MATH PREREQUISITES

The course assumes that students are familiar with high-school mathematics. The following exercises provide a sample of the math tools used throughout the course.

■ 1. Growth rates.

(a) Variable x initially has a value of 42. It then increases to 46. Compute the increase in x as a percentage of the initial value.

Answer: $(46-42)/42 = 4/42 = .095238095238095238 09 \approx 9.5\%$

(b) Variable x's initial value is $x_1 = 80$. It then increases by 7%. What is its new value x_2?

Answer: The new value is given by $x_2 = x_1 + 7\% \times x_1 = (1+7\%)\,x_1 = (1+.07)\times 80 = 85.6$.

■ 2. Scientific notation.

(a) Write the number 12,000,000 in scientific (or exponential) notation.

Answer: $12,000,000 = 1.2\text{E}07$

(b) Write the result of 2.4E12/12E08 in decimal notation.

Answer: $2.4\text{E}12/12\text{E}08 = (2.4 \times 1,000,000,000,000)/(12 \times 10,000,000) = 2,000$.

■ 3. Average. The variable x takes on the following values: 4, 10, 7, 8, 2, 14, 5, 10. Determine the average value x.

Answer: There are 8 different values of x. The average of x is given by

$$\overline{x} = (4+10+7+8+2+14+5+10)/8 = 7.5$$

■ 4. Expected value. With probability 80%, x takes the value 40. With probability 20%, x takes the value 60. What is the expected value of x?

Answer: The expected value of x is given by $\mathbb{E}(x) = .8 \times 40 + .2 \times 60 = 44$.

■ 5. Functions.

(a) Suppose that $f(x) = x^2$. Determine the value of $f(2)$.

Answer: $f(2) = 2^2 = 4$.

(b) Suppose that $f(x) = \sqrt{x}$. Determine $f(3) - f(2)$.

Answer: $f(3) - f(2) = \sqrt{3} - \sqrt{2} = 1.7320508075688772 - 1.4142135623730951 \approx 0.32$.

(c) Suppose that $f(x) = 2 + x$. Derive the expression of the inverse function $x = g(y)$.

Answer: Solving $y = 2 + x$ with respect to x we get $x = y - 2$. Therefore, $g(y) = y - 2$.

■ **6. Systems of equations.** Determine the values x, y that solve the following system of equations

$$\begin{aligned} x + 2\,y &= 10 \\ 3\,x - 2\,y &= 2 \end{aligned}$$

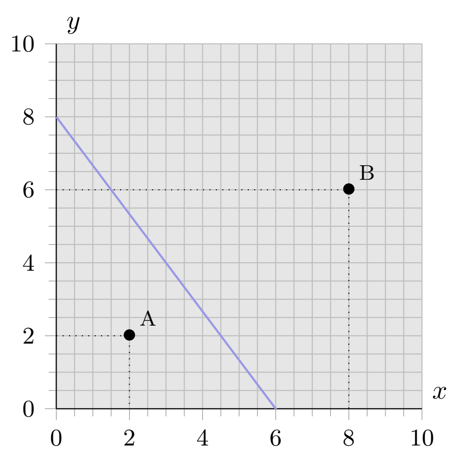
Answer: From the first equation, we get $x = 10 - 2\,y$. Substituting into the second equation, we get $3\,(10 - 2\,y) - 2\,y = 2$. Solving with respect to y, we get $30 - 8\,y = 2$, or $y = 28/8 = 3.5$. Substituting in the equation $x = 10 - 2\,y$, we get $x = 10 - 2 \times 3.5 = 3$.

■ **7. Inequalities.** Solve the inequality $-2\,y + 4\,x > 5$ with respect to y.

Answer:

$$\begin{aligned} -2\,y + 4\,x &> 5 \\ -2\,y &> 5 - 4\,x \\ 2\,y &< -5 + 4\,x \\ y &< -\frac{5}{2} + 2\,x \end{aligned}$$

■ **8. Graphing.** Consider the following graph.



(a) Determine the slope of the blue line.

Answer: Let (x_1, y_1) and (x_2, y_2) be the coordinates of the intercepts in the x and y axis, respectively. The slope is then given by

$$s = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0 - 8}{6 - 0} = -\frac{8}{6} = -\frac{4}{3}$$

(b) Determine the equation of the line going through points A and B.

Answer: The general formula for a line going through two points — except when the line is vertical — is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$$

In the present case, let point A correspond to coordinates with subscript 1 and point B to coordinates with subscript 2. Then we have

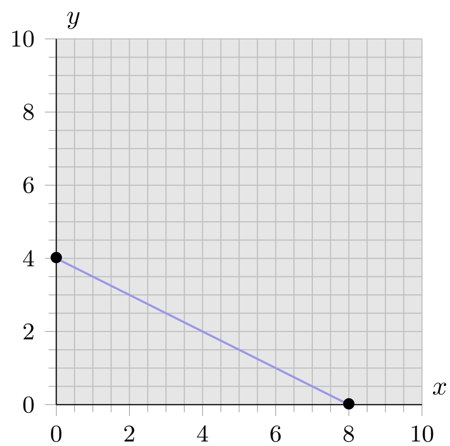
$$y - 2 = \frac{6-2}{8-2}\,(x-2)$$

or simply

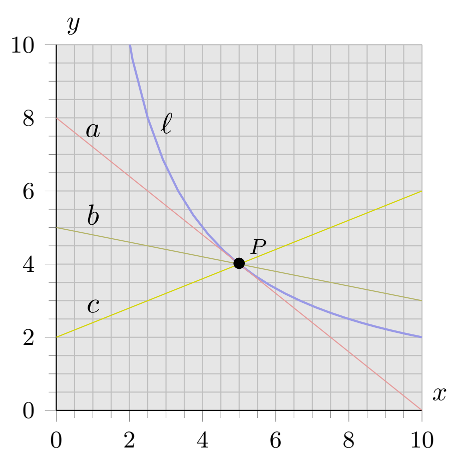
$$y = \tfrac{2}{3} + \tfrac{2}{3}\,x$$

(c) Plot the line $y = 4 - x/2$.

Answer: One way to plot the line is to find the intercepts in the x and y axes and then to draw a line through them. The x (resp. y) axis intercept is set by equating y (resp. x) to zero. y=0 implies $x = 8$, whereas $x = 0$ implies $y = 4$.



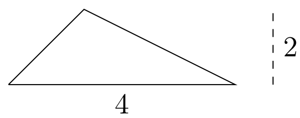
■ 9. Tangent lines. Consider the graph below. Which of the straight lines — a, b, or c — is tangent to ℓ at point P? What is the slope of the tangent? If, starting from point P, the value of x increased by 1, what is the change in the value of y as we move along the tangent?



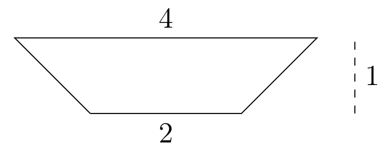
Answer: Line a is tangent to ℓ at point P. The tangent's slope is given by $-8/10$, or simply $-4/5$. Starting from point P and moving along the tangent, if the value of x is increased by 1, then the value of y drops by $-4/5$.

■ 10. Polygon area. Determine the area of the following polygons, where the numbers represent lengths:

(a)



(b)



Answer: In case (a) (triangle), the area is given by

$$A = \frac{1}{2} \times 4 \times 2 = 4$$

In case (b) (trapeze), the area is given by

$$A = \frac{1}{2} \times (2 + 4) \times 1 = 3$$