Algorithmic and High Frequency Trading in Dynamic Limit Order Markets

Alejandro Bernales
Universidad de Chile

Motivation

High Frequency Trading (HFT) is intended/designed to be: FAST

Fig 3. Human traders

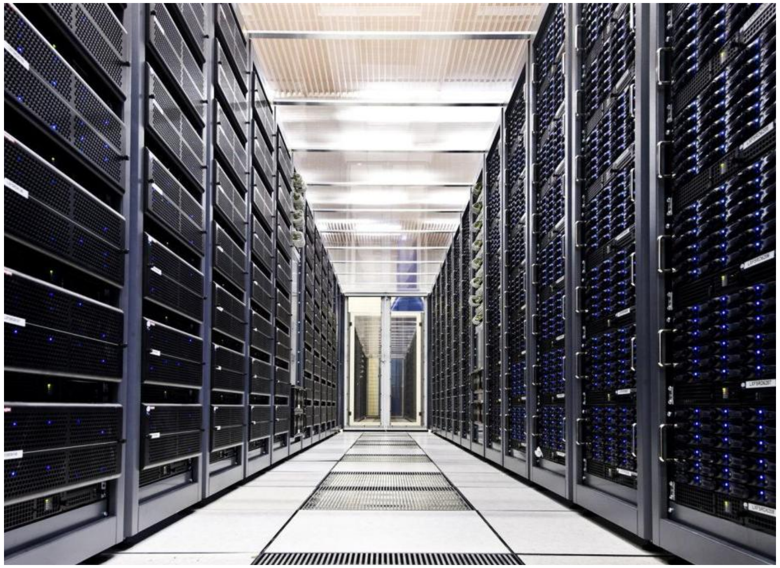


Fig 4. HFT traders

- HFT technological transformation.
  - Sophisticated computers quickly process information and automatically submit orders utilizing superfast connections to the exchanges.
- **Who will be the winner between a human trader and a HFT trader?**
- 73% of the trading volume on the U.S. stock market in 2009 can be attributed to HFT (Hendershott et al., 2012; and Brogaard, 2010).

Motivation

Speed Advantages of HFT

- HFT traders have speed advantages over other agents:
  - Informational advantage: fast access and quick analysis of market information.
  - Trading speed advantage: low-latency transmission of orders and prompt modifications to previous trading decisions.

- There is a growing theoretical literature on HFT.
  - HFT is characterized :
    - independently through the informational advantage.
    - independently through the trading speed advantage.

- There are no many theoretical studies in which HFT traders have an simultaneously both characteristics.

  - **The main goal of our paper is to fill this gap.**

Motivation

What is the structure of exchanges which incorporate HFT?

- Exchanges in which HFT takes place are fully, or at least partially, organized as limit order book markets.
  - E.g. BATS U.S. Stock Exchange, NYSE, NASDAQ, London Stock Exchange, NYSE Euronext, BATS Chi-X Europe.

- **Microstructure features** and the endogenous dynamics of limit order markets **have to be considered when evaluating the effects of HFT** on market quality and stability.

Research Aims: The Big Picture of my Study (THE INTUITIONS)

Objective:

- I introduce a dynamic equilibrium model in which HFT traders have an effective trading speed advantage and an informational advantage.
- The model describes the evolution of a limit order market.

How?

- The model is a stochastic sequential game with endogenous trading decisions.
- Two types of agents: fast and slow traders.
- Fast traders have speed advantages in terms of analysing information and the low-latency transmission of orders.
- I obtain a stationary Markov-perfect equilibrium using Pakes and McGuire's (2001, Econometrica) algorithm given the analytical intractability of the model.

Why?

- No theoretical studies of HFT exist in which the main speed advantages (information advantage and trading speed advantage) of this technology are studied at the same time.

- I simulate a complete limit order book.
  - In the BIS Foresight study (2012): "*simulation tools and techniques could enable central regulatory authorities to judge the stability of particular financial markets, given knowledge of the structure of those markets*".

Research Aims: The Big Picture of my Study (THE INTUITIONS)

Findings:

- We find that HFT improves market quality 'only' under specific conditions and changes trading behavior of 'traditional' agents.
  - HFT traders prefer to act as liquidity suppliers when they represent the majority.
  - If HFT traders are the minority, they have a 'predatory' behavior through market orders by 'picking-off' limit orders coming from the big crowd of slow traders; which induces a damage in the liquidity of the system.
- HFT reduces waiting costs but finally damages slow traders profits.
- Fast traders with only informational (trading speed) advantages increase (reduces) the global welfare.
  - Nevertheless, there is a positive synergy between informational and trading speed advantages of fast traders; when they both advantages the system welfare increase even more than when fast traders have only an informational superiority.
- The maximum system welfare is obtained when the percentage of fast traders is around 70%, which in fact is congruent to the current U.S. stock trading volume reported in the empirical literature.

Research Aims: The Big Picture of my Study (THE INTUITIONS)

Findings:

- We show that that AT in general reduces microstructure noise since it mitigates the cognitive limits of human beings.

- We also perform some policy exercises using the dynamic features of our model.
  - A latency restriction and a cancelation fee for fast traders have harmful impacts on market quality.
    - However, a cancelation fee may be better policy since it may induce that fast traders behave more as liquidity suppliers.

  - Moreover, we find that that fast traders may have incentives to trade in assets that are more volatile or when there is economic period in which there is a high volatility.
    - This explains the abnormal trading behaviors agents with AT technology observed in the ‘flash crash’.

The Model

- I consider a dynamic limit order market in continuous time with a single asset.
  - The fundamental value asset, v_t, follows a random walk (drift zero and volatility σ).

- Two types of risk-neutral agents: fast (HFT) traders and slow traders.
  - Fast and slow traders arrive following Poisson process at rate λ (on average x% of agents will be HFT traders).

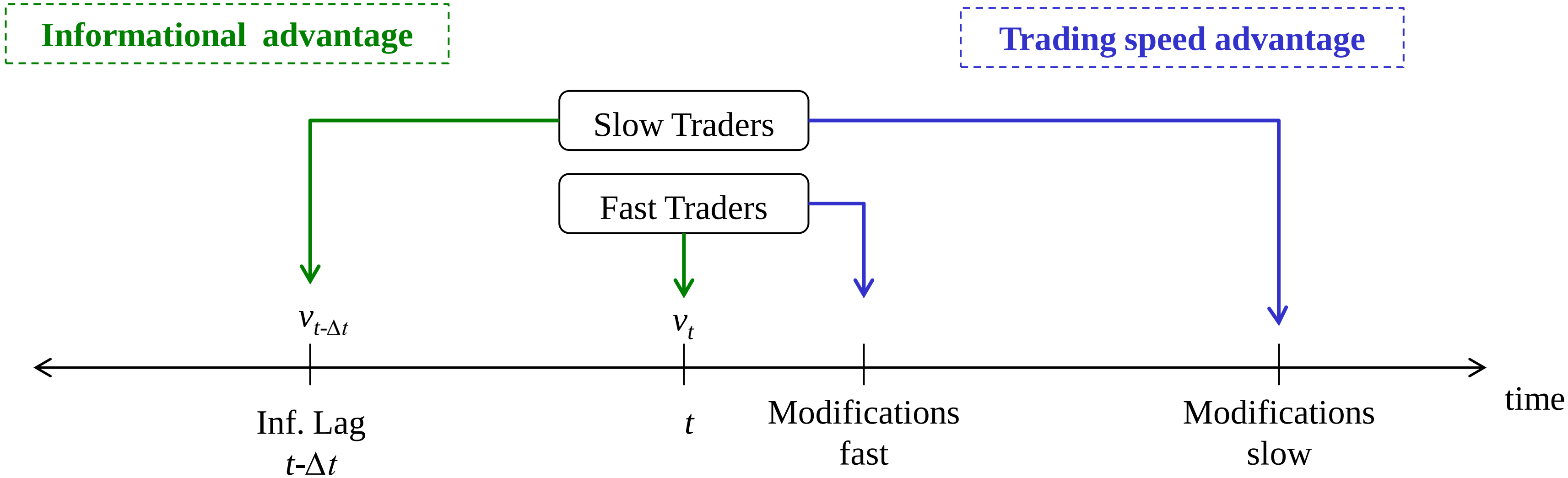
- Traders can submit limit orders and market orders.
  - Traders can re-enter at the market to modify unexecuted limit orders.

- Traders have intrinsic values to trade (private values) which gives additional heterogeneity.
  - The private value α of an agent is drawn from a distribution F_α.

The Model

- **Informational advantage** of HFT:
  - Fast traders observe v_t ; while slow traders observe $v_{t\text{-}\Delta t}$.
  - BUT, slow traders can learn from the information revealed by fast traders in the market activity to improve their accuracy of $E(v_t)$.

- **Trading speed advantage** of HFT:
  - Traders cannot immediately modify their unexecuted limit orders after a change in the market conditions due to cognition limits; thus trading decisions are 'sticky' (Biais, Hombert and Weill, 2012).
    - Fast and slow traders re-enter following the Poisson processes at rate λ_r^{HFT} and λ_r^{ST} , respectively ($\lambda_r^{HFT} > \lambda_r^{ST}$).



The Model

- Trading decisions are endogenous and depend on market conditions (states).

- A trader has to make several initial trading decisions:
  - To submit an order or to wait.
  - A buy order or a sell order.
  - A market order or a limit order.
  - Price in the case of limit order .

- Traders, after submitting a limit order, have to make additional trading decisions when they re-enter.
  - Cancelling an unexecuted limit order or retaining the order without changes.
  - After cancelling, traders can submit a new order or decide to wait.
  - If a trader decides to submit a new order after a cancellation, she has to decide: to buy or to sell; to submit a limit or a market order; and to select the price in the case of a limit order.

- Limit orders:
  - Delaying cost ρ_d (cost of not executing immediately).
  - Cancellation cost c_{canc}.

- Traders can trade one share and they leave the market forever after the execution of their orders.

The Model

- The limit order book L_t is described by a discrete set of prices, denoted as $\{p^i\}_{\mathrm{i}=(-\infty,\infty)}$.
  - The tick size is d.
  - There is a backlog of outstanding orders to buy or to sell, $l_{t,i}$, which are associated with each price p^i (i.e. this is the depth at the price p^i).
  - The book respects the time and price priorities for the execution of limit orders.

The Model

- The expected value of an order executed prior to a re-entry at time h_r is:

$$\pi(h_r, \tilde{a}, s) = \int_0^{h_r} \int_{-\infty}^{\infty} e^{-\rho_d t} \big((\alpha + v - \tilde{p})\tilde{x}\big)\, \eta(h_r|\tilde{a}, s)\gamma(v|\sigma, s)\,dv\,dt$$

where:

- $(\alpha + v - \tilde{p})\tilde{x}$: Instantaneous payoff of an order where $\tilde{x}$= 1 (buy), $\tilde{x}$ = -1 (sell), or $\tilde{x}$ = 0 (none order).
- $\eta(\cdot)$: Prob. that an order is executed at time h_r given the trader takes the action $\tilde{a}$ in the state s.
- $\gamma(\cdot)$: Density function of v which depends on σ and the state s (the state s incorporates the type of trader).

- The Bellman equation for the agent's problem is:

$$V(s) = \max_{\tilde{a}\in\Gamma(s)} \int_0^{\infty} \left[\pi(h_r, \tilde{a}, s) + e^{-\rho_d h_r} \int_{s_{h_r}\in S} \left(V(s_{h_r}) - \tilde{z}_{s_{h_r}} c_{canc}\right) \psi(s_{h_r}|\tilde{a}, s, h_r)\,ds_{h_r}\right] dR(h_r|s)$$

where:

- $\Gamma(s)$: Set of possible actions that a trader can take given the state s.
- $\psi(\cdot)$: Probability that the agent observes the state s_{hr}.
- $R(\cdot)$: Distribution of re-entry time which depends on the type of trader.
- $\tilde{z}_{s_{h_r}} = 1$ if the optimal decision in the state s_{hr} is a cancellation and $\tilde{z}_{s_{h_r}} = 0$ in any other case.

Model Parameterization

- I assume the following plausible parameter values:
  - σ = 0.50 on an annual basis (Zhang, 2010).
  - Arrival rates: λ = 1/0.040 (Cont, 2011).
  - Re-entering rates: λ_r^{ST} = 1/0.600 (Trimmel and Poelzl , 2012) and λ_r^{HFT} = 1/0.120.
  - The distribution of the private value is assumed to be discrete with support {-8,-4,0,4,8} in ticks and {0.15,0.35,0.65,0.85,1.00} as the cumulative distribution function (Hollifield *et al*., 2006).
  - Slow traders observe the fundamental value of the asset with a lag, Δt, equal to 0.800 seconds.
  - Delaying cost ρ_d = 0.03 (Goettler *et al*., 2009).

Results

How do traders execute orders? (through what kind of order?)

	How do traders execute orders? (through what kind of orders?)			Prob. of being picked-off
	Market orders	Limit order	Total	Limit order
	Base case: Slow traders and fast traders in the market			
Slow traders	53.509%	46.491%	100.00%	42.406%
Fast traders	48.495%	51.505%	100.00%	27.159%
	Only slow traders in the market			
Slow traders	50.000%	50.000%	100.00%	21.580%

- **Observation.** *HFT induces changes in the submission behaviour of slow traders due to their disadvantages in analysing information and quickly modifying previous trading decisions.*

Results

How do traders execute orders? (through what kind of order?)

	% of Traders in the Market ST: 80% and FT: 20%		% of Traders in the Market ST: 60% and FT: 40%		% of Traders in the Market ST: 40% and FT: 60%		% of Traders in the Market ST: 20% and FT: 80%	
	Market order	Limit order	Market order	Limit order	Market order	Limit order	Market order	Limit order
	Porcentage of type of orders executed per trader							
ST	49.348%	50.652%	49.597%	50.403%	52.280%	47.720%	55.964%	44.036%
FT	52.532%	47.468%	51.045%	48.955%	48.478%	51.522%	48.508%	51.492%
Total	50.000%	50.000%	50.000%	50.000%	50.000%	50.000%	50.000%	50.000%

- **Observation.** *HFT traders prefer to act as liquidity suppliers when they represent the majority.*
- *If HFT traders are the minority, they have a 'predatory' behavior through market orders by 'picking-off' limit orders coming from the big crowd of slow traders; which induces a damage in the liquidity of the system.*

Results

Depth of the book

	N. of limit orders at the ask	N. of limit orders sell side	N. limit orders sell side (efectively traded)	N. of cancelat. / N. of traders
	Base case: Slow traders and fast traders in the market			
Slow and Fast traders	2.163	6.335	1.728	1.252 (21.9% Slow T.; 78.1% Fast T.)
	Only slow traders in the market			
Slow traders	1.967	6.139	2.914	0.554

I observe the market every 10 minutes. The bid-ask spread is measured in ticks. I show only the number of orders at the ask price since the model is symmetric in both sides of the book.

- **Observation.** *HFT reduces the depth of the limit order book due to cancelations.*

Results

Average payoffs per trader

The values are measured in ticks. Standard errors are less than 0.0009 for fast traders, while standard errors for slow traders are less than 0.0037.

	Fast trd.: Δt = 0.8 ; λr = 1/0.6 Slow trd.: Δt = 0.8; λr = 1/0.6 HFT: none advantages	Fast trd.: **Δt = 0.0** ; λr = 1/0.6 Slow trd.: Δt = 0.8; λr = 1/0.6 HFT: only inform. advan.	Fast trd.: Δt = 0.8 ; **λr = 1/0.12** Slow trd. Δt = 0.8; λr = 1/0.6 HFT: only trad. speed. advan.	Fast trd.: **Δt = 0.0** ; **λr = 1/0.12** Slow trd.: Δt = 0.8; λr = 1/0.6 HFT: both advantages	Fast trd.: **Δt = 0.0** ; **λr = 1/0.12** Slow trd.: **Δt = 0.0**; **λr = 1/0.12** HFT: all fast traders
Slow trd.(A)	3.764	3.668	3.711	3.662	3.738
Fast trd. (B)	3.764	3.815	3.753	3.826	3.738
B-A	0.000	0.146	0.042	0.164	0.000
Total	3.754	3.771	3.741	3.777	3.738

This case is equivalent to the scenario with only slow traders in the market.

- **Observation.** *It is true that HFT induces some economic damage to slow traders.*

Results

- To understand the payoffs, we can calculate the gains from trade.
- Following Hollifield et al (2006, JF): "Estimating the Gains from Trade in Limit-Order Markets"
  - In our model, when we have a transaction we have gains from trade (GFT):

$$GFT = (\alpha_{buy} + v - \tilde{p})e^{-\rho_d t_{buy}} + (-\alpha_{sell} - v + \tilde{p})e^{-\rho_d t_{sell}}$$

  - We can rewrite this equation as:

$$GFT = \overbrace{\alpha_{buy} - \alpha_{sell}}^{\text{Private Values}} + \overbrace{\alpha_{buy}(e^{-\rho_d t_{buy}} - 1)}^{\text{Waiting Costs Buyer}} - \overbrace{\alpha_{sell}(e^{-\rho_d t_{sell}} - 1)}^{\text{Waiting Costs Seller}}$$
$$\underbrace{+ (v - \tilde{p})\, e^{-\rho_d t_{buy}}}_{\text{Money Transfer Buyer}} \underbrace{- (v - \tilde{p})e^{-\rho_d t_{sell}}}_{\text{Money Transfer Seller}}$$

Results

	Waiting cost			Money transfer			% Limit orders per trader type		
	Private value |α|			Private value |α|			Private value |α|		
	0	4	8	0	4	8	0	4	8
	Base case: Slow traders and fast traders in the market								
Slow trd.(A)	0.000	-0.199	-0.082	0.159	-0.280	-0.564	84.07%	42.64%	14.05%
Fast trd. (B)	0.000	-0.318	-0.178	0.462	-0.050	-0.371	69.65%	54.07%	29.95%
B-A	0.000	-0.119	-0.096	0.302	0.231	0.193	-14.42%	11.43%	15.89%
	Only slow traders in the market								
Slow trd.	0.000	-0.267	-0.191	0.367	-0.163	-0.389	71.26%	48.51%	30.73%

$$GFT = \overbrace{\alpha_{buy} - \alpha_{sell}}^{\text{Private Values}} + \overbrace{\alpha_{buy}\left(e^{-\rho d t_{buy}} - 1\right)}^{\text{Waiting Costs Buyer}} - \overbrace{\alpha_{sell}\left(e^{-\rho d t_{sell}} - 1\right)}^{\text{Waiting Costs Seller}}$$

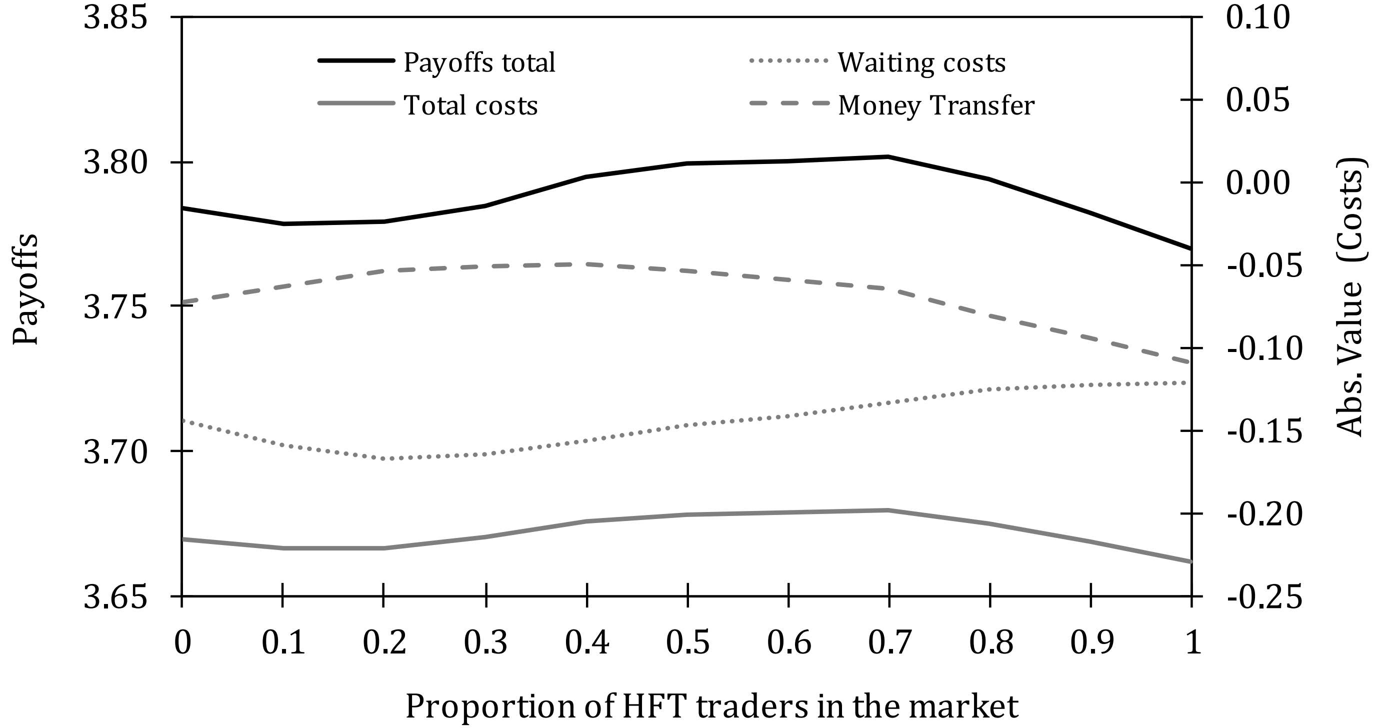
$$+ \underbrace{(v - \tilde{p})\, e^{-\rho d t_{buy}}}_{\text{Money Transfer Buyer}} - \underbrace{(v - \tilde{p})\, e^{-\rho d t_{sell}}}_{\text{Money Transfer Seller}}$$

- **Observation.** *Fast traders have a larger waiting cost than slow traders due to the high submission of limit orders from HFT traders.*

- **Observation.** *Speculators* (*private value*=0) *has positive value in the "money transfer"; while other traders (private value* ≠ 0) make profit mainly through the private values.

Results

Total payoff and costs per trader with different proportion of HFT traders in the market



$$GFT = \overbrace{\alpha_{buy} - \alpha_{sell}}^{\text{Private Values}} + \overbrace{\alpha_{buy}\left(e^{-\rho_d t_{buy}} - 1\right)}^{\text{Waiting Costs Buyer}} - \overbrace{\alpha_{sell}\left(e^{-\rho_d t_{sell}} - 1\right)}^{\text{Waiting Costs Seller}}$$
$$+ \underbrace{(v - \tilde{p})\, e^{-\rho_d t_{buy}}}_{\text{Money Transfer Buyer}} - \underbrace{(v - \tilde{p}) e^{-\rho_d t_{sell}}}_{\text{Money Transfer Seller}}$$

Results

Microstructure noise || Belief errors of slow traders || Bid-ask Spread

	Microstructure noise	Belief errors of slow traders	Bid-ask spread
	Mean $\lvert v_t - p_t \rvert$	Mean $\lvert E(v_t) - v_t \rvert$	Mean bid-ask
	Base case: Slow traders and fast traders in the market		
Slow and Fast traders	0.503	0.398	1.453
	Only slow traders in the market		
Slow traders	1.328	1.189	1.614

I observe the market every 10 minutes. All values are in ticks. Microstructure noise is defined as v_t - p_t (i.e. fundamental value of the asset minus the transaction price).

- **Observation.** *Microstructure noise is reduced by the presence of HFT participants.*
  - HFT mitigates the cognitive limits of human beings.

- **Observation.** *The learning process followed by slow traders reduces their belief errors regarding v_t in the presence of HFT participants.*

- **Observation.** *HFT reduces the bid-ask spread.*

Conclusions

- I introduce a dynamic equilibrium model in which HFT traders have an effective trading speed advantage and an informational advantage.
- The model describes the evolution of a limit order market.
- We find that HFT improves market quality ‘only’ under specific conditions and changes trading behavior of 'traditional' agents.
- AT traders prefer to act as liquidity suppliers (demanders) when they represent the majority (minority) of investors.
- AT reduces waiting costs but finally damages slow traders profits.
- In some scenarios, AT decreases liquidity and global welfare.
- AT traders prefer volatile assets, and cancelation fees may be better policy instruments than latency restrictions to control AT activity.

Algorithmic and High Frequency Trading in Dynamic Limit Order Markets

Alejandro Bernales
Universidad de Chile

Equilibrium

- **Equilibrium algorithm:**
  - Pakes and McGuire's (2001, Econometrica) algorithm:

    Intuitions:
    - Traders follow an updating process by playing in the game.
    - Traders start with beliefs about the expected payoffs of different actions given a state.
    - Traders update their expected payoffs when they decide to follow an action and observe its realized payoff.
    - The equilibrium is reached when the expected payoffs and optimal trading decisions of each trader type in a given state s^* are exactly the same as those which occur if a similar trader observes s^* in the future.

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s}+1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s}+1} e^{-\rho_d(t_r-t)}\left(U_{t_r}\left(\tilde{a}^{**}\middle|s_{t_r}\right) - \tilde{z}_{\tilde{a}^{**}} c_{canc}\right)$$

where $n_{\tilde{a}^*,s}$ is a counter that increases by one when the action $\tilde{a}^*$ is taken in the state *s*.

Equilibrium

- **Existence**
  - The state space is given by the five-tuple: (v, *trader type*, α, L_t , potential trading action)
  - We impose some assumptions to make the state-space finite and countable:
    - Discretization of v:
      - » $\{v^j\}_{j=(0,\infty)}$ where $v^{ij+1} - v^j = d$ (d is the same tick size as the book)
    - We put always the center of the book L_t at v:
      - » $p^0 = v$
    - Traders cannot submit orders very far away from the fundamental value:
      - » $v - kd < p^i < v + kd$
      - » k is a integer high enough that even very ‘unaggressive’ strategies can never go outside the grid of prices.
  - Using the same arguments as Goettler et al. (2005, JF), since the state-space if finite and countable from Riader (1979) this game has a Markov-perfect Equilibrium.

- **Uniqueness**
  - We do not prove uniqueness.
  - We verify that the equilibrium appears to be computationally unique. We start the algorithm at different initial values, and ensure that it converges to the same equilibrium.

Learning process of slow traders regarding v_t

- **Slow traders**:

  $$E(v_t \mid s) = v_{t-\Delta t} + \varphi(s)$$

  in which $\phi(s)$ is the adjustments that a slow trader has to apply to $v_{t-\Delta t}$, given that she observe the state *s* in order to improve the accuracy of her beliefs about v_t.

- **Fast traders**:

  $$E(v_t \mid s) = v_t$$