

The Common Factor in Idiosyncratic Volatility: Quantitative Asset Pricing Implications*

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First Draft: November 2012

This Draft: March 2014

Abstract

We document a strong factor structure in firm-level volatility of idiosyncratic cash flow growth and returns. If markets are incomplete, these factors can be priced. An increase in idiosyncratic volatility represents a worsening of the investment opportunity set for the average investor. As predicted by the theory, we find that exposure to the common factor in idiosyncratic volatility (CIV) in stock returns is indeed priced in the cross-section of U.S. stocks. Stocks that tend to appreciate when CIV rises earn relatively low average returns and thus appear to be valuable hedges. The calibrated model is able to match the high degree of comovement in idiosyncratic volatilities, the CIV beta spread, and a host of other asset price moments. We provide new empirical evidence on the common volatility factor structure of firm- and household-level income growth.

JEL: E3, E20, G1, L14, L25

Keywords: Firm volatility, Idiosyncratic risk, Cross-section of stock returns

*We thank John Campbell, Rob Engle and conference/seminar participants at the AFA, Chicago Booth, McGill, NYU, University of Washington, and WFA for helpful comments and suggestions.

1 Introduction

This paper presents three central findings. First, we document a strong factor structure in firm-level volatility of idiosyncratic cash flow growth and returns. Second, in a model with incomplete risk sharing, we show that these factors in firm-level idiosyncratic volatility are valid candidate asset pricing factors. Third, we find that exposure to the common factor in idiosyncratic volatility (CIV) is indeed priced in the cross-section of U.S. stocks. Stocks that tend to appreciate when CIV rises earn relatively low average returns and thus appear to be valuable hedges.

Statistically, we can decompose the returns on any assets into the common factors and residuals. As shown in Ross (1976)' APT, any of these common factors are valid candidate asset pricing factors. The idiosyncratic residuals will not be priced, because they can be diversified away. However, in a world with non-traded assets, such as human wealth, these residuals cannot be diversified away. In this environment, we find that any factors driving the common variation in the volatility (or other higher order moments) of the residuals could be valid asset pricing factors.

We start by documenting the factor structure in the volatility of the residuals for stock returns. We estimate annual realized return volatilities of more than 20,000 CRSP stocks over the 1926-2010 sample. The first principal component explains 36% of total variation in the volatility panel.¹ The firm-level volatility factor structure is effectively unchanged after accounting for common factors in returns. We examine residuals from models such as the Fama-French (1993) three-factor model and statistical factor decompositions using several principal components. Stock return residuals from these models possess an extremely high degree of common variation in their second moments. The first principal component in the

¹ At first glance this may not appear surprising. Many finance theories posit that returns are linear functions of common factors. Prominent examples include the CAPM (Sharpe 1964), ICAPM (Merton 1973), APT (Ross 1976) and the Fama and French (1993) model. If the factors themselves have time-varying volatility, then firm-level volatility will naturally inherit a factor structure as well.

annual panel of *residual* volatilities explains 35% of the panel’s variation.² At the firm-level, there appears to be little distinction between total and idiosyncratic volatility. They possess effectively the same volatility factor structure.

This strong volatility comovement does not arise from omitted common factors – factor model residuals are virtually uncorrelated on average. We further show that comovement in volatilities is not only a feature of returns, but also of the volatility of firm-level cash flows. We estimate volatilities of firm “fundamentals” (sales growth) using quarterly Compustat data. Despite the fact that these volatility estimates are noisier and less frequently observed than stock returns, we again find a strong factor structure among firms’ fundamental volatilities. We also find that the common factor in fundamental volatility follows the same low frequency patterns as the common factor in idiosyncratic return volatilities – the two share a correlation of 64.6%. This suggests that return volatility patterns identified in this paper are not attributable to shocks to investor preferences or other sources of pure discount rate variation, but rather they measure the volatility of persistent idiosyncratic cash flow growth shocks driven by firm-level productivity and demand innovations.

The most important source of persistent, idiosyncratic shocks experienced by households and investors has to be the employer/firm and the labor income that households derive from the firm.³ For example, when workers possess firm-specific capital, shocks to firm value are also shocks to workers’ human wealth. And while firms provide employees with some temporary insurance against idiosyncratic productivity shocks, workers have little protection against persistent shocks⁴ which ultimately affect compensation either through wages or layoffs.⁵

²These findings are symptomatic of the fact that residual volatility accounts for the vast majority of the volatility in a typical stock’s return – 91% at the daily frequency and 67% at the monthly frequency according to the Fama-French model.

³While there many other sources of idiosyncratic risk (e.g., health risk, family-related risk such as divorce), these types of risks are unlikely to have a factor structure in the volatility.

⁴See, e.g., Berk, Stanton, and Zechner (2010) and Lustig, Syverson, and Nieuwerburgh (2011).

⁵Other sources of household exposure to firm-level productivity shocks include under-diversified equity positions in own-employer stock and the influence of firm performance on local residential real estate.

Several pieces of evidence collectively suggest that the common factor in idiosyncratic return variance measures idiosyncratic risk faced by households. For example, individual income data from the U.S. Social Security Administration shows that the cross-sectional dispersion in household earnings growth computed by Guvenen, Ozkan, and Song (2014) rises and falls with the CIV factor in stock returns – annual CIV innovations have a correlation of 53% with changes in household earnings growth dispersion ($t = 3.4$). Similarly, changes in CIV share a correlation of 34% with changes in annual employment growth dispersion ($t = 2.7$) for the set of U.S. public firms in Compustat. Also, sector-level employment data covering both private and publicly-listed firms shows that changes in CIV and changes in the cross-sectional standard deviation of sector-level employment growth have a correlation of 44% ($t = 2.0$).

Households cannot completely insulate their consumption from persistent shocks to their labor income. That is the consensus view in the literature (Blundell, Pistaferri, and Preston (2008)). Heathcote, Storesletten, and Violante (2009) estimate that more than 40% of permanent labor income shocks are passed through to household consumption. As a result, the volatility of the households' consumption growth distribution in equilibrium will inherit the same factor structure as the volatility in firm-level returns and cash-flow growth. As the dispersion of the cross-section of firm-level growth rates rises, investors face more idiosyncratic risk (which they can only imperfectly hedge) and the dispersion of consumption growth rates across investors increases as a result.

Motivated by these empirical facts, we derive the asset pricing implications of idiosyncratic volatility comovement in a heterogeneous agent incomplete markets model. In our specification, idiosyncratic investor consumption growth possesses the same volatility factor structure as firm-level cash flow growth. An increase in the common idiosyncratic volatility of firms represents a deterioration of the investment opportunity set for the average investor, whose individual consumption growth has now become riskier. In our model, CIV indexes the cross-sectional variance of the household consumption growth distribution, which is a priced

state variable because households have a desire to hedge against increases in cross-sectional volatility.

The poor quality of household consumption data has proved a challenge in evaluating the asset pricing predictions of this class of incomplete markets models. Survey data on consumption are not reliable. Koijen, Nieuwerburgh, and Vestman (2013) compare actual to reported consumption data for a panel of Swedish households, and they find large discrepancies. In the U.S., existing sources (PSID and CEX) give produce conflicting pictures of the evolution of consumption inequality. Furthermore, several authors report a growing discrepancy between survey and aggregate consumption data from NIPA (see, e.g., Attanasio, Battistin, and Leicester (2004)).

The incomplete markets model gives us a license to sidestep survey consumption data and instead use innovations to firm-level CIV measured from stock returns as an asset pricing factor.⁶ We focus on the cross-sectional asset pricing implications: Differences in firms' return exposure to CIV shocks translate into differences in expected returns. This prediction is testable by regressing firm-level returns on CIV shocks to arrive at a CIV beta. According to the model, higher CIV beta stocks will have lower returns on average due to their ability to hedge investors' uninsurable consumption risk.

The data strongly supports this prediction. Portfolios formed on past CIV betas capture a large spread in average returns. The top CIV beta quintile earns average returns 6.4% per annum lower than firms in the bottom quintile. We show that this fact is not due to high CIV beta firms having high betas on market variance, nor it is driven by differences in the idiosyncratic variance levels across CIV portfolios.⁷

The last question we tackle is whether the large return spread based on CIV betas is

⁶Given well-known difficulties of measuring the cross-sectional dispersion in investors' consumption growth, (see Vissing-Jorgensen (2002) and Brav, Constantinides, and Geczy (2002) for recent examples) this proxy could be of independent interest to the consumption literature.

⁷Therefore, our findings do not resolve the idiosyncratic volatility puzzle documented by Ang et al. (2006). Rather, we find that the CIV beta spread and the idiosyncratic volatility spread both survive in bivariate sorts.

quantitatively consistent with our model for reasonable parameter values. We calibrate our model to match the cross-sectional spread in CIV betas, as well as the overall equity risk premium on the market portfolio and the risk-free rate. Our calibration respects the observed dispersion in individual consumption growth. In the model, all cross-sectional differences in expected returns arise from heterogeneity in the CIV shock exposure. While not directly targeted, the model generates the observed spread in expected return across CIV beta portfolios. It also generates the right amount of return volatility for these portfolios.

Other Related Literature This paper relates to several strands of research. A range of representative agent models have explored the role of aggregate consumption growth volatility for explaining a host of asset pricing stylized facts. In such models, the representative agent is willing to sacrifice a portion of her expected returns for insurance against a rise in aggregate volatility, but she does not seek to hedge against idiosyncratic volatility which is fully diversifiable – even if idiosyncratic volatility possesses a common factor. As explained by Campbell (1993), aggregate volatility is a priced state variable provided that the agent has a preference for early or late resolution of uncertainty. See Bansal, Kiku, Shaliastovich, and Yaron (2014) and Campbell, Giglio, Polk, and Turley (2014) for two recent examples.⁸

Mankiw (1986) and Constantinides and Duffie (1996) explored counter-cyclical CIV in consumption growth as a mechanism to increase the equilibrium equity premium. Our model follows the approach of Constantinides and Duffie (1996) in modeling investors with an incentive to hedge against increases in idiosyncratic volatility, giving rise to the common component in idiosyncratic volatility as a priced state variable. In our model, investors seek to hedge against idiosyncratic volatility shocks even if they are indifferent about the timing of uncertainty resolution (though early resolution of uncertainty magnifies the price of CIV risk). Constantinides and Ghosh (2013) explore the asset pricing implications of counter-cyclical left-skewness in the cross-sectional distribution of household consumption growth,

⁸The literature on the pricing of market volatility risk includes Coval and Shumway (2001), Ang, Hodrick, Xing, and Zhang (2006), Adrian and Rosenberg (2008), among many others.

but they do not study the association with the common idiosyncratic volatility of firms, which is the focus of our paper. In related work, Storesletten, Telmer, and Yaron (2004) document evidence of counter-cyclical variation in idiosyncratic labor income variance, while Guvenen, Ozkan, and Song (2014) find counter-cyclical skewness in income data.

Idiosyncratic return volatility has been studied in a number of asset pricing contexts. Campbell, Lettau, Malkiel, and Xu (2001) examine secular variation in average idiosyncratic volatility, but do not study its cross section properties.⁹ Wei and Zhang (2006) study aggregate time series variation in fundamental volatility. Bekaert, Hodrick, and Zhang (2012) find comovement in average idiosyncratic volatility across countries. We analyze comovement among volatilities at the firm-level for both returns and fundamentals. Our focus is on the joint dynamics of the entire panel of firm-level volatilities, which we show is a prominent empirical feature of returns and cash flow growth rates. Furthermore, our volatility results are coupled with new asset pricing facts and quantitatively rationalized in an economic model.

Gilchrist and Zakrajsek (2010) and Atkeson, Eisfeldt, and Weill (2013) study the distribution of volatility across firms to understand credit risk and debt prices. Engle and Figlewski (2012) document a common factor in option-implied volatilities since 1996, and Barigozzi, Brownlees, Gallo, and Veredas (2010) and Veredas and Luciani (2012) examine the factor structure in realized volatilities of intra-daily returns since 2001. Bloom, Floetotto, Jaimovich, and Terry (2012) show that firm-specific output growth volatility is broadly counter-cyclical. Kelly, Lustig, and VanNieuwerburgh (2013) propose a simple model in which firms are connected to other firms in a customer-supplier network, and show that size-dependent network formation generates a common factor in firm-level idiosyncratic volatility.

The rest of the paper is organized as follows. Section 2 describes the CIV factor in U.S. stock returns and firm-level cash flows. Section 3 provides evidence linking our return-based

⁹Several papers explore this fact in more detail, such as Bennett, Sias, and Starks (2003), Irvine and Pontiff (2009), and Brandt, Brav, Graham, and Kumar (2010).

CIV factor to dispersion in household income shocks. In section 4, we demonstrate that CIV is a priced factor in the cross-section of stock returns. Section 5 describes the heterogeneous agent model with CIV as priced state variable. Section 6 calibrates the model and discusses its quantitative fit for asset price data. Section 7 concludes.

2 The Factor Structure in Volatility

In this section we study idiosyncratic volatility in the annual panel of U.S. public firms. We first discuss data and how we construct volatilities, then describe the behavior of the volatility panel.

2.1 Data Construction

We construct annual volatility of firm-level returns and cash flow growth. Return volatility is estimated using data from the CRSP daily stock file from 1926-2010. It is defined as the standard deviation of a stock’s daily returns within the calendar year.¹⁰ We refer to these estimates as “total” return volatility.

Idiosyncratic volatility is the focus of our analysis. Idiosyncratic returns are constructed within each calendar year τ by estimating a factor model using all observations within the year. Our factor models take the form

$$r_{i,t} = \gamma_{0,i} + \boldsymbol{\gamma}'_i \mathbf{F}_t + \varepsilon_{i,t} \quad (1)$$

where t denotes a daily observation in year τ . Idiosyncratic volatility is then calculated as the standard deviation of residuals $\varepsilon_{i,t}$ within the calendar year. The result of this procedure is a panel of firm-year idiosyncratic volatility estimates. The first return factor model that we

¹⁰A firm-year observation is included if the stock has a CRSP share code 10, 11 or 12 and the stock has no missing daily returns within the year.

consider is the market model, specifying that \mathbf{F}_t is the return on the CRSP value-weighted market portfolio. The second model specifies \mathbf{F}_t as the 3×1 vector of Fama-French (1993) factors.¹¹ The third return factor model we use is purely statistical and specifies \mathbf{F}_t as the first five principal components of the cross section of returns within the year.¹²

Total fundamental volatility in year τ is estimated for all CRSP/Compustat firms using the 20 quarterly year-on-year sales growth observations for calendar years $\tau - 4$ to τ .¹³ We also estimate idiosyncratic volatility of firm fundamentals based on factor specifications. Since there is no predominant factor model for sales growth in the literature, we only consider principal component factors. The approach is the same as in equation (1), with the exception that the left hand side variable is sales growth and the data frequency is quarterly. \mathbf{F}_t contains the first K principal components of growth rates within a five-year window ending in year τ , where K equals one or five, and residual volatility in year τ is the standard deviation of model residuals over the five-year estimation period. The sales growth volatility panel covers 1975-2010.

2.2 The Cross Section Distribution of Volatility

In Figure 1 we plot histograms of the empirical cross-sectional distribution of firm-level volatility (in logs). Panel A shows the distribution of total return volatility pooling all firm-years from 1926-2010. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution, and each figure reports

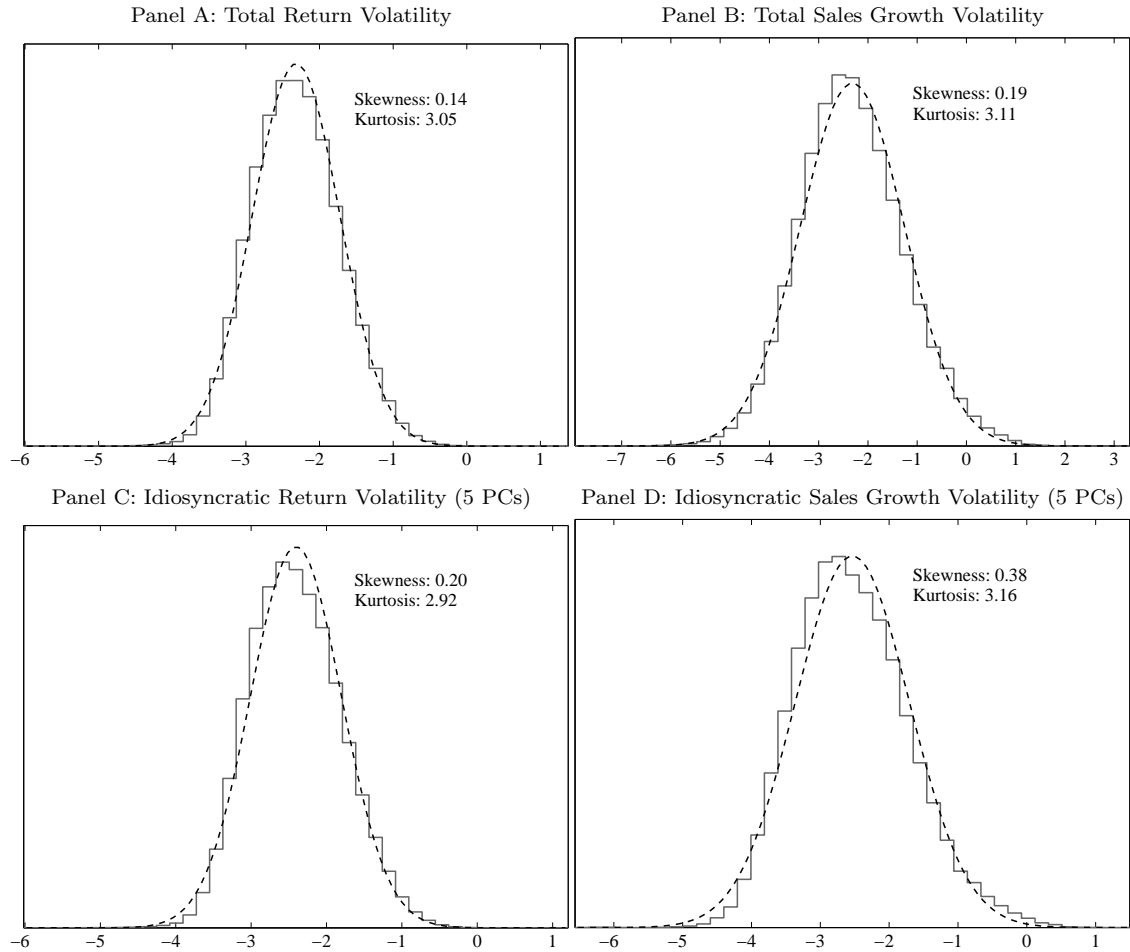
¹¹Our estimates diverge slightly from the standard Fama-French model in which returns in excess of the risk free rate are the left-hand side variables, and the excess market return is the first factor. We use gross returns on the left-hand side, and the gross market return as the first factor.

¹²Robustness tests using 10 PCs produce quantitatively similar results, hence we focus our presentation on 5 PCs. If the true underlying factor structure is non-linear then our regression models are misspecified. In the principal components approach, non-linear dependence can be captured to some extent through the inclusion of additional components. When using five or ten components, we find qualitatively identical results to those from lower-dimension factor models such as Fama-French, which suggests that model misspecification is unlikely to explain our findings. Results are also qualitatively unchanged when we allow for GARCH residuals in factor model regressions.

¹³The data requirements for a non-missing sales growth volatility observation in year τ are analogous to those for returns: We use all Compustat firms linked to a CRSP and possessing share code 10, 11 or 12, and require a firm to have no missing observations in the 20 quarter window ending in year τ .

Figure 1: Log Volatility: Empirical Density Versus Normal Density

The figure plots histograms of the empirical cross section distribution of annual firm-level volatility (in logs) pooling all firm-year observations. Panel A shows total return volatility, calculated as the standard deviation of daily returns for each stock within a calendar year. Panel B shows total sales growth volatility, calculated as the standard deviation of quarterly year-on-year sales growth observations in a 20 quarter window. Panels C and D show idiosyncratic volatility based on the five principal components factor model. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.



the skewness and kurtosis of the data in the histogram. Panel B shows the distribution of log total sales growth volatility pooling all firm-years from 1975-2010. Panels C and D plot histograms of idiosyncratic return and sales growth volatility based on the five principal components factor model.

Our estimates reveal that the cross-sectional distribution of estimated firm-level volatility is lognormal to a close approximation. Log volatilities demonstrate only slight skewness (less

than 0.4) and do not appear to be leptokurtic (kurtosis between 2.9 and 3.2). The cross section volatility distribution also appears lognormal in one-year snapshots of the cross section, as shown in Appendix Figure A1 for the 2010 calendar year. An attractive implication of this result is that dynamics of the entire cross-sectional distribution of firm volatility can be described with only two time-varying parameters: the cross-sectional mean and standard deviation of log volatility.¹⁴

2.3 Common Secular Patterns in Firm-Level Volatility

2.3.1 Return Volatility

Firm-level volatilities share an extraordinary degree of common time variation. Panel A of Figure 2 plots annual firm-level total return volatility, averaged within start-of-year size quintiles. Stocks of all sizes demonstrate very similar secular time series volatility patterns. The same is true of industry groups. Panel B reports average return volatilities among the stocks in the five-industry SIC code categorization provided on Kenneth French’s web site.

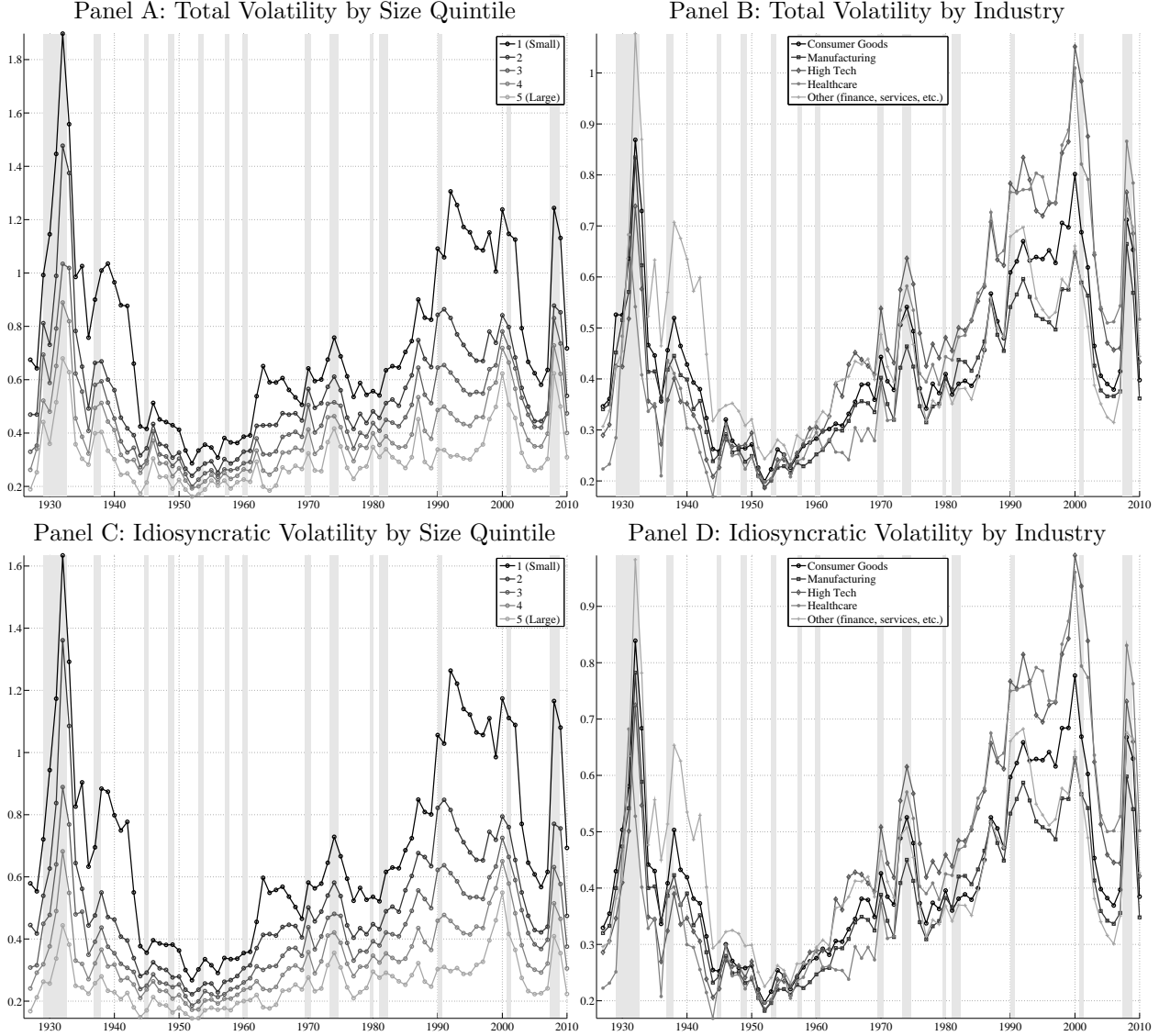
The common time series variation of total return volatilities by size and industry groups is perhaps unsurprising given that firm-level returns are believed to have a substantial degree of common return variation, as evidenced by the predominance of factor-based models of individual stock returns. If returns have common factors and the volatility of those factors varies over time, then firm-level variances will also inherit a factor structure.

What is surprising is that volatilities of residuals display the same degree of common variation after removing common factors from returns. Panels C and D of Figure 2 plot average idiosyncratic volatility within size and industry groups based on residuals from a five principal components factor model for returns. The plots show that the same dynamics appear for all groups of firms when considering idiosyncratic rather than total volatility.

¹⁴The distribution of idiosyncratic return volatility estimated from the market model and the Fama-French three-factor model are qualitatively identical to those shown in Figure 1.

Figure 2: Total and Idiosyncratic Return Volatility by Size and Industry Group

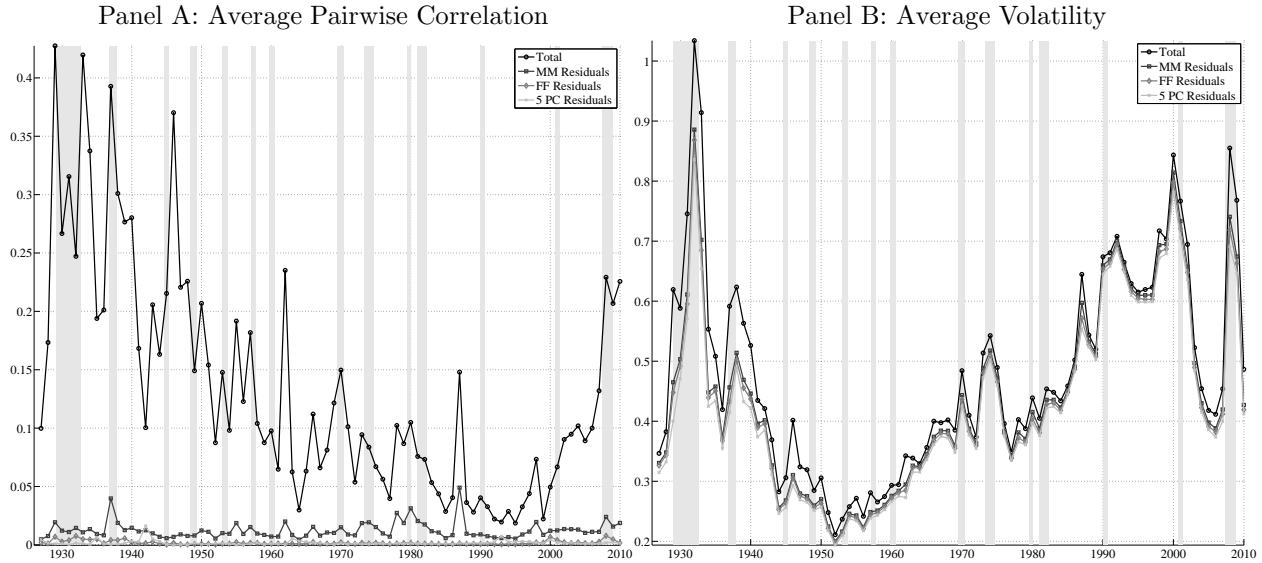
The figures plot annualized firm-level volatility averaged within size and industry groups. Within each calendar year, volatilities are estimated as the standard deviation of daily returns for each stock. Panel A shows firm-level total return volatility averaged within market equity quintiles. Panel B shows total return volatility averaged within the five-industry categorization of SIC codes provided on Kenneth French's web site. Panels C and D report the same within-group averages of firm-level idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals from a five factor principal components model for daily returns.



The correlation between average idiosyncratic volatility within size quintiles one and five is 81%. The lowest correlation among idiosyncratic volatilities of the five industry groups is 65%, which corresponds to the health care industry versus the “other” category (including construction, transportation, services, and finance).

Figure 3: Volatility and Correlation of Return Factor Model Residuals

Panel A shows the average pairwise correlation for total and idiosyncratic returns within each calendar year. Panel B shows the cross-sectional average annualized firm-level volatility each year for total and idiosyncratic returns. Idiosyncratic volatility is the standard deviation of residuals from the market model, the Fama-French three-factor model, or a five factor principal components model for daily returns within each calendar year.



This common variation in idiosyncratic volatility cannot be explained by comovement among factor model residuals, for instance due to omitted common factors. Panel A of Figure 3 shows that raw returns share substantial common variation, with an average pairwise correlation of 13% over the 1926-2010 sample, and occasionally exceeding 40%. However, the principal components model captures nearly all of this common variation at the daily frequency, as average correlations among its residuals are typically less than 0.2%, and are never above 0.9% in a year. The same is true for the market and Fama-French models. Moving to a higher number of principal components, such as 10, has no quantitative impact on these results. Indeed, the Fama-French model and the five principal component model appear to absorb all of the comovement in returns, making omitted factors an unlikely explanation for the high degree of commonality in idiosyncratic volatilities.

Despite the absence of comovement among residual return realizations, Panel B of Figure 3 shows that average idiosyncratic volatility from various factor models is nearly the same as

average volatility of total returns. In the typical year, only 11% of average total volatility is accounted for by the five principal components factor model, while average log idiosyncratic volatility inherits 89% of the average total volatility level. The same is true for the market model and Fama-French model, with 8% and 9% of average volatility explained by common factors respectively.¹⁵

The analyses in Figures 2 and 3 use return volatility estimated from daily data within each year.¹⁶ We find very similar results when estimating volatility from 12 monthly return observations within each year. First, firm-level total and idiosyncratic return volatilities share a high degree of comovement. The average pairwise correlation of idiosyncratic volatility among size quintiles is 84% annually, while the average correlation of idiosyncratic volatility for industry groups is 83% (based on residuals from the five principal component model). Second, the vast majority of correlation among monthly returns is absorbed by common factors. Total return correlations are 30% on average based on monthly data, dropping to 0.4% for monthly Fama-French model residuals. Third, most of the average firm’s volatility is left unexplained by common factors – the ratio of average Fama-French residual volatility to average total volatility is 67%. See Appendix Figure A2 for additional detail.

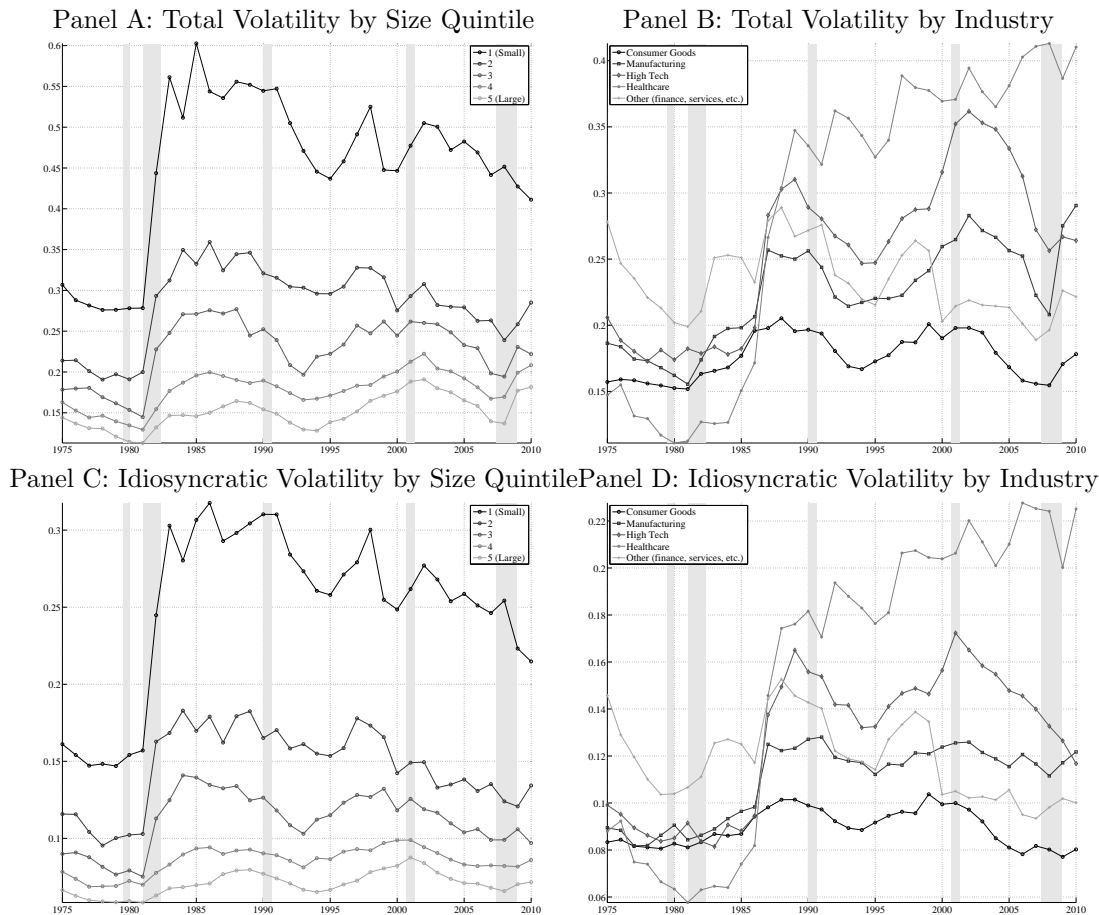
The strong comovement of return volatility is also a feature of portfolio returns. Figure A3 in the Appendix reports average volatility and average pairwise correlations for total and residual returns for 100 Fama-French size and value portfolios. Portfolio return volatilities show a striking degree of comovement across the size and book-to-market spectrum, even after accounting for common factors. Like the individual stock results above, factor models remove the vast majority of common variation in returns, thus common volatility patterns are unlikely to be driven by omitted common return factors.

¹⁵We calculate the percent of average volatility explained by common factors as one minus the ratio of average factor model residual volatility to average total volatility, where averages are first computed cross-sectionally, then averaged over the full 1926-2010 sample.

¹⁶It is standard practice in the literature to estimate idiosyncratic volatility from daily data. See, for example, Ang et al. (2006).

Figure 4: Log Total and Idiosyncratic Sales Growth Volatility by Size and Industry Group

The figures plot firm-level log volatility averaged within size and industry groups. Within each calendar year, volatilities are estimated as the standard deviation of four quarterly year-on-year sales growth observations for each stock. Panel A shows firm-level log total volatility averaged within market equity quintiles. Panel B shows log total return volatility averaged within the five-industry categorization of SIC codes provided on Ken French's website. Panels C and D report the same within-group averages of firm-level log idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals (four observation within each year) from a five factor principal components model for quarterly sales growth. The components are estimated in a five year rolling window ending in the year that the residual volatility is calculated.

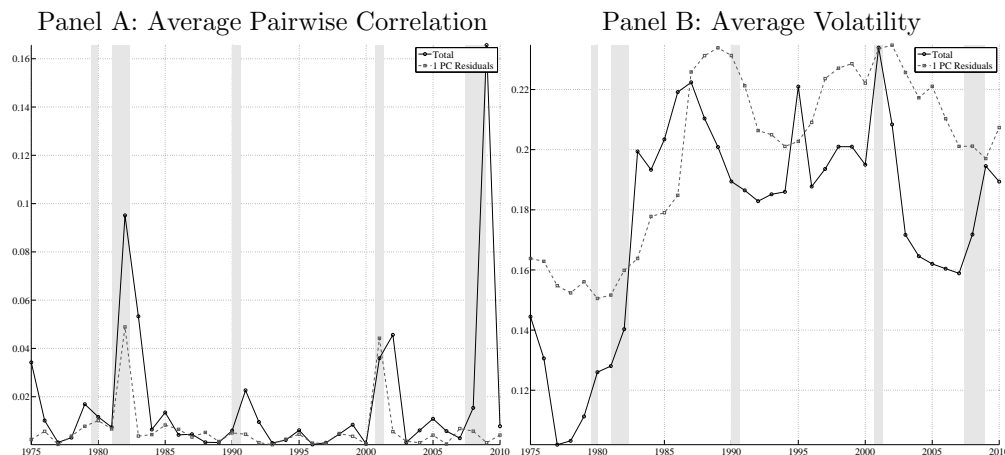


2.3.2 Fundamental Volatility

Figure 4 reports average yearly sales growth volatility by size quintile and five-industry category in Panels A and B. As in the case of returns, firm sales growth data display a high degree of volatility commonality – the average pairwise correlation among size and industry groups is 85% and 53%, respectively. Panels C and D show within-group average

Figure 5: Volatility and Correlation of Total and Idiosyncratic Sales Growth

Panel A shows the average pairwise correlation for total and idiosyncratic sale growth within a 20 quarter window through the end of each calendar year. Panel B shows cross section average firm-level volatility each year for total and idiosyncratic sales growth. Idiosyncratic volatility is the standard deviation of residuals from a one factor principal components model for quarterly sales growth. The components are estimated in a five year rolling window ending in the year that the residual volatility is calculated.



idiosyncratic volatility estimated from a five factor principal components model for sales growth. Comovement in residual sales growth volatility is nearly identical to total sales growth volatility. This is because total sales growth rates have minimal average pairwise correlation (1.6% annually), as shown in Figure 5. After accounting for one sales growth principal component, the average pairwise correlation drops to 0.7%. This fundamental volatility behavior is not specific to sales growth, but is also true of other measures of firm fundamentals such as cash flows. For example, the average variance of firms' net income growth is 67.4% correlated with that of sales growth, and the common net income volatility factor explains panel variation in firm-level volatility with an R^2 of 28.5% on average.¹⁷ In summary, strong comovement is not unique to return volatilities, but also appears to be a feature of fundamental volatility.

¹⁷See appendix Table A1 Panels B and C for volatility factor model estimates based on net income growth and EBITDA growth.

2.4 Volatility Factor Model Estimates

We next estimate one-factor models for firm-level volatility. We consider total volatility as well as idiosyncratic volatility estimated from a Fama-French three factor model or a five factor principal component model. In all cases, time series regressions are run firm-by-firm with volatility as the left-hand side variable. The factor in each set of regressions is defined as the equal-weighted average of the left-hand size volatility measure. This is approximately equal to the first principal component of a given volatility panel, but avoids the principal components complications arising from unbalanced panels.

Panel A of Table 1 reports volatility factor model results for daily return volatilities. Columns correspond to the method used to construct return residuals. The average univariate time series R^2 is 36.2% for the total volatility model, and close to 35% for the idiosyncratic volatility models. The pooled panel OLS R^2 is between 33% and 35% (relative to a volatility model with only firm-specific intercepts). For portfolios rather than individual stock returns, we find even higher volatility factor model R^2 values. Based on the Fama-French 100 size and value portfolios, the average univariate R^2 is 70.8% for total volatility, 49.7% for market model residual volatility, and 39.4% for Fama-French model residual volatility (see Panel A of appendix Table A1).

Panel A of Figure 6 plots the level of average annualized idiosyncratic volatility (denoted CIV) against the volatility of the value-weighted market portfolio (MV). The series possess substantial common variation, particularly associated with the deep recessions at the beginning and end of the sample (correlation of 63.8% in levels). Panel B reports changes in CIV, as well as residuals from a regression of CIV changes on changes in MV. The two sets of innovations share a correlation of 67.0%, indicating that the behavior of idiosyncratic volatility is in large part distinct from that of market volatility. The asset pricing tests of the next section document important additional differences in the behavior of CIV and MV.

In Panel B of Table 1, we show volatility factor model estimates for sales growth volatility.

Table 1: Volatility Factor Model Estimates

The table reports estimates of annual volatility one-factor regression models. In each panel, the volatility factor is defined as the equal-weighted cross section average of firm volatilities within that year. That is, all estimated volatility factor models take the form: $\sigma_{i,t} = \text{intercept}_i + \text{loading}_i \cdot \bar{\sigma}_{.,t} + e_{i,t}$. Columns represent different volatility measures. For returns (Panel A), the columns report estimates for a factor model of total return volatility and idiosyncratic volatility based on residual returns from the market model, the Fama-French model, or the five principal component models. For sales growth (Panel B), the columns report total volatility or idiosyncratic volatility based on sales growth residuals from the one and five principal components model (using a rolling 20 quarter window for estimation). The last column of Panel B reports results when using only the four quarterly growth observations within each calendar year to estimate total volatility. We report cross-sectional averages of loadings and intercepts as well as time series regression R^2 averaged over all firms. We also report a pooled factor model R^2 , which compares the estimated factor model to a model with only firm-specific intercepts and no factor.

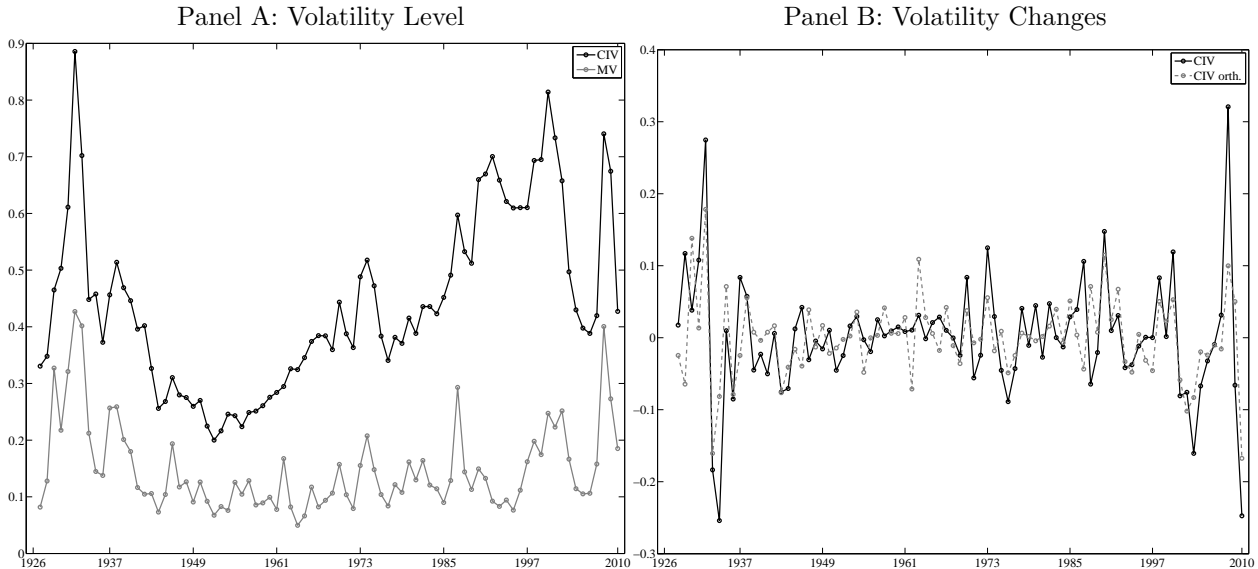
	Panel A: Returns			
	Total	MM	FF	5 PCs
Loading (average)	1.012	1.024	1.032	1.031
Intercept (average)	0.006	0.005	0.004	0.004
R^2 (average univariate)	0.362	0.347	0.346	0.348
R^2 (pooled)	0.345	0.337	0.339	0.347

	Panel B: Sales Growth			
	Total (5yr)	1 PC (5yr)	5 PCs (5yr)	Total (1yr)
Loading (average)	0.885	1.149	1.249	0.884
Intercept (average)	0.044	-0.018	-0.024	0.030
R2 (average univariate)	0.293	0.299	0.299	0.178
R2 (pooled)	0.303	0.315	0.304	0.168

The first three columns report results based on panels of total volatility and idiosyncratic volatility from one and five principal component models. The last column reports model estimates for an annual volatility panel that estimates volatility from only four quarterly year-on-year sales growth observations within each year (as opposed to the rolling 20 quarter window used elsewhere). Due to the small number of observations used to construct sales growth volatility, we might expect poorer fit in these regressions, yet the results are closely in line with those for return volatility. The time series R^2 for raw and idiosyncratic growth rate volatility ranges between 17.8% and 29.9% on average. The pooled R^2 reaches as high as 31.5%.

Figure 6: Common Idiosyncratic Volatility: Levels and Changes

Panel A plots the average annual volatility of the CRSP value-weighted market return (MV) and the cross-sectional average volatility of market model residuals (CIV). Panel B plots annual changes in CIV, and CIV changes orthogonalized against annual changes in MV.



We also find that the common factor in fundamental volatility follows the same low frequency patterns as the common factor in idiosyncratic return volatilities, sharing a correlation of 64.6% with the common factor in Fama-French residual return volatility. This suggests that return volatility patterns identified in this section are not attributable to discount rate shocks, but rather they measure the volatility of persistent idiosyncratic cash flow growth shocks at the firm level. If the shocks were largely transitory, they would have only a minor impact on returns.

3 Idiosyncratic Risk of the Firm and the Household

The evidence presented in Section 2 indicates that firm-level idiosyncratic volatilities possess a high degree of comovement that is aptly described by a factor model. The commonality in firms' idiosyncratic risks hints at the possibility that income and consumption growth realizations experienced by households, while idiosyncratic, are also subject to common vari-

ation in their second moments. That is, households may face common fluctuations in their idiosyncratic *risks*, even though their individual consumption growth realizations themselves may be (conditionally) independent. This seems plausible since shocks to households labor income, human capital and financial capital derive in large part from shocks to their employers.

In this section we investigate the empirical association between fluctuations in firm-level idiosyncratic volatility and idiosyncratic consumption and income risk faced by households. Individual household consumption and individual firm performance are potentially linked through a number of channels. First, households may be directly exposed to the equity risk of their employers. A large theoretical literature beginning with Jensen and Meckling (1976) predicts that management will hold under-diversified positions in their employers' stock for incentive reasons. This prediction is born out empirically (Demsetz and Lehn (1985), Murphy (1985), Morck, Shleifer, and Vishny (1988), Krole (1995), and others). Benartzi (2001) and Cohen (2009) show that non-manager employees also tend to overallocate wealth to equity of their employer and offer behavioral interpretations for this phenomenon.¹⁸

Firm-specific human capital (Becker 1962) is a second potential channel tying household idiosyncratic outcomes to those of the firm. As noted by Hashimoto (1981), "The standard analysis of firm-specific human capital argues that the cost of and the return to the investment will be shared by the worker and the employer." The typical employee's wealth is dominated by her human capital (Lustig, Van Nieuwerburgh, and Verdelhan 2013), implying that shocks to an employer's firm value and (human) wealth shocks of its employees move in tandem. This mechanism is empirically documented in Neal (1995) and Kletzer (1989, 1990). These studies also emphasize that the firm-specific human capital mechanism leads to protracted and potentially permanent income impairment following job displacements, consistent with the evidence in Ruhm (1991) and Jacobson, LaLonde, and Sullivan

¹⁸A large literature including French and Poterba (1991), Coval and Moskowitz (1999) and Calvet, Campbell, and Sodini (2007) indicate that the typical household is exposed to idiosyncratic stock risk above and beyond that due to overallocation of savings to the equity of one's employer.

(1993). Furthermore, the probability of job displacement, defined by Kletzer (1998) as “a plant closing, an employer going out of business, a layoff from which he/she was not recalled,” is directly tied to firm performance. In addition to job loss risk, the empirical findings of Brown and Medoff (1989) suggest that employees at larger firms enjoy a wage premium, presenting a mechanism through which employee income shocks may be correlated with idiosyncratic firm shocks. Theoretical work of skilled labor compensation in Harris and Holmstrom (1982), Berk, Stanton, and Zechner (2010) and Lustig, Syverson, and Nieuwerburgh (2011) finds that employers optimally insure some, but not all productivity shocks, leaving employee compensation subject to firm-level shocks.

Our empirical analysis suggests that common idiosyncratic return volatility is a plausible proxy for idiosyncratic risk faced by individual consumers. We present four new results consistent with this interpretation.

Our first result documents a significant association between shocks to firm-level idiosyncratic risk and data on idiosyncratic household income risk. Our measure of idiosyncratic firm risk is the equally-weighted average of firm-level market model residual return variance (denoted CIV in Figure 7).¹⁹ The highest quality household income growth data come from the U.S. Social Security Administration. While not publicly available, Guvenen, Ozkan, and Song (2014) report cross-sectional summary statistics each year from 1978-2011. The most relevant statistic to our analysis are the standard deviation and the difference between the 90th and 10th percentiles of the cross-sectional earnings growth distribution for male individuals (see Table A.8 of their data appendix for additional details). A large literature documents that shocks to individual labor income growth translate into shocks to individual consumption growth because of incomplete risk sharing.²⁰ In Figure 7, we plot yearly changes in CIV alongside yearly changes in the standard deviation and interdecile range of

¹⁹Results are quantitatively similar and qualitatively the same if using an alternative definition such as variance of Fama-French model residuals. We compute a monthly, quarterly and annual version of average idiosyncratic variance to conform with various data sources that are available at one these frequencies.

²⁰Blundell, Pistaferri, and Preston (2008) and Heathcote, Storesletten, and Violante (2009) show that permanent shocks to labor income end up in consumption, while transitory shocks are partially insurable.

Figure 7: CIV and Individual Consumer Income Growth

The figure compares yearly changes in CIV with yearly changes in the standard deviation and interdecile range of the individual earnings growth distribution. CIV is the equal-weighted average of firm-level market model residual return variance each year. Individual earnings data is from the U.S. Social Security Administration and summarized by Guvenen, Ozkan, and Song (2014). Each series is standardized to have equal mean and variance for ease of comparison.



the earning growth distribution (all series are standardized to have zero mean and unit variance for comparison). CIV innovations have a correlation of 53.2% ($t = 3.4$) with changes in idiosyncratic income risk measured by the cross-sectional standard deviation of earnings growth, and a correlation of 43.2% ($t = 2.6$) with changes in the interdecile range.

Our second finding is that changes in CIV are significantly associated with employment risk. We calculate firm-level employment growth rates growth for U.S. publicly-listed firms from 1975-2010 using Compustat data for number of employees. Then, to proxy for employment risk, we calculate the cross-sectional interquartile range of employment growth rates each year. Changes in CIV share a correlation of 33.5% with changes in employment growth dispersion ($t = 2.7$).²¹ The drawback of Compustat data is its restriction to the universe

²¹We use the interquartile range to avoid undue influence of extreme observations in the employment growth distribution. Results are qualitatively similar but only marginally significant if we measure employ-

of public firms. The Federal Reserve reports monthly total employment for over 100 sectors beginning in 1991, aggregating both private and public firms, and we use this data to calculate log employment growth for each sector-year. Changes in CIV and changes in the cross-sectional standard deviation of sector-level employment growth have a correlation of 44.2% ($t = 2.0$).

A large fraction of household wealth is invested in residential real estate, leaving individuals exposed to idiosyncratic wealth shocks deriving from fluctuations in the value of their homes.²² Local house prices also reflect local labor market conditions (Van Nieuwerburgh and Weill 2010). Our third and fourth findings relate CIV to the cross-sectional dispersion of house price growth and wage-per-job growth across metropolitan areas. House price data are from the Federal Housing Financing Agency and wage data from NIPA’s Regional Economic Information System. The merged data set contains annual information from 1969-2009 for 386 regions. The correlation between innovations in CIV and innovations in the cross-sectional standard deviation of house price growth is 23.2% per quarter ($t = 2.6$), and the correlation with innovations in the cross-sectional standard deviation of per capita wage growth is 16.6% per quarter ($t = 1.9$). This evidence offers further support of a link between the cross-sectional income distribution of firms and of households.

4 CIV and Expected Stock Returns

In this section, we document that firms’ exposures to CIV shocks help explain cross-sectional differences in average stock returns. Then, Section 5 rationalizes these asset pricing findings and the empirical association between CIV and household income risk in an equilibrium incomplete markets model with heterogeneous agents.

ment risk with standard deviation rather than interquartile range – the correlation with CIV changes is 23.7% ($t = 1.8$).

²²For the median household with positive primary housing wealth in the 2010 wave of the Survey of Consumer Finance, the primary residence represents 61% of all assets. For 25% of households it represents 90% or more of all assets.

Our asset pricing analysis is conducted using monthly returns, so the results of this section use the monthly version of common idiosyncratic variance described earlier.²³ Each month, we estimate a regression of daily individual firm returns on the value-weighted market return for all CRSP firms with non-missing data that month. We then calculate CIV in levels as the equal-weighted average of market model residual variance across firms. From here we construct monthly CIV changes and orthogonalize these changes against changes in the monthly variance of the market portfolio. The residuals from this regression serve as the asset pricing factor for our empirical tests. The orthogonalization disentangles our CIV exposures from the market variance exposures studied by Ang, Hodrick, Xing, and Zhang (2006).

For each month from January 1963 until December 2010, we regress monthly individual firm stock returns on orthogonalized CIV innovations using a trailing 60-month window.²⁴ Based on these CIV betas, we sort stocks into CIV quintile portfolios and construct value-weighted portfolio returns over the subsequent month.

Value-weighted average returns on CIV beta-sorted portfolios are reported in Table 2. The first row of Panel A shows that average returns are decreasing in CIV beta. Stocks in the first quintile have low/negative CIV betas and thus tend to lose value when CIV rises. In contrast, stocks in Q5 tend to hedge CIV rises, paying off in high volatility states. The monthly spread between highest and lowest quintiles is -6.4% (annualized) with a t-statistic of -3.4 . The spread in average returns is robust to controlling for market returns, SMB, and HML, as shown in rows 2 and 3 of Panel A.

In Panel B, we report average returns for portfolios sorted independently on CIV beta and market variance beta, where rows correspond to the market variance beta dimension.²⁵

²³The monthly nature of our return tests also highlight an attractive feature of CIV estimated from returns – it is a plausible proxy for idiosyncratic household income risk while being easily observable at high frequencies.

²⁴We use all CRSP stocks with share codes 10, 11, and 12, and include a stock in portfolio sorts if it had no missing monthly returns in the 60-month estimation window.

²⁵Our market variance beta calculation exactly mirrors the rolling CIV beta calculation described above.

Table 2: Value-weighted Portfolios Formed on CIV Beta

The table reports average return and alphas (in annual percentages) for value-weighted portfolio sorts on the basis of monthly CIV beta for the 1963-2010 sample. Panel A reports average returns and alphas in one-way sorts using all CRSP stocks. Panel B shows average returns in independent two-way sorts on CIV beta and market variance beta. Panel B shows average returns in independent two-way sorts on CIV beta and idiosyncratic stock variance.

	CIV beta						
	1 (Low)	2	3	4	5 (High)	5-1	$t(5-1)$
Panel A: One-way sorts on CIV beta							
$E[R]$	15.23	12.39	11.71	10.55	8.80	-6.44	-3.42
α_{CAPM}	3.38	1.47	1.14	0.27	-1.95	-5.33	-2.91
α_{FF}	2.32	0.84	0.94	0.22	-1.97	-4.28	-2.33
Panel B: Two-way sorts on CIV beta and MV beta							
MV beta							
1 (Low)	16.05	14.50	11.72	11.60	9.37	-6.69	-2.55
2	14.47	13.42	11.55	11.49	10.25	-4.22	-1.91
3	16.67	12.98	13.51	11.27	10.91	-5.76	-2.48
4	17.17	11.26	10.81	9.26	9.12	-8.05	-2.95
5 (High)	14.48	12.88	10.84	10.86	8.72	-5.76	-1.96
5-1	-1.57	-1.63	-0.87	-0.73	-0.64		
$t(5-1)$	-0.54	-0.52	-0.29	-0.25	-0.22		
Panel C: Two-way sorts on CIV beta and idiosyncratic variance							
Idios. var.							
1 (Low)	14.11	11.83	12.69	11.45	8.57	-5.54	-2.70
2	15.88	12.94	12.19	10.47	9.81	-6.07	-3.00
3	17.00	14.90	12.30	11.83	10.66	-6.34	-2.69
4	15.10	12.77	10.96	11.56	9.73	-5.37	-2.37
5 (High)	7.13	8.10	7.04	3.74	1.19	-5.94	-2.14
5-1	-6.98	-3.73	-5.65	-7.71	-7.38		
$t(5-1)$	-2.05	-1.08	-1.53	-2.14	-1.94		

This provides a comparison with the market variance factor tests of Ang, Hodrick, Xing, and Zhang (2006). High CIV beta stocks continue to earn substantially lower average returns within each market beta quintile. The Q5 minus Q1 CIV beta spread ranges from -4.2% to -8.0% per year depending on the market variance beta quintile, and is significant for all

market beta quintiles (at the 5% level for four quintiles out of five and at the 10% level for the other). CIV betas and market betas are distinct characteristics: Their cross section correlation, averaged over the years 1963-2010, is 2.5%.

Panel C reports bivariate sorts based on CIV beta and stock-level idiosyncratic volatility, providing a comparison to the idiosyncratic volatility puzzle of Ang, Hodrick, Xing, and Zhang (2006). The CIV beta is between -5.4% and -6.3% per year depending on the idiosyncratic volatility quintile, and has a t -statistic of at least two in all idiosyncratic volatility quintiles. The idiosyncratic volatility effect survives double sorts as well, and is significant at the 10% level in three out of five CIV beta quintiles. Thus, the cross-sectional return pattern we document is to a large extent complementary to the idiosyncratic volatility return pattern.

We report a range of robustness tests in the appendix. Table A2 reports similar findings for equal-weighted rather than value-weighted portfolio returns. Table A3 Panels A and B show results for the 1985-2010 and 1963-1985 subsamples that are consistent with our findings in Table 2, though the Fama-French alpha loses significance in the early subsample. Panels C and D of Table A3 provide further evidence that the results are not due to CIV betas proxying for market variance betas. Results are only slightly weaker if CIV changes are not orthogonalized against changes in market variance (Panel C), while one-way sorts on market variance beta alone produce no discernible spread in average returns (Panel D). Together, these imply that variation in common idiosyncratic volatility, as opposed to market volatility, is the primary driver of average return differences in Table 2.

5 Model with CIV in Household Consumption

The null of perfect insurance is soundly rejected in household consumption data. Instead, households only partially hedge their consumption and labor income shocks.²⁶ Motivated by this and the CIV facts presented in Sections 3 and 4, we develop an incomplete markets model in which investors' income and consumption volatility are exposed to a common idiosyncratic volatility factor.²⁷

Uninsurable idiosyncratic risk simply lowers the risk-free rate in a large class of incomplete market models and need not affect risk premia per se.²⁸ However, if idiosyncratic risk is endowed with a factor structure that covaries with aggregate outcomes, then risk premia may be affected as well. A large theoretical literature has explored counter-cyclical variation in idiosyncratic risk as a potential explanation for the equity premium puzzle and other asset pricing phenomena, exemplified by Mankiw (1986) and Constantinides and Duffie (1996).²⁹ We explore the quantitative asset pricing implications of CIV in household consumption growth for the cross-section of stock returns. While we allow for counter-cyclical variation in the CIV, it is not crucial for our results.

In our model, the common idiosyncratic volatility factor, denoted σ_{gt}^2 , is the key state variable driving residual stock return volatility and household consumption growth. This link is supported by the empirical analysis in Section 2. We show that innovations to this factor carry a negative price of risk. Stocks with more negative exposure with respect to this innovation (a more negative CIV beta) in turn carry a higher risk premium. To keep the

²⁶E.g., Cochrane (1991) and Attanasio and Davis (1996). In particular, a large fraction of permanent income innovations are passed through to consumption (Blundell, Pistaferri, and Preston (2008)).

²⁷We abstract from changes in the persistence of labor income shocks (Blundell, Pistaferri, and Preston (2008)) or changes in the risk sharing technology (e.g., Krueger and Perri (2006) and Lustig and Van Nieuwerburgh (2005)) which may drive a wedge between the cross-sectional volatility of consumption and labor income growth at lower frequencies.

²⁸E.g. Grossman and Shiller (1982) and Krueger and Lustig (2010).

²⁹These models possess two key features. First, consumers are heterogeneous and their differences are summarized by the cross-sectional distribution of individual consumption growth, and second consumption insurance markets are incomplete. Related theoretical analyses of asset prices in heterogeneous agents setting with imperfect insurability of idiosyncratic consumption risk include Lucas (1994), Heaton and Lucas (1996), Storesletten, Telmer, and Yaron (2007), Lustig and Van Nieuwerburgh (2007), among others.

model simple and to highlight the differences with the existing literature, aggregate stock market volatility and aggregate consumption growth volatility are assumed to be constant over time.

5.1 Preferences

There is a unit mass of atomless agents, each having Epstein-Zin preferences. Let $U_t(C_t)$ denote the utility derived from consuming C_t . The value function of each agent takes the following recursive form:

$$U_t(C_t) = \left[(1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}.$$

where $\theta \equiv (1 - \gamma)/(1 - \frac{1}{\psi})$. The time discount factor is δ , the risk aversion parameter is $\gamma \geq 0$, and the inter-temporal elasticity of substitution (IES) is $\psi \geq 0$. When $\psi > 1$ and $\gamma > 1$, then $\theta < 0$ and agents prefer early resolution of uncertainty.

5.2 Technology

Aggregate labor income is defined as I_t . There is a large number of securities in zero or positive net supply. The combined total (and per capita) dividends is D_t . Aggregate dividend income plus aggregate labor income equals aggregate consumption: $C_t = I_t + D_t$. Individual consumption is given by $S_t^j C_t$, where S^j denotes agent j 's consumption share, and individual labor income is defined by

$$I_{j,t} = S_t^j C_t - D_t$$

All agents can trade in all securities at all times and are endowed with an equal number of all securities at time zero. Labor income risk is, however, uninsurable. As in Constantinides and Ghosh (2013), given the symmetric and homogeneous preferences, households choose not to trade away from their initial endowments. That is, autarky is an equilibrium and

individual j 's equilibrium consumption is $C_{j,t} = I_{j,t} + D_t = S_t^j C_t$.

Following Constantinides and Duffie (1996), we think of this consumption process $C_{j,t}$ as the post-trade consumption that obtains after households have exhausted all insurance options and the temporary innovations to labor income have been smoothed out. What remains are the permanent innovations to income that are not directly insurable and are not insured by the government. These permanent shocks are then passed on to consumption.³⁰

We use lowercase symbols to denote logs. We impose the same idiosyncratic volatility factor structure on investor consumption growth and firm dividend growth by adopting the following specification for consumption growth in aggregate and for each agent j :

$$\Delta c_{t+1}^a = \mu_g + \sigma_c \eta_{t+1} + \phi_c \sigma_g w_{g,t+1} \quad (2)$$

$$\Delta s_{t+1}^j = \sigma_{g,t+1} v_{t+1}^j - \frac{1}{2} \sigma_{g,t+1}^2 \quad (3)$$

$$\sigma_{g,t+1}^2 = \sigma_g^2 + \nu_g (\sigma_{gt}^2 - \sigma_g^2) + \sigma_w \sigma_g w_{g,t+1}. \quad (4)$$

All shocks are i.i.d. standard normal and mutually uncorrelated. Individual consumption growth is $\Delta c_{t+1}^j = \Delta c_{t+1}^a + \Delta s_{t+1}^j$. While aggregate consumption growth is homoscedastic, household consumption growth is not. The cross-sectional mean and variance of the consumption share process are:

$$\mathbb{E}_j [\Delta s_{t+1}^j] = -\frac{1}{2} \sigma_{g,t+1}^2, \quad \mathbb{V}_j [\Delta s_{t+1}^j] = \sigma_{g,t+1}^2.$$

where $\mathbb{E}_j [\cdot]$ and $\mathbb{V}_j [\cdot]$ are expectation and variance operators over the cross-section of households. Thus, the process $\sigma_{g,t+1}$ measures the cross-sectional standard deviation of consumption share growth.³¹ The mean consumption share in levels is one ($\mathbb{E}_j [S_t^j] = 1$).

³⁰As mentioned above, this is borne out in the data. See Blundell, Pistaferri, and Preston (2008) and Heathcote, Storesletten, and Violante (2009).

³¹Our results are robust to changes in the timing of the consumption share growth process in equation (3). The consumption share growth dispersion could be either $\sigma_{g,t}$ or $\sigma_{g,t+1}$. This changes the expressions in the model as well as the calibration, but it doesn't affect our quantitative results. We use the current timing, because dividend growth in equation 5 depends on the lagged consumption share growth dispersion,

We only allow for permanent innovations. Both individual consumption growth and firm dividend growth follow random walk processes. These innovations are the hardest to insure against for households, and they are the main drivers of returns for firms.

Aggregate consumption growth may be exposed to the innovation in the cross-sectional standard deviation. That is, when $\phi_c < 0$, our model activates the standard Mankiw (1986) and Constantinides and Duffie (1996) mechanism: counter-cyclical cross-sectional variation in idiosyncratic risk. We allow for this channel, but it is not crucial for our results.

Dividend growth of firm i is given by:

$$\Delta d_{t+1}^i = \mu_i + \chi^i (\sigma_{gt}^2 - \sigma_g^2) + \varphi_i \sigma_c \eta_{t+1} + \phi_i \sigma_g w_{g,t+1} + \kappa_i \sigma_{gt} e_{t+1}^i + \zeta_i \sigma_{it} \varepsilon_{t+1}^i \quad (5)$$

$$\sigma_{i,t+1}^2 = \sigma_i^2 + \nu_i (\sigma_{it}^2 - \sigma_i^2) + \sigma_{iw} w_{i,t+1}. \quad (6)$$

Dividend growth for each stock i is subject four shocks. The first two are the systematic shocks that drive aggregate consumption growth ($\sigma_c \eta_{t+1}$ and $\sigma_g w_{g,t+1}$). The second two are idiosyncratic. The $\sigma_{gt} e_{t+1}^i$ shock captures common idiosyncratic volatility at the firm level. It is orthogonal to all other shocks in the economy yet shares a common volatility. The $\sigma_{it} \varepsilon_{t+1}^i$ shock allows for firm-specific idiosyncratic variance. Taking these two shocks together, firm i 's idiosyncratic risk is $\mathbb{V}_j [\kappa_i \sigma_{gt} e_{t+1}^i + \zeta_i \sigma_{it} \varepsilon_{t+1}^i] = \kappa_i^2 \sigma_{gt}^2 + \zeta_i^2 \sigma_{it}^2$, capturing the factor structure in idiosyncratic variance documented in Section 2.

Idiosyncratic shocks drop out of the market portfolio's dividend process. The conditional variance of dividend growth on the market portfolio is therefore constant because the first two shocks have constant variances. All time variation in the conditional variance of dividend growth of stock i arises from time variation in its idiosyncratic variance. Evidently, the σ_{gt}^2 process is proportional to the common idiosyncratic variance factor, hence the model links the CIV factor to the cross-sectional dispersion of consumption share growth. We now show that positive innovations in the CIV factor ($w_{g,t+1} > 0$) are associated with a deterioration σ_{gt} , and not on its contemporaneous value. This is consistent with CIV as state variable.

in the investment opportunity set and carry a negative price of risk. Assets whose returns are low exactly when the cross-sectional volatility is high must pay higher risk premia.

5.3 Claim to Individual Consumption Stream

We start by pricing a claim to individual consumption growth, using the individual's own intertemporal marginal rate of substitution. We conjecture that the log wealth-consumption ratio of agent j is linear in the state variable σ_{gt}^2 , and does not depend on any agent-specific characteristics: $wc_t^j = \mu_{wc} + W_{gs} (\sigma_{gt}^2 - \sigma_g^2)$. We verify this conjecture evaluating the Euler equation for the consumption claim of agent j : $E_t[M_{t+1}^j R_{t+1}^j] = 1$, where M_{t+1}^j is agent j 's stochastic discount factor (SDF). Under symmetric preferences, this conjecture implies that the individual wealth-consumption ratio does not depend on agent-specific attributes, only on aggregate objects. We denote the return to agent j 's consumption claim by r_{t+1}^j . The log stochastic discount factor is a function of consumption growth and the return to the consumption claim:

$$\begin{aligned} m_{t+1}^j &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1}^j + (\theta - 1) r_{t+1}^j \\ &= \mu_s + \left[(\theta - 1) W_{gs} (\nu_g - \kappa_1^c) + \frac{1}{2} \gamma \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) \\ &\quad - \gamma \sigma_c \eta_{t+1} - \gamma \sigma_{g,t+1} v_{t+1}^j + \left[(\theta - 1) W_{gs} \sigma_w - \gamma \phi_c + \frac{1}{2} \gamma \sigma_w \right] \sigma_g w_{g,t+1} \end{aligned}$$

where μ_s is the unconditional mean of the stochastic discount factor.³²

³²The intermediate steps are provided in the appendix, along with all other derivations. The appendix shows that the coefficient W_{gs} is given by:

$$W_{gs} = - \frac{\gamma \nu_g \left(1 - \frac{1}{\psi}\right)}{2 (\kappa_1^c - \nu_g)}$$

If the IES ψ exceeds 1, then $W_{gs} < 0$. Hence, higher consumption share dispersion leads to a lower wealth-consumption ratio.

5.4 Aggregate SDF

Since all agents can invest in all risky assets, the Euler equation has to be satisfied for any two agents j and j' and for every stock i (with returns orthogonal to the agents' idiosyncratic income shocks v^j and $v^{j'}$). This implies that the average SDF must also price all financial assets if all the individual SDFs price the return R_{t+1}^i :

$$1 = \mathbb{E}_t [M_{t+1}^j R_{t+1}^i] = \mathbb{E}_t [\mathbb{E}_j (M_{t+1}^j R_{t+1}^i)] = \mathbb{E}_t [\mathbb{E}_j (M_{t+1}^j) R_{t+1}^i] = \mathbb{E}_t [M_{t+1}^a R_{t+1}^i] .$$

We can write the expression for the average log real stochastic discount factor as:

$$m_{t+1}^a = \mu_s + \frac{1}{2}\gamma^2\sigma_g^2 + s_{gs}(\sigma_{gt}^2 - \sigma_g^2) - \lambda_\eta\sigma_c\eta_{t+1} - \lambda_w\sigma_g w_{g,t+1},$$

where the loadings are given by:

$$s_{gs} = \frac{1}{2}\gamma\nu_g \left(\frac{1}{\psi} + 1 \right), \quad \lambda_\eta = \gamma, \quad \lambda_w = -\frac{1}{2}\gamma(1+\gamma)\sigma_w + \gamma\phi_c + \frac{\gamma\nu_g \left(\frac{1}{\psi} - \gamma \right)}{2(\kappa_1^c - \nu_g)}\sigma_w.$$

Hence, there are two priced sources of aggregate risk in our model: shocks to aggregate consumption growth (which carry a price of risk λ_η equal to the coefficient of relative risk aversion) and shocks to the idiosyncratic volatility factor. The latter carries a negative price of risk, λ_w , indicating that an increase in the cross-sectional volatility of consumption growth is bad news for the stand-in agent. All three terms in the λ_w expression are negative, provided that the agent has a preference for early resolution of uncertainty. The first term captures precautionary motives against changes in consumption risk sharing and the second term compensates for exposure to counter-cyclical cross-sectional variation in idiosyncratic risk.³³ Both of these terms appear even when utility is time-additive. The third term requires a preference for early resolution of uncertainty. With Epstein-Zin preferences, the average

³³Negative aggregate consumption growth episodes tend to occur when the cross-sectional volatility increases provided that $\phi_c < 0$.

investor cares not only about the current dispersion (as in Mankiw 1986), but also about the future dispersion of consumption growth. The size of this effect is governed by the persistence of the idiosyncratic volatility factor, ν_g .

We also derive expressions for Sharpe ratios and the interest rate. The maximum Sharpe ratio in the economy is larger when these risk prices are higher and shocks are more volatile, $\max SR_t = \sqrt{\lambda_\eta^2 \sigma_c^2 + \lambda_w^2 \sigma_g^2}$. The risk-free interest rate is:

$$r_t^f = -\mu_s - \frac{1}{2}\gamma^2 \sigma_g^2 - \frac{1}{2}\lambda_\eta^2 \sigma_c^2 - \frac{1}{2}\lambda_w^2 \sigma_g^2 - s_{gs} (\sigma_{gt}^2 - \sigma_g^2)$$

Interest rates contain the usual impatience and intertemporal substitution terms. They also capture the precautionary savings motive: when idiosyncratic risk is high, agents increase savings thereby lowering interest rates.

5.5 Firm Stock Return

Turning to the pricing of the dividend claim defined by equation (5), we guess and verify that its log price-dividend ratio is affine in the common and idiosyncratic variance terms, $pd_t^i = \mu_{pdi} + A_{gs}^i (\sigma_{gt}^2 - \sigma_g^2) + A_{is}^i (\sigma_{it}^2 - \sigma_i^2)$. Log returns are approximated as $r_{t+1}^i = \Delta d_{t+1}^i + \kappa_0^i + \kappa_1^i pd_{t+1}^i - pd_t^i$.

Innovations in individual stock returns and the return variance reflect the additional sources of idiosyncratic risk:

$$r_{t+1}^i - \mathbb{E}_t [r_{t+1}^i] = \beta_{\eta,i} \sigma_c \eta_{t+1} + \beta_{gs,i} \sigma_g w_{g,t+1} + \kappa_i \sigma_{gt} e_{t+1}^i + \zeta_i \sigma_{it} \varepsilon_{t+1}^i + \kappa_1^i A_{is}^i \sigma_{iw} w_{i,t+1} \quad (7)$$

$$\mathbb{V}_t [r_{t+1}^i] = \beta_{\eta,i}^2 \sigma_c^2 + \beta_{gs,i}^2 \sigma_g^2 + (\kappa_1^i A_{is}^i)^2 \sigma_{iw}^2 + \kappa_i^2 \sigma_{gt}^2 + \zeta_i^2 \sigma_{it}^2 \quad (8)$$

where

$$\beta_{\eta,i} \equiv \varphi_i, \quad \beta_{gs,i} \equiv \kappa_1^i A_{gs}^i \sigma_w + \phi_i. \quad (9)$$

Innovations in stock returns contain two sources of aggregate risk and three sources of idiosyncratic risk (equation 7). The variance of idiosyncratic stock returns are driven by the common σ_{gt} and firm-specific σ_{it} processes (equation 8). In the empirical section, we demonstrated the presence of a large first principal component in both total and residual stock returns, and showed that it was the same component in both. We also demonstrated that total and residual volatility at the firm-level were nearly identical. This model generates these features. It associates the common component in residual variance with changes in the cross-sectional dispersion of consumption growth across agents. Times of high cross-sectional consumption volatility are times of high idiosyncratic (and total) stock return variance.

The expression for the equity risk premium on an individual stock is:³⁴

$$\mathbb{E}_t \left[r_{t+1}^i - r_t^f \right] + .5\mathbb{V}_t[r_{t+1}^i] = \beta_{\eta,i}\lambda_{\eta}\sigma_c^2 + \beta_{gs,i}\lambda_w\sigma_g^2. \quad (10)$$

The first term is the standard consumption CAPM term. The second term is a new term which compensates investors for movements in the cross-sectional (income and) consumption distribution, today and in the future. Stocks that have low returns when the cross-sectional vol of consumption growth increases ($\beta_{gs,i} < 0$) are risky and carry high expected returns because $\lambda_w < 0$.

There are three mechanisms by which a stock's $\beta_{gs,i}$ may be negative. First, if χ^i is sufficiently negative, $A_{gs}^i < 0$ and $\beta_{gs,i} < 0$. A negative χ_i means that periods of high average idiosyncratic volatility are bad economic times that are associated with low future dividend growth. We present empirical support for this mechanism below. In particular, we show that CIV negatively forecasts dividend growth.

Second, if positive innovations to the cross-sectional volatility coincide with low dividend growth realizations ($\phi_i < 0$), then that stock has a lower (more negative) $\beta_{gs,i}$. Given the negative price of risk, it will carry a higher expected return. Hence, the contemporaneous

³⁴The coefficients of the price-dividend equation are obtained from the Euler equation. These and additional detail for the equity premium derivation are provided in the appendix.

cash-flow effects ($\phi_c < 0$ and $\phi_i < 0$) increase the equity risk premium, all else equal.

Third, a stock with lower exposure to the common idiosyncratic risk term κ_i will have a lower beta and therefore carry a higher expected return, all else equal. Intuitively, when there is a positive shock to the volatility factor, the quantity of idiosyncratic risk goes up more so for stocks with greater exposure κ_i . As a result of the convexity in the relation between growth and terminal value, this in turn raises the price more of those high volatility stocks, increasing their beta to the volatility factor, and lowering their equilibrium expected return.³⁵

The idiosyncratic stock return variance is:

$$\mathbb{V}_t \left[r_{t+1}^{idio,i} \right] = (\kappa_1^i A_{is}^i)^2 \sigma_{iw}^2 + \kappa_i^2 \sigma_{gt}^2 + \zeta_i^2 \sigma_{it}^2,$$

and, as in section 3, the common idiosyncratic variance (CIV) factor is defined as the equally-weighted average of the idiosyncratic return variance,

$$CIV_t \equiv \mathbb{E}_i \left[\mathbb{V}_t \left[r_{t+1}^{idio,i} \right] \right] = \bar{\kappa}^2 \sigma_{gt}^2 + \bar{\zeta}^2 \sigma_i^2,$$

where we define $\bar{\kappa}^2 \equiv \mathbb{E}_i[\kappa_i^2]$ and $\bar{\zeta}^2 \equiv \mathbb{E}_i[\zeta_i^2]$. The last term is constant in a (large) cross-section of stocks by virtue of the i.i.d. nature of the σ_{it} processes.³⁶ Thus, in terms of the dynamics, CIV is proportional to the consumption growth share dispersion process σ_{gt}^2 , where $\bar{\kappa}^2$ is the constant of proportionality. This result allows us to use innovations in the volatility factor constructed from stock returns to test our asset pricing mechanism empirically rather than measure the cross-sectional dispersion of investor consumption growth directly.

³⁵A similar convexity effect is explored by Pastor and Veronesi (2003, 2009).

³⁶In computing CIV in the model, we ignore the cross-sectional average of $(\kappa_1^i A_{is}^i)^2 \sigma_{iw}^2$, which is very small in our calibration.

6 Quantitative Implications of the CIV Model

In the section, we use our model to evaluate if the average return spreads across CIV beta sorted portfolios documented in Section 4 are quantitatively consistent with the extent of idiosyncratic volatility comovement documented in Section 2.

Table 3 shows our parameter choices; the model is calibrated to data for the 1963-2010 period and simulated at an annual frequency. Risk aversion γ is set to 15 and the intertemporal elasticity of substitution ψ is set to 2.³⁷ The time discount factor δ is set to produce a mean real risk-free rate of 1.5% per year, given all other parameters. The model produces a risk-free rate with modest volatility of 2.4% per year. Mean consumption growth μ_g is 2% per year and σ_c is 2% per year. We set ϕ_c equal to -0.04 to capture the negative correlation between aggregate consumption growth and the cross-sectional volatility of consumption growth. Aggregate consumption growth volatility is modest at 2.5% per year.

We set the mean of the cross-sectional dispersion in consumption growth, σ_g , to 38%. This is the value for the cross-sectional dispersion in consumption growth we measure in the Consumption Expenditure Data (CEX) for the period 1984-2011. The persistence of the cross-sectional dispersion process, ν_g , is set to 0.6 per year, a value equal to the annual persistence of the CIV factor in the data. This choice implies that our main state variable moves at business cycle rather than at much lower frequencies. We set σ_w to 0.74%. This ensures that σ_{gt}^2 remains positive. The time series standard deviation of σ_{gt} is 0.46%. The model results in a negative market price of “CIV risk,” λ_w , of -1.54 and a substantial maximum conditional Sharpe ratio of 0.66.

As shown in Section 4, stocks whose returns have a more negative exposure to CIV innovations earn higher average returns. To represent the typical stock in each of the CIV beta sorted quintile portfolios, we solve our model for five assets that differ in terms of their

³⁷The high value for γ is needed to generate a high equity risk premium for the market portfolio. It is not required to generate cross-sectional dispersion in risk premia across CIV beta-sorted portfolios. A richer model with time variation in market variance would allow us to match the equity risk premium with lower risk aversion, but at the expense of higher model complexity.

Table 3: Calibration Parameters

This table lists the parameters of the model. The last panel discusses the calibration of five stock portfolios, sorted from lowest volatility (Q1) to highest volatility (Q5). The market portfolio is indicated by the letter M.

Preferences						
δ	0.19	γ	15	ψ	2	
Aggregate Consumption Growth Process						
μ_g	0.02	σ_c	0.02	ϕ_c	-0.0395	
Consumption Share Process						
σ_g	0.38	ν_g	0.6	σ_w	0.0074	
Dividend Growth Process						
σ_i	0.004	ν_i	0.15	σ_{iw}	1.5e-06	
Parameter	Q1	Q2	Q3	Q4	Q5	M
μ_i	7.42%	3.44%	5.45%	5.42%	6.35%	5.20%
φ_i	8.61	8.61	8.61	8.61	8.61	8.61
ϕ_i	-0.31	-0.20	-0.15	-0.11	-0.03	-0.12
χ^i	-0.42	-0.15	-0.16	-0.15	-0.00	-0.15
κ_i	1.34	1.00	0.91	0.91	1.10	-
ζ_i	94.39	71.31	65.10	64.92	78.57	-

cash-flow growth process (equation 5). We also consider the market portfolio, which is an asset whose cash flow growth has no idiosyncratic shocks. We set mean dividend growth μ_i equal to the values observed for the CIV beta-sorted portfolios and the market portfolio in the data.

We set φ_i , a standard consumption leverage parameter, equal to 8.61 for all portfolios. By setting this parameter equal for all portfolios, we impose that all differences in risk premia across portfolios arise from differences in exposure to the $w_{g,t+1}$ shocks. The choice of 8.61 is such that the model matches the equity risk premium for the market portfolio of 5.50% exactly, given all other parameters. The contribution to the equity risk premium from the η -term, the first term in equation (10), is 5.16% per year. The other parameters we hold fixed across portfolios are the parameters governing the σ_{it} process in equation (6). We set σ_i to 0.4%, ν_i to 0.15, and σ_{iw} to 1.5e-6. The persistence of σ_{it} is much lower than that of σ_{gt} , conforming to the data. We choose σ_{iw} such that σ_{it} never becomes negative in simulations. Finally, the value for σ_i is chosen to match the mean of the observed CIV process of 0.254,

given all other parameters.

The four key parameters for each quintile portfolio are ϕ_i , χ_i , κ_i , and ζ_i . We pin down these four parameters to match four moments. The first is the CIV beta, $\beta_{gs,i}$, in equation (9). The second is the slope of a regression of dividend growth on lagged CIV, ensuring that the model respects the dividend growth predictability patterns observed in the data. The third and fourth moments are the slope and the R^2 from a regression of idiosyncratic stock return variance on the CIV factor:

$$\mathbb{V}_t \left[r_{t+1}^{idio,i} \right] = a^i + b^i CIV_t + \nu_{t+1}^i. \quad (11)$$

While these are four simultaneous equations, χ_i mostly affects the dividend growth predictability slope, κ_i governs the portfolios' return variance exposure to CIV, ζ_i affects the R^2 of the regression in (11), while ϕ_i is chosen to match $\beta_{gs,i}$ given the other three parameters. The last four rows of Table 3 show the chosen values for these four parameters for each portfolio.

Table 4 summarizes the quantitative results. The post-formation CIV betas in the data are reported in row 5.³⁸ They are monotonically increasing from very negative for Q1 (-0.18) to positive for Q5 (0.11). The spread in betas is 0.30. The model matches the betas exactly. The model also matches the slope and R^2 of the regression (11), as reported in rows 14-17. The U-shaped pattern in the regression slopes translates in a U-shaped pattern for the κ_i coefficients.

Rows 18 and 19 show that the model matches the dividend growth predictability slopes exactly. The data show a monotonically increasing pattern in the dividend growth predictability slope, which the model accommodates via a negative value for χ_i for portfolio Q1, a less negative χ_i for the intermediate portfolios, and a zero slope for portfolio Q5.³⁹

³⁸The table reports rescaled betas, where the scaling ensures that the innovation volatility of CIV is the same in model and data.

³⁹These regression slopes are statistically significant in the data for Q1, Q3, and the market portfolio.

In addition to the monotonically increasing pattern in χ_i , matching the pattern in the CIV betas requires a monotonically increasing pattern in the ϕ_i parameter. Fitting the cash-flow predictability evidence alongside the asset pricing moments is important given that we documented in Section 2 that the factor structure in idiosyncratic volatility is present both in cash flow and return volatility data.

The main result of the calibration exercise is that the model is able to match the excess returns on the CIV beta-sorted portfolios. It generates a monotonically declining pattern in value-weighted excess stock return from Q1 to Q5 (row 2). The model exactly matches the return spread between portfolio 5 and portfolio 1 of -6.44% per year in the data (row 1). In the data, the latter has a t-statistic of -3.42 (recall Table 3). As rows 3 and 4 make clear, the common level of the equity risk premium comes from compensation for η -risk, while the entire cross-sectional slope in excess returns is due to differential exposure to the w_g -risk. The stocks in portfolio Q1 (Q5) have negative (positive) exposure to the CIV factor. Their returns fall (increase) when the cross-sectional vol increases, making them risky (a hedge). As a result, they carry the highest (lowest) risk premia.

The model accurately matches total return volatilities of the CIV beta-sorted portfolios, shown in rows 7 and 8 of Table 4. Annual return volatilities (standard deviations) for the typical stock in each of the quintile portfolios range from 46% to 67%. They are highest for portfolios Q1 and Q5, displaying a U-shape. The market portfolio has a volatility of 15.7% in the data and 17.2% in the model. Rows 9-13 break down total return volatility into its five components. As in the data, most of total return volatility is idiosyncratic return volatility. In particular, the common idiosyncratic and firm-specific idiosyncratic components contribute about equally to firm volatility (rows 11 and 12). The model matches the persistence of the various volatility components as well as the relative amount of variation that comes from the common and the firm-specific volatility components.

Table 4: Calibration Results

This tables reports moments from the model and compares them to the data. The first two rows report the average excess return in model and data. The next two rows split out the equity risk premium into a contribution representing compensation for η risk and a compensation for w_g risk. Rows 5 and 6 report the adjusted CIV betas in data and model. Rows 7 and 8 report stock return volatilities in data and model, followed by a breakdown of volatility into its five components in rows 9-13 (see equation 8). Since the variance but not the volatility components are additive, we calculate the square root of each variance component, and then rescale all components so they sum tot total volatility. Rows 14 and 15 report the slope of regression (11), $\frac{Cov(\mathbb{V}_t[r_{t+1}^{idio,i}], \mathbb{V}_t[r_{t+1}^{IV}])}{Var[\mathbb{V}_t[r_{t+1}^{IV}]]} = \frac{\kappa_i^2}{\bar{\kappa}^2}$, in the data and in the model, multiplied by 100. Rows 16 and 17 report the R-squared of this regression for both data and model, multiplied by 100. Rows 18 and 19 report the slope of a predictive regression of annual dividend growth on one-year lagged CIV. The model is simulated at annual frequency for 60,000 periods. All moments in the data are expressed as annual quantities and computed from the January 1963 to December 2010 sample.

	Moment		Q1	Q2	Q3	Q4	Q5	M
1	Excess Ret.	Data	9.96	7.12	6.44	5.28	3.52	5.50
2		Model	9.15	6.78	5.69	4.67	2.71	5.50
3		η risk	5.16	5.16	5.16	5.16	5.16	5.16
4		w_g risk	3.99	1.62	0.52	-0.50	-2.45	0.34
5	Beta $\beta_{gs,i}$	Data	-0.18	-0.07	-0.02	0.02	0.11	-0.02
6		Model	-0.18	-0.07	-0.02	0.02	0.11	-0.02
7	Return Vol.	Data	67.04	52.16	47.30	46.47	54.39	15.67
8		Model	65.91	50.56	46.49	46.47	55.10	17.23
9		η risk	10.01	10.02	10.14	10.14	9.98	16.67
10		w_g risk	3.97	1.61	0.53	0.50	2.43	0.56
11		e_i risk	29.50	22.07	20.28	20.31	24.14	-
12		ε_i risk	21.97	16.60	15.33	15.30	18.22	-
13		w_i risk	0.46	0.26	0.22	0.22	0.32	-
14	Eq. (11) slope	Data	1.55	0.87	0.71	0.72	1.04	-
15		Model	1.58	0.89	0.73	0.73	1.07	-
16	Eq. (11) R^2	Data	17.70	17.10	16.80	17.00	16.90	-
17		Model	17.70	17.10	16.80	17.00	16.90	-
18	Div. predict	Data	-0.37	-0.13	-0.14	-0.13	-0.00	-0.13
19		Model	-0.37	-0.13	-0.14	-0.13	-0.00	-0.13

7 Conclusion

We document strong comovement of individual stock return volatilities. Removing common variation in returns has little effect on volatility comovement, as residual return volatility possesses effectively the same volatility factor structure as total returns, despite the fact that these residuals are uncorrelated. The distinction between stock total volatility and

idiosyncratic volatility is tiny – almost all return variation at the stock level is idiosyncratic. Volatility comovement is not only a feature of returns, but also of firm-level cash flows. We find a strong factor structure in sales growth volatilities, both for total and idiosyncratic sales growth rates.

We explore the asset pricing implications of these findings in a model with heterogeneous investors whose consumption risk is linked to firms' idiosyncratic cash flow risk. CIV is a priced state variable: Increases in CIV lead to an increase in the dispersion of consumption growth across households and are associated with high marginal utility for the average investor. Stocks whose returns rise with CIV hedge against deterioration in the investment opportunity set and thus earn low average returns. Sorting stocks into portfolios based on their exposure to the CIV, we find that stocks with more negative betas carry higher average returns. The calibrated model quantitatively matches the observed return spread and volatility facts for plausible parameter values.

Our work suggests a link between the cross-sectional volatility in firms' returns and cash flow growth and the cross-sectional volatility in household consumption growth. This link seems plausible. A large literature argues that shocks to firms have important effects on both the labor income and financial income of their employees. Another literature documents that households cannot or do not fully insure against their labor income shocks. We provide several new pieces of evidence in support of this link. A valuable direction for future work is to provide further evidence on the joint dynamics of the distributions of firm output and household income and consumption.

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A Empirical Appendix

Figure A1: Log Volatility: Empirical Density Versus Normal Density (2010 Snapshot)

The figure plots histograms of the empirical cross section distribution of annual firm-level volatility (in logs) for the 2010 calendar year. Panel A shows total return volatility, calculated as the standard deviation of daily returns for each stock within a calendar year. Panel B shows total sales growth volatility, calculated as the standard deviation of quarterly year-on-year sales growth observations in a 20 quarter window. Panels C and D show idiosyncratic volatility based on the five principal components factor model. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.

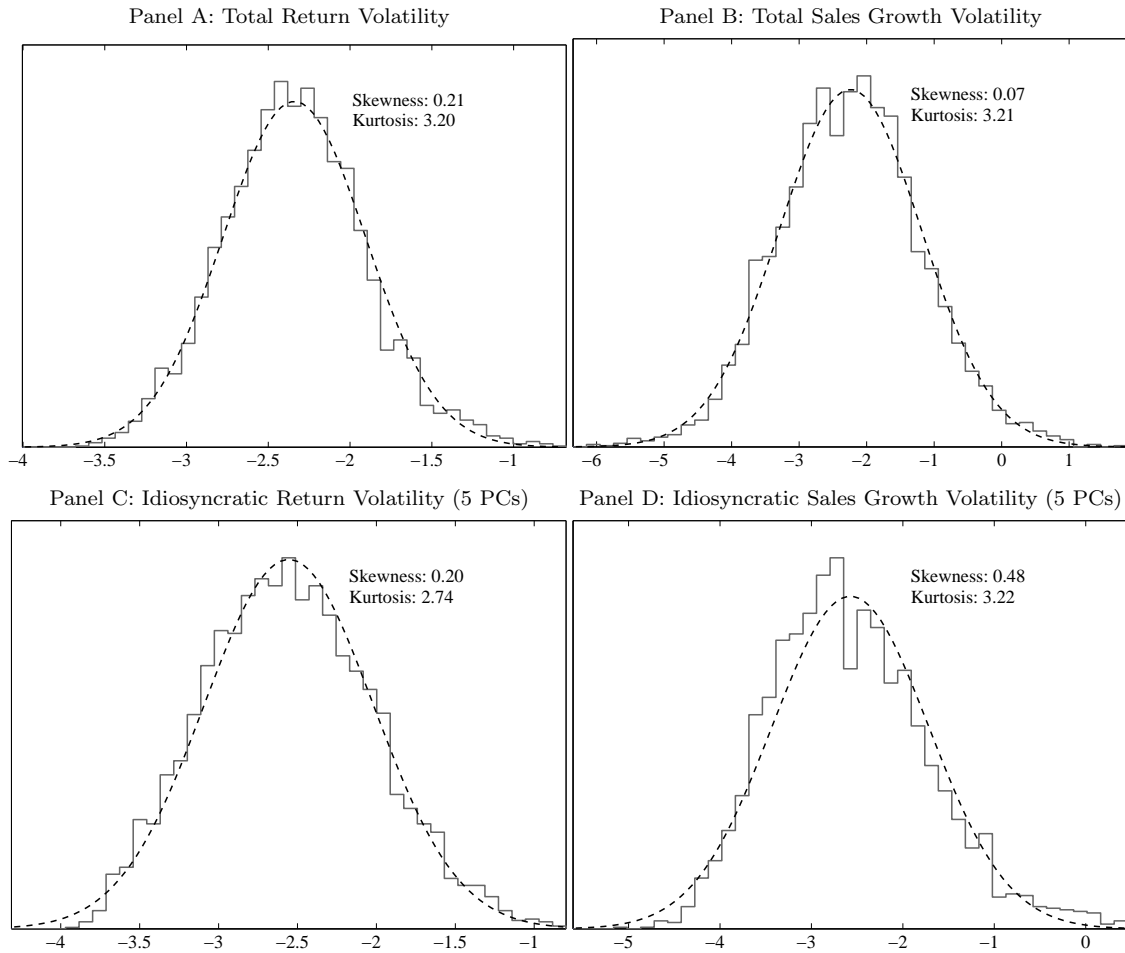


Figure A2: Volatility and Correlation of Monthly Returns

The figure repeats the analysis of Figure 3 using monthly return observations within each calendar year, rather than daily.

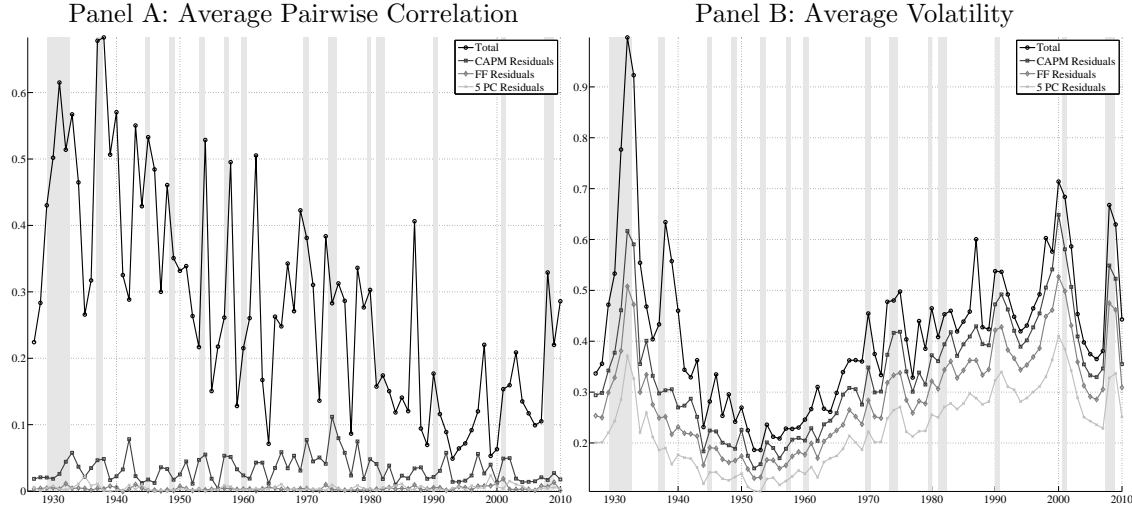


Figure A3: Volatility of 100 Size and Value Portfolios

The figures plot volatility of total and idiosyncratic returns on 100 size and value portfolios. Within each calendar year, total return volatilities are estimated from daily returns for each portfolio (Panel A), while idiosyncratic return volatility is the standard deviation of residuals from the three factor Fama-French model (Panel B) estimated within each calendar year. Panel C shows average pairwise correlation for total and idiosyncratic returns on 100 size and value portfolios within each calendar year

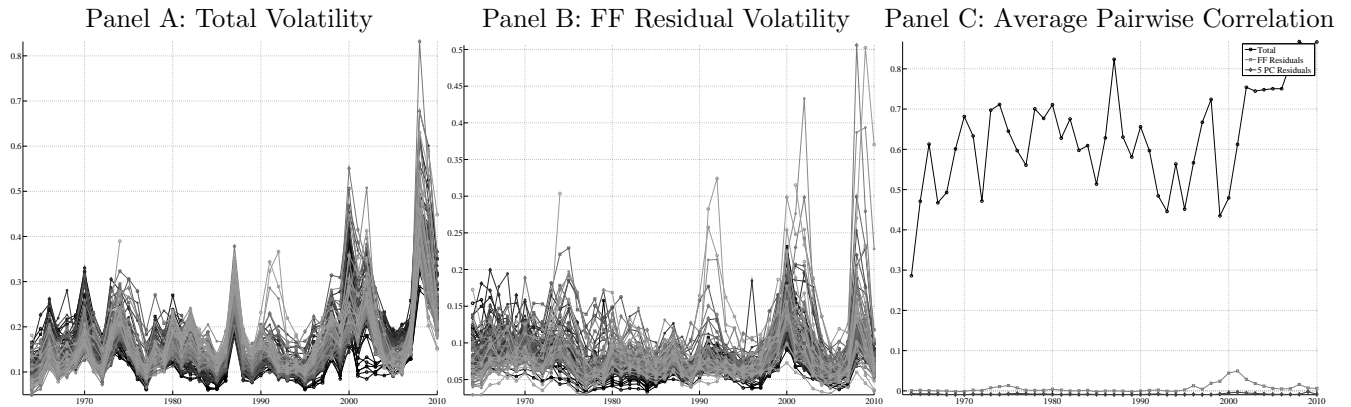


Table A1: Volatility Factor Model Estimates

The table reports estimates of annual volatility one-factor regression models. In each panel, the volatility factor is defined as the equal-weighted cross section average of firm volatilities within that year. That is, all estimated volatility factor models take the form: $\sigma_{i,t} = \text{intercept}_i + \text{loading}_i \cdot \bar{\sigma}_{.,t} + e_{i,t}$. Columns represent different volatility measures. For the 100 Fama-French returns (Panel A), the columns report estimates for a factor model of total return volatility and idiosyncratic volatility based on residual returns from the market model, the Fama-French model, or the five principal component models. For net income growth (Panel B) and EBITDA growth (Panel C), the columns report total volatility or idiosyncratic volatility based on sales growth residuals from the one and five principal components model (using a rolling 20 quarter window for estimation). The last column of Panels B and C reports results when using only the four quarterly growth observations within each calendar year to estimate total volatility. We report cross-sectional averages of loadings and intercepts as well as time series regression R^2 averaged over all firms. We also report a pooled factor model R^2 , which compares the estimated factor model to a model with only firm-specific intercepts and no factor.

Panel A: Portfolio Returns				
	Total	MM	FF	5 PCs
Loading (average)	1.000	0.999	1.000	0.999
Intercept (average)	0.000	0.000	0.000	0.000
R2 (average univariate)	0.708	0.497	0.394	0.450
R2 (pooled)	0.691	0.454	0.375	0.470

Panel B: Net Income Growth				
	Total (5yr)	1 PC (5yr)	5 PCs (5yr)	Total (1yr)
Loading (average)	1.142	0.809	0.839	1.079
Intercept (average)	-0.053	-0.007	-0.011	-0.031
R2 (average univariate)	0.285	0.270	0.269	0.199
R2 (pooled)	0.273	0.257	0.252	0.169

Panel C: EBITDA Growth				
	Total (5yr)	1 PC (5yr)	5 PCs (5yr)	Total (1yr)
Loading (average)	0.842	0.934	0.884	0.990
Intercept (average)	0.065	-0.021	-0.009	0.017
R2 (average univariate)	0.294	0.281	0.280	0.191
R2 (pooled)	0.261	0.269	0.259	0.152

Table A2: Equal-weighted Portfolios Formed on CIV Beta

The table reports average return results (in annual percentages) for equal-weighted portfolio sorts on the basis of monthly CIV beta for the 1963-2010 sample. Panel A reports average returns and alphas in one-way sorts using all CRSP stocks. Panel B shows average returns in independent two-way sorts on CIV beta and market variance beta. Panel B shows average returns in independent two-way sorts on CIV beta and idiosyncratic stock variance.

	CIV beta						
	1 (Low)	2	3	4	5 (High)	5-1	$t(5-1)$
Panel A: One-way sorts on CIV beta							
$E[R]$	17.17	16.22	14.76	13.77	12.07	-5.10	-3.51
α_{CAPM}	5.23	5.21	4.11	3.18	0.93	-4.30	-3.02
α_{FF}	1.03	1.20	0.48	-0.18	-1.89	-2.93	-2.09
Panel B: Two-way sorts on CIV beta and MV beta							
MV beta							
1 (Low)	16.46	15.58	14.45	13.72	13.16	-3.29	-1.74
2	17.12	16.58	14.64	14.54	13.38	-3.74	-2.32
3	18.01	16.32	15.87	14.32	13.08	-4.93	-3.00
4	19.92	15.69	14.17	13.41	12.48	-7.44	-3.85
5 (High)	16.48	15.99	13.62	12.87	12.40	-4.08	-1.83
5-1	0.02	0.41	-0.82	-0.85	-0.77		
$t(5-1)$	0.01	0.20	-0.39	-0.42	-0.31		
Panel C: Two-way sorts on CIV beta and idiosyncratic variance							
Idios. var.							
1 (Low)	16.27	14.83	14.00	13.50	11.92	-4.34	-3.33
2	18.90	17.44	15.09	14.59	13.41	-5.49	-4.13
3	18.84	18.88	17.28	15.65	14.37	-4.46	-3.20
4	18.87	16.18	16.52	15.78	14.37	-4.49	-3.22
5 (High)	13.22	11.87	9.74	10.26	8.93	-4.29	-2.48
5-1	-3.04	-2.96	-4.26	-3.24	-2.99		
$t(5-1)$	-1.01	-1.02	-1.51	-1.11	-0.90		

Table A3: Portfolios Formed on CIV Beta – Robustness

The table reports average return results for value-weighted portfolio sorts in annual percentages. Panels A and B report one-way sorts on CIV beta using all CRSP stocks in the 1985-2010 and 1963-1985 subsamples, respectively. Panel C reports sorts on CIV betas estimated from CIV changes that have *not* been orthogonalized with respect to changes in market variance in the full 1963-2010 sample. Panel D reports sorts on market variance beta in the full 1963-2010 sample.

	1 (Low)	2	3	4	5 (High)	5-1	$t(5-1)$
Panel A: One-way sorts on CIV beta, 1985-2010							
$E[R]$	16.59	14.55	13.31	10.56	9.13	-7.47	-2.69
α_{CAPM}	3.42	2.68	2.14	0.26	-1.54	-4.96	-1.94
α_{FF}	3.14	2.43	1.92	-0.06	-1.85	-4.99	-2.03
Panel B: One-way sorts on CIV beta, 1963-1985							
$E[R]$	14.08	10.41	10.38	11.00	8.85	-5.23	-2.20
α_{CAPM}	3.00	-0.06	-0.03	0.44	-2.19	-5.19	-2.17
α_{FF}	0.80	-1.58	-0.37	0.95	-1.43	-2.22	-0.94
Panel C: One-way sorts on CIV beta, no orthogonalization							
$E[R]$	14.81	12.75	11.60	10.32	9.70	-5.11	-2.53
α_{CAPM}	2.66	1.43	0.97	0.13	-0.68	-3.34	-1.77
α_{FF}	1.97	0.97	0.68	0.00	-0.98	-2.96	-1.63
Panel D: One-way sorts on MV beta							
$E[R]$	11.06	11.76	12.15	9.86	10.64	-0.42	-0.17
α_{CAPM}	-1.51	0.41	1.46	-0.30	0.84	2.34	1.09
α_{FF}	-1.20	0.29	1.10	-0.85	-0.13	1.06	0.58

B Model Appendix

Starting from the budget constraint for agent j :

$$W_{t+1}^j = R_{t+1}^j(W_t^j - C_t^j).$$

The beginning-of-period (or cum-dividend) total wealth W_t^j that is not spent on consumption C_t^j earns a gross return R_{t+1}^j and leads to beginning-of-next-period total wealth W_{t+1}^j . The return on a claim to consumption, the *total wealth return*, can be written as

$$R_{t+1}^j = \frac{W_{t+1}^j}{W_t^j - C_t^j} = \frac{C_{t+1}^j}{C_t^j} \frac{WC_{t+1}^j}{WC_t^j - 1}.$$

We use the Campbell (1991) approximation of the log total wealth return $r_t^j = \log(R_t^j)$ around the long-run average log wealth-consumption ratio $\mu_{wc} \equiv E[w_t^j - c_t^j]$:

$$r_{t+1}^j = \kappa_0^c + \Delta c_{t+1}^j + wc_{t+1}^j - \kappa_1^c wc_t^j,$$

where the linearization constants κ_0^c and κ_1^c are non-linear functions of the unconditional mean log wealth-consumption ratio μ_{wc} :

$$\kappa_1^c = \frac{e^{\mu_{wc}}}{e^{\mu_{wc}} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log(e^{\mu_{wc}} - 1) + \frac{e^{\mu_{wc}}}{e^{\mu_{wc}} - 1} \mu_{wc}.$$

The return on a claim to the consumption stream of agent j , R^j , satisfies the Euler equation under her stochastic discount factor:

$$\begin{aligned} 1 &= \mathbb{E}_t \left[M_{t+1}^j R_{t+1}^j \right] \\ 1 &= \mathbb{E}_t \left[\mathbb{E}_j \left[M_{t+1}^j R_{t+1}^j \right] \right] \\ 1 &= \mathbb{E}_t \left[\mathbb{E}_j \left[\exp\{m_{t+1}^j + r_{t+1}^j\} \right] \right] \\ 1 &= \mathbb{E}_t \left[\exp\{ \mathbb{E}_j (m_{t+1}^j + r_{t+1}^j) + \frac{1}{2} \mathbb{V}_j (m_{t+1}^j + r_{t+1}^j) \} \right] \end{aligned} \tag{12}$$

where the second equality applies the law of iterated expectations, and the last equality applies the cross-sectional normality of consumption share growth.

Wealth-consumption ratio We combine the approximation of the log total wealth return with our conjecture for the wealth-consumption ratio of agent j , $w_t^j = \mu_{wc} + W_{gs} (\sigma_{gt}^2 - \sigma_g^2)$, and we solve for the coefficients μ_{wc} and W_{gs} by imposing the Euler equation for the consumption claim.

First, using the conjecture, we compute the individual log total wealth return r_{t+1}^j :

$$\begin{aligned} r_{t+1}^j &= \kappa_0^c + \Delta c_{t+1}^j + wc_{t+1}^j - \kappa_1^c wc_t^j \\ &= r_0^c + \left[W_{gs} (\nu_g - \kappa_1^c) - \frac{1}{2} \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) + \sigma_c \eta_{t+1} + (\phi_c + W_{gs} \sigma_w - \frac{1}{2} \sigma_w) \sigma_g w_{g,t+1} + \sigma_{g,t+1} v_{t+1}^j \end{aligned}$$

where $r_0^c = \kappa_0^c + \mu_g + (1 - \kappa_1^c) \mu_{wc} - \frac{1}{2} \sigma_g^2$.

Second, Epstein and Zin (1989) show that the log real stochastic discount factor is

$$\begin{aligned} m_{t+1}^j &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1}^j + (\theta - 1) r_{t+1}^j \\ &= \mu_s + \left[(\theta - 1) W_{gs} (\nu_g - \kappa_1^c) + \gamma \frac{1}{2} \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) \\ &\quad - \gamma \sigma_c \eta_{t+1} - \gamma \sigma_{g,t+1} v_{t+1}^j + \left[(\theta - 1) W_{gs} \sigma_w - \gamma \phi_c + \frac{1}{2} \gamma \sigma_w \right] \sigma_g w_{g,t+1} \end{aligned}$$

where $\mu_s = \theta \log \delta - \gamma \mu_g + (\theta - 1) [\kappa_0^c + (1 - \kappa_1^c) \mu_{wc}] + \gamma \frac{1}{2} \sigma_g^2$ is the unconditional mean log SDF.

Third, using the individual stochastic discount factor and total wealth return expressions, we can compute elements of equation 12. Specifically, we have that:

$$\begin{aligned} \mathbb{E}_j \left(m_{t+1}^j \right) &= \mu_s + \left[(\theta - 1) W_{gs} (\nu_g - \kappa_1^c) + \gamma \frac{1}{2} \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) \\ &\quad - \gamma \sigma_c \eta_{t+1} + \left[(\theta - 1) W_{gs} \sigma_w - \gamma \phi_c + \frac{1}{2} \gamma \sigma_w \right] \sigma_g w_{g,t+1} \\ \mathbb{E}_j \left(r_{t+1}^j \right) &= r_0^c + \left[W_{gs} (\nu_g - \kappa_1^c) - \frac{1}{2} \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) \\ &\quad + \sigma_c \eta_{t+1} + (\phi_c + W_{gs} \sigma_w - \frac{1}{2} \sigma_w) \sigma_g w_{g,t+1} \\ \mathbb{V}_j \left[m_{t+1}^j + r_{t+1}^j \right] &= (1 - \gamma)^2 (\sigma_g^2 + \nu_g (\sigma_{gt}^2 - \sigma_g^2) + \sigma_w \sigma_g w_{g,t+1}) \end{aligned}$$

Putting all together, the equations above imply that:

$$\begin{aligned} \mathbb{E}_j \left(m_{t+1}^j + r_{t+1}^j \right) + \frac{1}{2} \mathbb{V}_j \left[m_{t+1}^j + r_{t+1}^j \right] &= \mu_s + r_0^c + \frac{1}{2} (1 - \gamma)^2 \sigma_g^2 \\ &\quad + \left[\theta W_{gs} (\nu_g - \kappa_1^c) + \frac{1}{2} \gamma (\gamma - 1) \nu_g \right] (\sigma_{gt}^2 - \sigma_g^2) \\ &\quad + (1 - \gamma) \sigma_c \eta_{t+1} + \left[\theta W_{gs} \sigma_w + (1 - \gamma) \phi_c + \frac{1}{2} \gamma (\gamma - 1) \sigma_w \right] \sigma_g w_{g,t+1} \end{aligned}$$

Finally, we can use the above equation to solve the Euler equation 12. Using log normal properties, we can take the expected value conditional on time t information and compute the Euler equation. Applying method of undetermined coefficients, the following equalities have to hold

$$0 = \mu_s + r_0^c + \frac{1}{2} (1 - \gamma)^2 \sigma_g^2 + \frac{1}{2} (1 - \gamma)^2 \sigma_c^2 + \frac{1}{2} \left[\theta W_{gs} \sigma_w + (1 - \gamma) \phi_c + \frac{1}{2} \gamma (\gamma - 1) \sigma_w \right]^2 \sigma_g^2 \quad (13)$$

and

$$W_{gs} = \frac{\nu_g \gamma (\gamma - 1)}{2 \theta (\kappa_1^c - \nu_g)} = - \frac{\gamma \nu_g \left(1 - \frac{1}{\psi} \right)}{2 (\kappa_1^c - \nu_g)}$$

Plugging the W_{gs} expression as well as κ_0^c and κ_1^c back into equation (13) implicitly defines a nonlinear

equation in one unknown (μ_{wc}), which can be solved for numerically, characterizing the average wealth-consumption ratio.

Stochastic discount factor Once we have solved for the individual stochastic discount factors, the common log real stochastic discount factor can be derived:

$$\begin{aligned} m_{t+1}^a &= \mathbb{E}_j \left[m_{t+1}^j \right] + \frac{1}{2} \mathbb{V}_j \left[m_{t+1}^j \right] \\ &= \mu_s + \frac{1}{2} \gamma^2 \sigma_g^2 + s_{gs} (\sigma_{gt}^2 - \sigma_g^2) - \lambda_\eta \sigma_c \eta_{t+1} - \lambda_w \sigma_g w_{g,t+1} \end{aligned}$$

where

$$\begin{aligned} s_{gs} &\equiv (\theta - 1) W_{gs} (\nu_g - \kappa_1^c) + \frac{1}{2} \gamma (1 + \gamma) \nu_g = \frac{1}{2} \gamma \nu_g \left(\frac{1}{\psi} + 1 \right), \\ \lambda_\eta &\equiv \gamma, \\ \lambda_w &\equiv (1 - \theta) W_{gs} \sigma_w + \gamma \phi_c - \frac{1}{2} \gamma (1 + \gamma) \sigma_w = \frac{\gamma \nu_g \left(\frac{1}{\psi} - \gamma \right)}{2 (\kappa_1^c - \nu_g)} \sigma_w + \gamma \phi_c - \frac{1}{2} \gamma (1 + \gamma) \sigma_w, \end{aligned}$$

The risk-free rate is given by

$$\begin{aligned} r_t^f &= -\mathbb{E}_t[m_{t+1}^a] - \frac{1}{2} \mathbb{V}_t[m_{t+1}^a] \\ &= -\mu_s - \frac{1}{2} \gamma^2 \sigma_g^2 - \frac{1}{2} \lambda_\eta^2 \sigma_c^2 - \frac{1}{2} \lambda_w^2 \sigma_g^2 - s_{gs} (\sigma_{gt}^2 - \sigma_g^2) \end{aligned}$$

Stock returns For individual firm's stock returns, we guess and verify that the log price-dividend ratio is linear on the state variables $\sigma_{g,t}^2$ and $\sigma_{i,t}^2$:

$$pd_t^i = \mu_{pdi} + A_{gs}^i (\sigma_{gt}^2 - \sigma_g^2) + A_{is}^i (\sigma_{it}^2 - \sigma_i^2)$$

As usual, returns are approximated as:

$$r_{t+1}^i = \Delta d_{t+1}^i + \kappa_0^i + \kappa_1^i pd_{t+1}^i - pd_t^i$$

where $\kappa_1^i = \frac{\exp(\mu_{pdi})}{1 + \exp(\mu_{pdi})}$ and $\kappa_0^i = \log(1 + \exp(\mu_{pdi})) - \kappa_1^i \mu_{pdi}$ are approximation constants. Plugging in the dividend growth equation as well as the price dividend expression, we get:

$$\begin{aligned} r_{t+1}^i &= r_0^i + [\chi_i - A_{gs}^i (1 - \kappa_1^i \nu_g)] (\sigma_{gt}^2 - \sigma_g^2) - A_{is}^i (1 - \kappa_1^i \nu_i) (\sigma_{it}^2 - \sigma_i^2) \\ &\quad + \varphi_i \sigma_c \eta_{t+1} + (\phi_i + A_{gs}^i \kappa_1^i \sigma_w) \sigma_g w_{g,t+1} + \kappa_i \sigma_{gt} e_{t+1}^i + \zeta_i \sigma_{it} \varepsilon_{t+1}^i + A_{is}^i \kappa_1^i \sigma_{iw} w_{i,t+1} \end{aligned}$$

where $r_0^i = \mu_i + \kappa_0^i + (\kappa_1^i - 1) \mu_{pdi}$.

Innovations in individual stock market return and individual return variance reflect the additional sources

of idiosyncratic risk:

$$\begin{aligned} r_{t+1}^i - \mathbb{E}_t[r_{t+1}^i] &= \beta_{\eta,i}\sigma_c\eta_{t+1} + \beta_{gs,i}\sigma_g w_{g,t+1} + \kappa_i\sigma_{gt}e_{t+1}^i + \zeta_i\sigma_{it}\varepsilon_{t+1}^i + \kappa_1^i A_{is}^i \sigma_{iw} w_{i,t+1} \\ \mathbb{V}_t[r_{t+1}^i] &= \beta_{\eta,i}^2\sigma_c^2 + \beta_{gs,i}^2\sigma_g^2 + (\kappa_1^i A_{is}^i)^2 \sigma_{iw}^2 + \kappa_i^2\sigma_{gt}^2 + \zeta_i^2\sigma_{it}^2 \end{aligned}$$

where

$$\begin{aligned} \beta_{\eta,i} &\equiv \varphi_i, \\ \beta_{gs,i} &\equiv \kappa_1^i A_{gs}^i \sigma_w + \phi_i, \end{aligned}$$

The expression for the equity risk premium on an individual stock is:

$$\mathbb{E}_t[r_{t+1}^i - r_t^f] + .5\mathbb{V}_t[r_{t+1}^i] = \beta_{\eta,i}\lambda_\eta\sigma_c^2 + \beta_{gs,i}\lambda_w\sigma_g^2.$$

The coefficients of the price-dividend equation are obtained from the Euler equation:

$$\begin{aligned} A_{gs}^i &= \frac{2s_{gs} + 2\chi_i + \kappa_i^2}{2(1 - \kappa_1^i\nu_g)} = \frac{2\chi_i + \kappa_i^2 + \left(1 + \frac{1}{\psi}\right)\gamma\nu_g}{2(1 - \kappa_1^i\nu_g)} \\ A_{is}^i &= \frac{\zeta_i^2}{2(1 - \kappa_1^i\nu_i)} \end{aligned}$$

and the constant μ_{pdi} is the mean log pd ratio which solves the following non-linear equation:

$$\begin{aligned} 0 &= r_0^i + \mu_s + \frac{1}{2}\gamma^2\sigma_g^2 + \frac{1}{2}(\beta_{gs,i} - \lambda_w)^2\sigma_g^2 + \frac{1}{2}(\beta_{\eta,i} - \lambda_\eta)^2\sigma_c^2 \\ &\quad + \frac{1}{2}\kappa_i^2\sigma_g^2 + \frac{1}{2}\zeta_i^2\sigma_i^2 + \frac{1}{2}(\kappa_1^i A_{is}^i)^2 \sigma_{iw}^2 \end{aligned}$$