$\omega^0, \ \omega^1 \text{ and } \omega' \text{ are each continuous and strictly increasing in } \lambda. \tag{A51}$$

In the rest of the proofs, $q \geq 0.5$. From (A48), $\omega^1(M,F,\lambda)$ depends on λ only through $u_M(M,F,\lambda)$ which, from (6), is strictly increasing in λ. From (A47), $\omega^0(M,F,\lambda)$ depends on λ only through $qu_M(M,F,\lambda)+(1-q)u_F(M,F,\lambda)$. Hence, the assumption that $q \geq 0.5$ and (6) yield that $\omega^0(M,F,\lambda)$ is strictly increasing in λ. From (A46), $\omega'(M,F,\lambda)$ is the maximum of the functions ω^0 and ω^1, and hence it is also strictly increasing in λ. Continuity follows from Lemma 1.

There are parents who conceive, and also those who do not. Hence, (A49), and (A50) imply that there is a value of λ for which $\omega'(M,F,\lambda) < 0$, and a value of λ for which $\omega'(M,F,\lambda) \geq 0$. This, (A51), and the intermediate value theorem, yield that $\lambda^C(M,F)$ exists such that

$$\omega'(M,F,\lambda^C(M,F)) = 0. \tag{A52}$$

(A51) and (A52) imply that $\omega'(M,F,\lambda) < 0$ for $\lambda < \lambda^C(M,F)$, and $\omega'(M,F,\lambda) \geq 0$ for $\lambda \geq \lambda^C(M,F)$. These two conclusions, combined with (A49) and (A50), yield the desired results (A44) and (A45).

For later use, we show that

$$\lambda^C(M, F) < \lambda^A(M, F). \tag{A53}$$

In this paragraph, $\lambda \in L^{10}$. Equivalently, from (A34), this paragraph deals only with the parents who do not abort. That is, it deals only with those values of λ for which $n^* = 1$ and $a^* = 0$. By assumption, the set L^{10} contains more than one value of λ. Now, consider $n^* = 1$. This and (A50) yield $\omega'(M, F, \lambda) \geq 0$. This, (A51) and (A52) yield $\lambda \geq \lambda^C(M, F)$. From (A51), not more than one value of λ can satisfy the equality in the preceding expression. Thus, given that L^{10} contains more than one value of λ, it must contain a value of λ such that $\lambda > \lambda^C(M, F)$. Next, consider $a^* = 0$. This and (A39) yield $u'(M, F, \lambda) \leq 0$. This, (A42) and (A43) yield $\lambda \leq \lambda^A(M, F)$. From (A42), not more than one value of λ can satisfy the equality in the preceding expression. Thus, given that L^{10} contains more than one value of λ, it must contain a value of λ such that $\lambda < \lambda^A(M, F)$. This, and the earlier demonstration that L^{10} contains a value of λ such that $\lambda > \lambda^C(M, F)$, yield that L^{10} contains a value of λ such that $\lambda^C(M, F) < \lambda^A(M, F)$, which is (A53).

Proposition 7 follows from (A36) to (A45) combined with (A53).

Recall that (A52) characterizes $\lambda^C(M, F)$. For later use, we simplify this characterization. Focus in this paragraph on $\lambda < \lambda^A(M, F)$. Then, from (A42) and (A43), $u'(M, F, \lambda) < 0$. This and (A38) yield $u_F(M, F, \lambda) > -\ell$. This and (A41) yield $\omega^0(M, F, \lambda) > \omega^1(M, F, \lambda)$. Hence, from (A46), $\omega'(M, F, \lambda) = \omega^0(M, F, \lambda)$. From (A53), $\lambda < \lambda^A(M, F)$ includes $\lambda^C(M, F)$. Thus, the equality $\omega'(M, F, \lambda) = \omega^0(M, F, \lambda)$ must hold for any value of λ that could potentially be $\lambda^C(M, F)$. Hence, instead of (A52), we can characterize $\lambda^C(M, F)$ by

$$\omega^0(M, F, \lambda^C(M, F)) = 0. \tag{A54}$$

Also, we can replace (A49) and (A50) by the following two expressions, which can be manipulated more easily.

$$n^* = 0 \text{ iff } \omega^0(M,F,\lambda) < 0 \text{ and} \tag{A55}$$

$$n^* = 1 \text{ iff } \omega^0(M,F,\lambda) \geq 0. \tag{A56}$$

For later use, we provide the following summary of the segments of λ corresponding to the sets defined in (A33) to (A35).

$$L^0(M,F) = [\lambda^{\min}, \lambda^C(M,F)),\ L^{10}(M,F) = [\lambda^C(M,F), \lambda^A(M,F)], \tag{A57}$$
$$\text{and } L^{11}(M,F) = (\lambda^A(M,F), \lambda^{\max}].$$

These follow from the definitions (A33) to (A35), (A44) and (A45), and (A49) and (A50).

■

Proof of expressions (23) to (25). We first prove (23) for $M' > M$, and $F' = F$. Let $\lambda^* \equiv \lambda^C(M,F)$ and $\lambda^{**} \equiv \lambda^C(M+1,F)$. From (A47), $\omega^0(M+1,F,\lambda) - \omega^0(M,F,\lambda) = qu_{MM}(M,F,\lambda) + (1-q)u_{MF}(M,F,\lambda) < 0$, where the last inequality is from (5), and from $u_{MM} \leq 0$, which is assumed in the rest of the proofs. For $\lambda = \lambda^*$, we thus have $\omega^0(M+1,F,\lambda^*) < \omega^0(M,F,\lambda^*)$. Combining this with $\omega^0(M,F,\lambda^*) = 0$, which defines λ^* from (A54), we obtain $\omega^0(M+1,F,\lambda^*) < 0$. From (A54), λ^{**} is defined by $\omega^0(M+1,F,\lambda^{**}) = 0$. The preceding two expressions and (A51) yield $\lambda^{**} > \lambda^*$. By iteration,

$$\lambda^C(M+k_1,F) > \lambda^C(M,F) \text{ for } k_1 \geq 1. \tag{A58}$$

We now prove (23) for $M' = M$, and $F' > F$. Let $\lambda^* \equiv \lambda^C(M,F)$ and $\lambda^{**} \equiv \lambda^C(M,F+1)$. From (A47), $\omega^0(M,F+1,\lambda) - \omega^0(M,F,\lambda) = qu_{MF}(M,F,\lambda) + (1-q)u_{FF}(M,F,\lambda) < 0$, where the last inequality is from (5). For $\lambda = \lambda^*$, we thus have $\omega^0(M,F+1,\lambda^*) < \omega^0(M,F,\lambda^*)$. Combining this with $\omega^0(M,F,\lambda^*) = 0$, which defines λ^* from (A54), we obtain $\omega^0(M,F+1,\lambda^*) < 0$. From (A54), λ^{**} is defined by $\omega^0(M,F+1,\lambda^{**}) = 0$. The preceding two expressions and (A51) yield $\lambda^{**} > \lambda^*$. By iteration,

$$\lambda^C(M, F + k_2) > \lambda^C(M, F) \text{ for } k_2 \geq 1. \tag{A59}$$

We next prove (24) for $M' > M$ and $F' = F$. Let $\lambda^* \equiv \lambda^A(M, F)$ and $\lambda^{**} \equiv \lambda^A(M + 1, F, \lambda)$. From (A38), $u'(M + 1, F, \lambda) - u'(M, F, \lambda) = -u_F(M + 1, F, \lambda) + u_F(M, F, \lambda) = -u_{MF}(m, F, \lambda) > 0$, where the last inequality is from (5). For $\lambda = \lambda^*$, we thus have $u'(M + 1, F, \lambda^*) > u'(M, F, \lambda^*)$. Combining this with $u'(M, F, \lambda^*) = 0$, which defines λ^* from (A43), we obtain $u'(M + 1, F, \lambda^*) > 0$. From (A43), λ^{**} is defined by $u'(M + 1, F, \lambda^{**}) = 0$. The preceding two expressions and (A42) yield $\lambda^{**} < \lambda^*$. By iteration,

$$\lambda^A(M + k_3, F) < \lambda^A(M, F), \text{ for } k_3 \geq 1. \tag{A60}$$

We finally prove (24) for $M' = M$ and $F' > F$. Let $\lambda^* \equiv \lambda^A(M, F)$ and $\lambda^{**} \equiv \lambda^A(M, F + 1, \lambda)$. From (A38), $u'(M, F + 1, \lambda) - u'(M, F, \lambda) = -u_{FF}(M, F) > 0$, where the last inequality is from (5). For $\lambda = \lambda^*$, we thus have $u'(M, F + 1, \lambda^*) > u'(M, F, \lambda^*)$. Combining this with $u'(M, F, \lambda^*) = 0$, which defines λ^* from (A43), we obtain $u'(M, F + 1, \lambda^*) > 0$. From (A43), λ^{**} is defined by $u'(M, F + 1, \lambda^{**}) = 0$. The preceding two expressions and (A42) yield $\lambda^{**} < \lambda^*$. By iteration,

$$\lambda^A(M, F + k_4) < \lambda^A(M, F), \text{ for } k_4 \geq 1. \tag{A61}$$

Recall that, by definition, $M' \neq M$, or $F' \neq F$, or both. We assume that (25) holds in the rest of the Appendix. Then, (A58) to (A61) yield

$$\lambda^C(M', F') > \lambda^C(M, F), \text{ and} \tag{A62}$$

$$\lambda^A(M', F') < \lambda^A(M, F). \tag{A63}$$

$$L^0(M', F') \subset L^0(M, F), \tag{A64}$$

$$L^{10}(M', F') \supset L^{10}(M, F), \text{ and} \tag{A65}$$

$$L^{11}(M', F') \subset L^{11}(M, F). \tag{A66}$$

Recall that (A33) to (A35) define the above sets in terms of the optimal parental choices. (A64) implies that there is no value of λ such that $n(M,F,\lambda)=0$ and $n(M,F,\lambda)=1$, and that there is at least one value of λ such that $n(M,F,\lambda)\neq 0$ (which means that $n(M,F,\lambda)=1$) and $n(M',F',\lambda)=0$. This yields the desired result (23). In (A66), $n(M',F',\lambda)=n(M,F,\lambda)=1$. Further, (A66) implies that if $a(M,F,\lambda)=1$ then $a(M',F',\lambda)=1$, and that there is at least one value of λ such that $a(M',F',\lambda)=1$ and $a(M,F,\lambda)=0$. This yields the desired result (24). ■

Proof of expression (27). Define

$$H^0(M,F)\equiv\int_{L^0(M,F)}dG(\lambda),\ H^{10}(M,F)\equiv\int_{L^{10}(M,F)}dG(\lambda),\text{ and}\qquad\text{(A67)}$$
$$H^{11}(M,F)\equiv\int_{L^0(M,F)}dG(\lambda).$$

These respectively are the fractions of parents who: (i) do not conceive, (ii) do not abort, and (iii) abort. From (26)

$$S(M,F)=\frac{q\{H^{10}(M,F)+H^{11}(M,F)\}}{(1-q)H^{10}(M,F)}=\frac{q}{1-q}\left[1+\frac{H^{11}(M,F)}{H^{10}(M,F)}\right].\qquad\text{(A68)}$$

Recall our assumption that the integral of $dG(\lambda)$ on any interval of positive length is positive. Thus, (A65) to (A67) imply that $H^{10}(M',F')<H^{10}(M,F)$, and that $H^{11}(M',F')>H^{11}(M,F)$. This and (A68) yields (27). ■