Stock Market Volatility during the 2008 Financial Crisis

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1. INTRODUCTION

From 2004 to early 2007, the financial markets had been very calm. The market volatility, as measured by the S&P 500 volatility and the VIX index, have been below long-term averages. However, the financial crisis of 2008 changed this: most asset classes experienced significant pullbacks, the correlation between asset classes increased significantly and the markets have become extremely volatile. During this time, the S&P 500 lost about 56% of its value from the October 2007 peak to the March 2009 trough and the VIX Index more than tripled, highlighting the leverage effect that Black (1976) described in his paper on the study of stock market volatility.

Figure 1: Historical values of S&P 500 Index and VIX

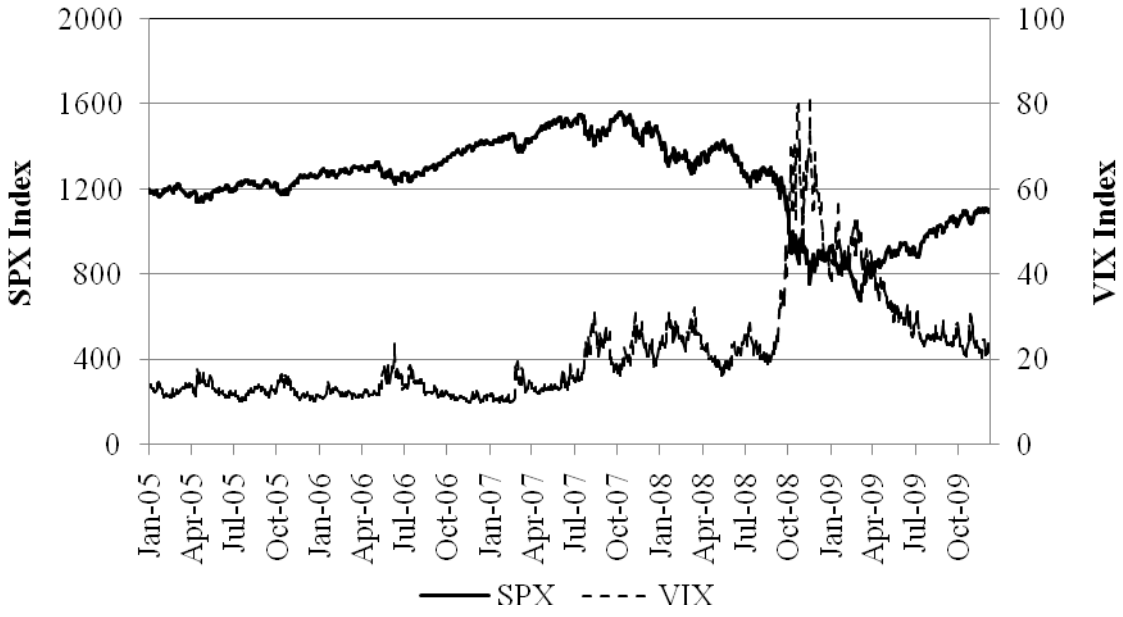

Figure 1: Daily closing levels of the S&P 500 Index (SPX) and the S&P 500 Volatility Index (VIX). The sample period is January 3, 2005 – December 11, 2009. Source: CBOE and Yahoo Finance

While the industry and academia have done extensive work on the stock market volatility and the negative relationship between stock returns and volatility over the years, we did not find any literature examining these subjects during the recent financial crisis. In this report, we study the stock market volatility and the behavior of various measures of volatility before, during and after the 2008 financial crisis, and whether the leverage effect was observed during this period. To explore the stock market volatility and different measures of volatility, we analyzed the volatility of S&P 500 returns, the VIX Index, VIX Futures, VXV Index, and S&P 500 Implied Volatility Skew. We also analyzed the implied volatility of Options on VIX Futures to study the behavior of "volatility of volatility" during the financial crisis. To study the leverage effect, we analyzed the relationship between S&P 500 returns, VIX Index and VIX Futures.

1.1 VIX Index

Since its introduction in 1993, VIX – the CBOE Volatility Index – became the benchmark for stock market volatility and is followed feverishly by both option traders and equity market participants. VIX measures the market's expectations of 30-day volatility, as conveyed by the market option prices. While the original VIX used options on the S&P 100 index, the updated VIX uses put and call options on the S&P 500 index. The new methodology estimates expected S&P 500 Index (SPX) volatility by averaging the weighted prices of SPX puts and calls over the entire range of strike prices. The components of VIX are near- and next-term put and call options, always in the first and second SPX contract months.

VIX has been dubbed as the "Fear Index" because it spikes during market turmoil or periods of extreme uncertainty. VIX reached its highest level ever during the major stock market decline in October 1987. Additionally, it has been shown that it is negatively correlated with the S&P 500 index – it rises when the index falls and vice versa.

1.2 VIX Futures

While the VIX index has a strong negative relationship with the S&P 500 Index, VIX is not a tradable asset. Hence, one cannot use the VIX index to protect against market declines. However, futures contracts on the VIX Index are available and market participants can use them as a hedging instrument. Unlike S&P 500, the futures contracts on VIX have an expiration date. The value of a particular VIX Futures contract corresponds to the markets expectation of the VIX Index value as of the expiration date of the contract. Since the maturity of the VIX Futures contract decreases every day, we decided to construct a VIX Futures contract with constant 30 day maturity for the purpose of this study. The fixed maturity VIX futures prices are constructed by using the market data of available contracts with linear interpolation technique.

Figure 2: VIX Futures - Open Interest and Contract Volume

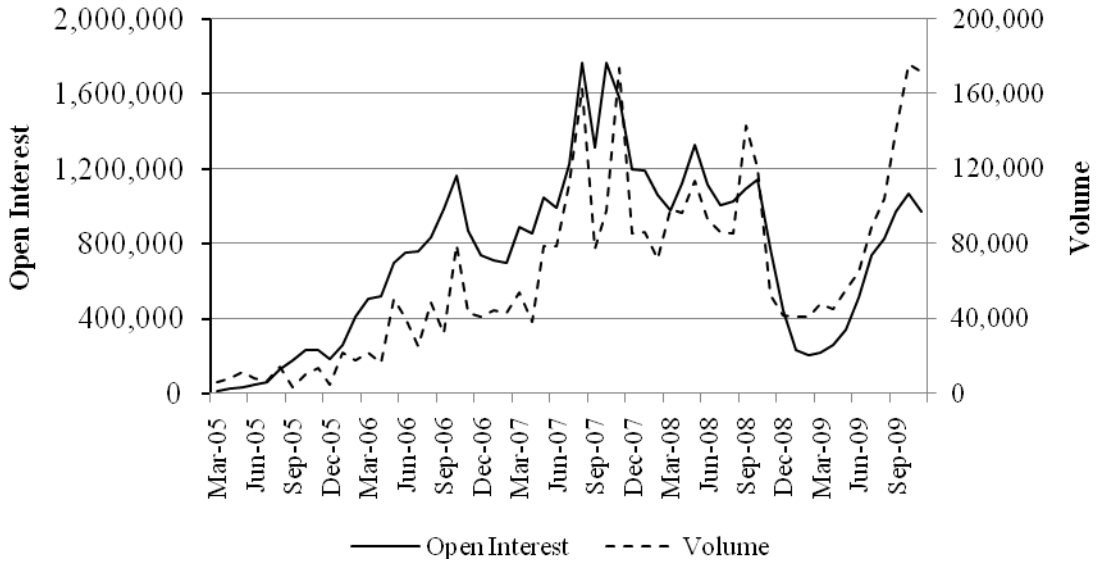

Figure 2: VIX Futures monthly open interest and volume. Plot shows increase in monthly volume and open interest of VIX Futures contracts since their introduction. The sample period is March 2005 – November 2009. Source: CBOE

1.3 VXV Index

While VIX is a measure of expected 30 days volatility of the S&P 500 Index, VXV measures the expected 3 month S&P 500 Index volatility. Conceptually, one can think of VIX as an indicator of near term event risk, because it captures the volatility that is associated with events that are expected to occur in the next 30 days. Using VIX and VXV indexes together, one can get good insight into the term structure of S&P 500 Index (SPX) options implied volatility.

Figure 3: Historical values of VIX and VXV Indexes

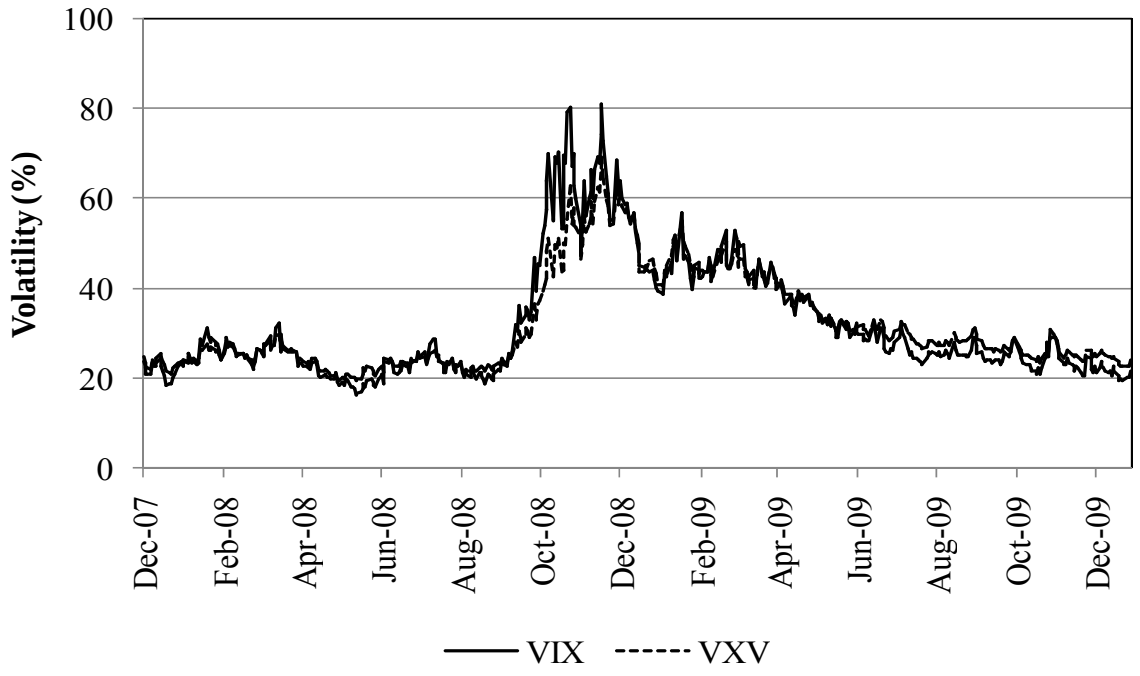

Figure 3: Daily closing values of VIX and VXV indexes. Plot shows strong correlation between the VIX and VXV Indexes. Additionally, the difference between VIX and VXV indexes was the highest just after the Lehman Brothers bankruptcy in September 2008. The sample period is December 4, 2007 – December 31, 2009. Source: CBOE

1.4 Implied Volatility Skew of S&P 500 Index Options

Several market participants use index options to either protect their investments or express their market views. Black-Scholes-Merton Model (BSM) is the industry standard for pricing equity and foreign exchange options. For a given stock or index, BSM assumes that the implied volatility is the same across option strike prices. However, several studies have shown that market prices for options do not reflect this constant volatility assumption and instead show a skew. Figures 4a and 4b show the S&P 500 Implied volatility skew and surface plots. Market participants define volatility skew in different ways; for the purpose of this report, we define it as the difference in implied volatilities of 30 days maturity S&P 500 Index options that are 90% moneyness and 110% moneyness. Moneyness is defined as:

% moneyness = Strike Price / Spot Price

Figure 4a: SPX Index 30 Days Implied Volatility Skew

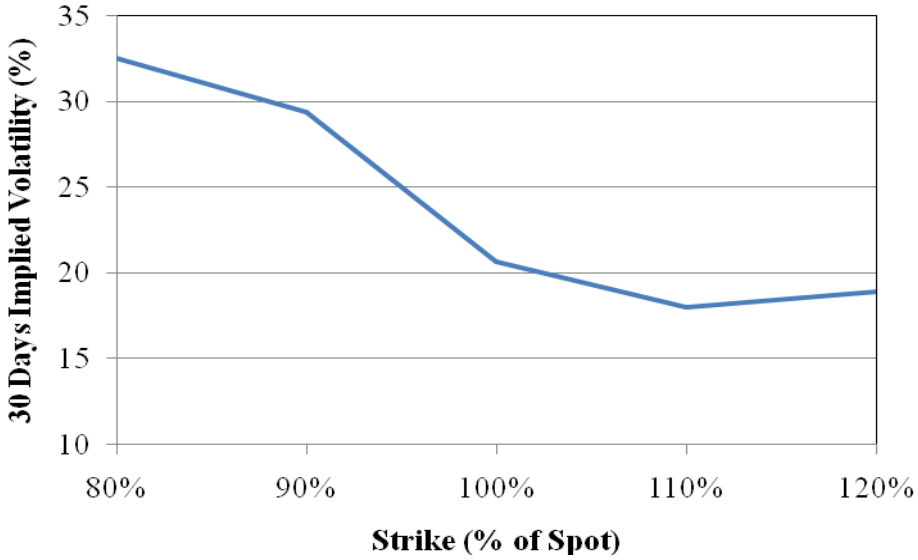

Figure 4a: S&P 500 Implied Volatility Skew on 11/30/2009. The skew referes to the pattern where the implied volatility of in-the-money options is higher than the implied volatility of at-the-money options. Source: Bloomberg

Figure 4b: SPX Index Implied Volatility Surface

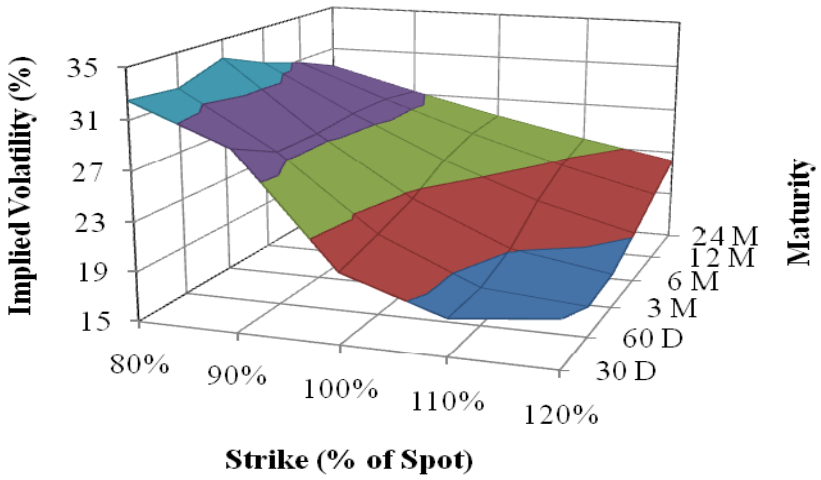

Figure 4b: S&P 500 Implied Volatility Surface on 11/30/2009. The implied volatility surface is a plot of implied volatility as a function of both strike price and time to maturity. It can also be described as a plot of volatility skews with different time to maturity. Source: Bloomberg

1.5 Implied Volatility of the Options on VIX

Since the introduction of options on VIX in 2006, VIX options have become very popular with investors trying to express their views on market volatility. VIX options are European style options and can only be exercised on the expiration date of the contract. The valuation of VIX options uses the expected, or forward, value of VIX on the expiration date and not the spot value of the VIX Index. Further, VIX options are priced differently from Stock or Index options. Stock or Index option pricing models assume that the underlying asset is lognormally distributed, whereas, VIX is not lognormal (in a lognormal world, the asset price can go to zero, but VIX cannot go to zero because it would mean that there is no volatility in S&P 500 Index). Another distinct feature of VIX options is very high implied volatility (i.e., very high volatility of volatility). Volatility of volatility, as defined here, is a measure of the volatility of the VIX

forward values. Put another way, this is a measure of how volatile markets views are about expected 30 day S&P 500 volatility on the expiration date of the contract.

Figure 5: Historical Trading Volumes of VIX Options

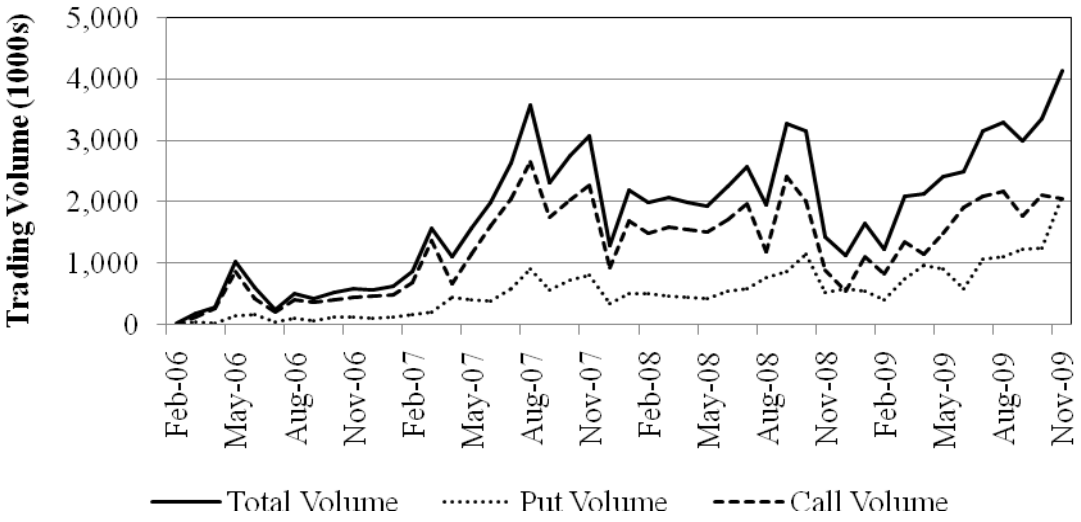

Figure 5: Monthly trading volumes for Put and Call Options on VIX. Total volume is the sum of put and call volumes. The increasing trading volume highlights the growing popularity of VIX Options. The sample period is February 2006 – November 2009. Source: CBOE

II. PREVIOUS WORK

Brenner and Galai (1989) first introduced the concept of volatility derivatives and the need for a volatility index. Moran and Dash (2007) demonstrated that VIX Futures contracts have a negative correlation to the S&P 500 returns and how they could be used in a hedging portfolio to improve the efficiency of investor portfolios. Further, they tested the behavior of the VIX Futures contracts during periods of high market volatility to demonstrate that the beneficial qualities of VIX exposure can be obtained through the use of VIX-linked Futures and Options contracts. Zhang, Shu and Brenner (2010) analyzed recent data to establish a theoretical relationship between VIX Futures and VIX and suggested a model that gives good VIX Futures prices under normal market conditions which could be used in pricing VIX Options.

Despite the popularity of the Black-Scholes model for pricing options, many researchers have shown that the model's constant volatility assumption across different strikes is inconsistent with market prices. It has been shown that the implied volatilities generally increase as the strike price decreases (Poon and Granger 2003). A popular explanation for the existence of volatility skew relate to the *Leverage Effect*. The *Leverage Effect* theory posits that as the stock index value decreases, the leverage of the market increases, which makes the equity more risky. Thus, the implied volatility increases for the lower strike prices.

Extensive research has been done on the leverage effect in the stock market returns since this phenomenon was first described by Black (1976). Whaley (2000) demonstrated the negative correlation between the S&P 500 returns and changes in the VIX Index. He showed that this negative relationship between S&P 500 returns and change in VIX is asymmetric – the index falls more when the VIX increases but it doesn't rise by as much when VIX falls. According to

Whaley (2000), the S&P index falls by 0.707% for a 100 bps increase in VIX and the S&P 500 index rises by 0.469% for a 100 bps drop in VIX.

III. DATA DESCRIPTION

For the purpose of our analysis, we reviewed daily data from January 2005 – November 2009. We divided this period into three distinct sub-periods called Pre-Crisis, Crisis and Post-Crisis. While there are different opinions about the exact date of the onset of the financial crisis, we have used March 17th 2008, the date on which US Investment Bank Bear Stearns & Co was taken over by JP Morgan, as the cutoff for our Pre-Crisis/Crisis periods. While it is difficult to exactly pinpoint when the crisis ended, we picked March 31st 2009 as the end date for the crisis because the S&P 500 index rebounded well from its lowest value by the end of March. Table 1 shows our assumptions regarding the study period dates.

Table 1: Classification of Study Period

Period	Start Date	End Date
Pre-Crisis	3-Jan-05	16-Mar-08
Crisis	17-Mar-08	31-Mar-09
Post-Crisis	1-Apr-09	30-Nov-09

While the dates for the Crisis and Post-Crisis periods are consistent throughout the report, the start date for the Pre-Crisis period is different for different measures of volatility due to data availability. We have provided this information along with the exhibits in this report. Table 2 shows the sources of data used in the analysis followed by a brief description of the data.

Table 2: Data sources

Data Description	Data Source	Website Link
S&P 500: Adjusted Close Values	Yahoo Finance	http://finance.yahoo.com/q/hp?s=SPX
VIX: Daily Closing Values	Chicago Board of Options Exchange (CBOE)	http://www.cboe.com/micro/VIX/historical.aspx
VXV: Daily Closing Values	Chicago Board of Options Exchange (CBOE)	http://www.cboe.com/micro/vxv/
VIX Futures: Daily Settle Values	CBOE Futures Exchange (CFE)	http://cfe.cboe.com/Products/historicalVIX.aspx
S&P 500: Implied Volatility Data	Bloomberg	
VIX Options: Call Options Prices	CBOE Market Data Express Service	http://www.marketdataexpress.com/

S&P 500 Index Data: We used the adjusted daily closing values of the SPX index as they incorporate the dividend yield. We assumed that the SPX daily returns are lognormal and used the percentage daily returns in estimating the negative correlation between index returns and volatility.

VIX and VXV Indexes: We used the daily closing values for both the indexes. VXV data is available from December 4, 2007 onwards. Thus, we used the data from the period December 2007 – December 2009 when analyzing VIX versus VXV.

VIX Futures: We considered using the daily "settle" values for the various VIX Futures contracts that were traded each day. However, the maturities of these contracts were not fixed and would decrease every day. So, we created a constant 30-day maturity VIX Futures contract through linear interpolation of available VIX Futures contracts with varying maturities.

S&P 500 Implied Volatility Skew Data: We obtained the implied volatilities of S&P 500 Index options that are 90% money, 100% money and 110% money from Bloomberg. We then obtained the volatility skew as the difference in implied volatilities of options at 90% money and 110% money. Appendix A provides details of the methodology that Bloomberg uses to estimate the implied volatilities for 30 days maturity options at a particular level of moneyness.

Implied Volatility of VIX Options (volatility of volatility): To study the volatility of volatility, we estimated the volatility implied by the VIX Options prices. Since there are no standard VIX Options pricing models, we decided to use the Black model for futures as a reasonable solution. For each trading date, we first mapped the available call option contracts to VIX Futures contracts such that the VIX Futures maturity is later than the options expiry date. For all VIX Futures contracts that satisfied this condition, we picked the one with the earliest maturity as the underlying for the VIX Options contract. Next, we picked option strike prices that straddle the VIX Futures closing values. Using the VIX Futures value as the price of the underlying, 1-month T-Bill rates as the riskless rate, the difference between the option expiry date and the current trading date as time to expiry, and option strikes and option prices from the selected call option contracts, we estimated the implied volatility of the VIX options.

We also estimated the volatility of VIX Index and the computed 30-day VIX Futures by calculating the standard deviation of percentage daily changes in their respective values.

IV. RESULTS and DISCUSSION

IV.1. Behavior of Stock market volatility and different measures of volatility

Table 3 below provides a summary of the stock market behavior, as measured by the S&P 500, during the study period. The results clearly show that the volatility of the index returns was significantly higher during the Crisis period compared to other periods.

Table 3: Summary Statistics for S&P 500 Index Performance

Period	S&P 500 Index Average Value	Annualized Volatility of S&P 500 Index Returns[2]
Pre-Crisis	1,335	13.4%
Crisis	1,098	43.6%
Post-Crisis	984	20.9%
All Data[1]	1,233	24.2%

1. 'All Data' corresponds to the time period January 2005 - November 2009

2. Annualized volatility is estimated by multiplying the standard deviation of daily returns by sqrt(250)

It is interesting to note that the average value of the S&P 500 index was higher during the Crisis period than that during the Post-Crisis period. However, this could be due to our selection of the dates for each period. If one were to analyze the performance of the SPX index from the time of the Lehman Brothers bankruptcy in September 2008 to the market bottom in March 2009, the core of the crisis, the average value of the index is 884, which is lower than the average value during the Post-Crisis period. Similarly, the annualized volatility of the SPX returns during the September 2008 – March 2009 period, the core of the crisis, is 56.9%. This confirms that the market volatility was significantly higher during the crisis period compared to other periods.

Tables 4a and 4b provides a summary of different volatility measures that we analyzed.

Table 4a: Performance Summary of Volatility Measures – Average Values

Period	VIX Average	VIX Futures Average[2]	VXV Average
Pre-Crisis	24.76	25.28	24.87
Crisis	36.85	34.70	35.39
Post-Crisis	30.70	29.80	32.93
All Data[1]	32.16	31.74	32.03

1. 'All Data' corresponds to the period December 4, 2007 - November 30, 2009
2. Refers to the 30 days constant maturity VIX Futures Index that we constructed

Table 4b: Performance Summary of Volatility Measures - Annualized Volatility of % Daily Changes[1]

Period	VIX	VIX Futures[2]	VXV
Pre-Crisis	96%	47%	64%
Crisis	128%	71%	86%
Post-Crisis	83%	48%	50%
All Data[3]	107%	61%	72%

1. Annualized volatility is estimated by multiplying the standard deviation of % daily changes by sqrt(250)
2. Refers to the 30 days constant maturity VIX Futures Index that we constructed
3. 'All Data' corresponds to the period December 4, 2007 - November 30, 2009

For all three volatility measures, the average values for the different periods were comparable. The average values for the Crisis period were 49%, 37% and 42% higher than the Pre-Crisis period averages for the VIX, VIX Futures and VXV respectively. These results make sense intuitively: VIX Futures are mean reverting and thus don't change as much as the VIX Index. Additionally, since VXV measures 90 day expected market volatility and incorporates the

expectations of the 30 day market volatility (VIX), it is expected to be more stable than the VIX Index. Further, the average values for the Post-Crisis period were 24%, 18% and 32% higher than the Pre-Crisis period averages for the VIX, VIX Futures and VXV respectively. These results show that even as the stock market rebounded from its March 2009 bottom, the average values for three volatility measures were still significantly higher than their Pre-Crisis averages.

Results from Table 4b provide evidence that these three measures of volatility were more volatile during the Crisis period compared to other periods. The annualized volatility values for the VIX, VIX Futures and VXV for the Crisis period were 34%, 50% and 33% higher than the Pre-Crisis period volatilities. For the Post-Crisis period, the volatilities of VIX, VIX Futures and VXV were 87%, 101% and 77% respectively of their Pre-Crisis period values.

The behavior of the volatility of VIX Futures was different from that of the volatility of VIX and VXV Indexes. During the Crisis, volatility of VIX Futures increased more than that of the other measures, whereas during the Post-Crisis period, the volatility of VIX Futures reverted to its Pre-Crisis level while VIX and VXV became more stable compared to Pre-Crisis. The reason for this behavior could be related to VIX Future's Pre-Crisis value. During the Pre-Crisis period, VIX Futures had the lowest volatility of all three measures: the volatility of VIX and VXV Indexes were 2.0 and 1.36 times that of the 30 days constant maturity VIX Futures.

Volatility of Volatility: Implied Volatility of VIX Options

Figure 6 shows a plot of the average monthly implied volatilities that were estimated using the At-The-Money VIX Call Options. Not surprisingly, the implied volatility of VIX Options was highest in October 2008, the month following the bankruptcy of Lehman Brothers. The spike in implied volatility in August 2007 could be related a specific action – the French

bank BNP Paribas decided to freeze redemptions from its structured products funds due to liquidity concerns, which resulted in a panic in the market.

Figure 6: Average Impied Volatility of ATM VIX Options

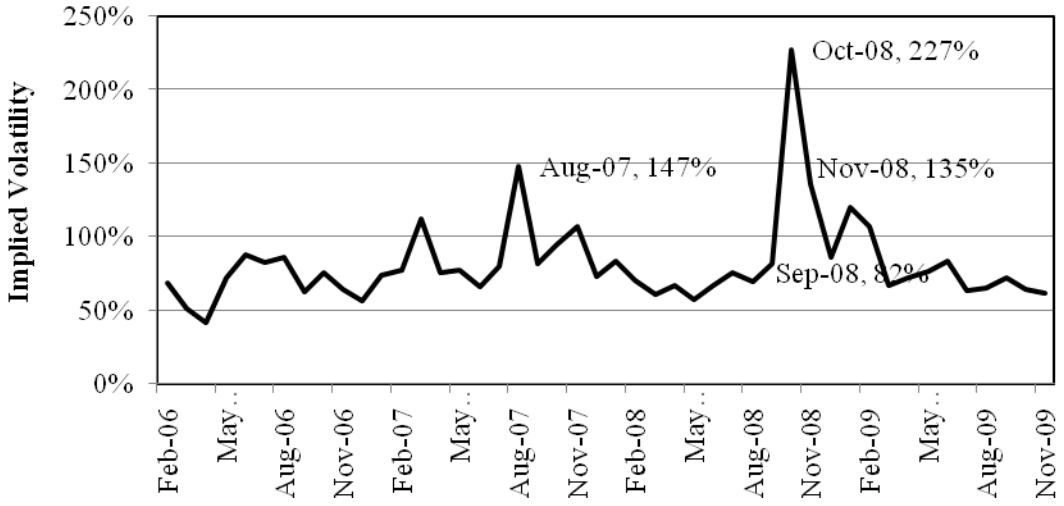

Figure 6: Average implied volatility of At-The-Money VIX Call Options. The August 2007 spike in implied volatility correspond to the problems with the BNP Paribas structured funds and the Oct 2008 peak corresponds to the market panic following the Lehman Brothers bankruptcy in September 2008. The sample period is February 2006 – November 2009. Source: CBOE MarketData Express Service

Table 5: Comparison of different measures of Volatility of Volatility

Period	VIX Options - Average Implied Volatility[1]	Volatility of VIX % Daily Changes	Volatility of VIX Futures % Daily Changes[2]
Pre-Crisis[3]	80%	119%	56%
Crisis	96%	128%	71%
Post-Crisis	69%	83%	48%
All Data[4]	83%	116%	59%

1. At-The-Money Call Options were used to estimate implied volatility using the Black model
2. Refers to the 30 days constant maturity VIX Futures Index that we constructed

3. Pre-Crisis volatility estimates for VIX and VIX Futures are different from that reported in Table 4b due to the different sample periods.
4. All-Data corresponds to the period February 24, 2006 - November 30, 2009

Data in Table 5 shows that the implied volatility of VIX options increased during the Crisis period. Further, as the market rebounded from its March 2009 lows, the implied volatility of VIX Options dropped to levels lower than were observed before the Crisis. From Figure 6, it is easy to see that, except for a few spikes, the average monthly implied volatilities were quite similar.

Results in Table 5 also show that the implied volatility of VIX Options is lower than the realized volatility of VIX for all periods. This difference is to be expected because the underlying for the VIP Options is VIX Forwards, which are less volatile than VIX due to mean reversion.

IV.2. Term Structure of Volatility: VIX vs VXV

Tables 4a and 4b showed the average values of the VIX and VXV Indexes and their annualized volatilities. Table 6 provides the correlation between these indexes and the statistical summary of the VIX:VXV ratio.

Table 6: Summary Statistics for the VIX:VXV Ratio

Period	Correlation between VIX and VXV	Average VIX:VXV Ratio	Standard Deviation of VIX:VXV Ratio	% Time VIX:VXV Ratio > 1[1]
Pre-Crisis	0.961	0.993	0.053	44%
Crisis	0.967	1.021	0.111	46%
Post-Crisis	0.984	0.928	0.043	2%
All Data[2]	0.969	0.983	0.095	30%

1. % Time is estimated as the % of days the ratio of closing values of VIX and VXV was greater than 1

2. 'All Data' refers to the time period December 4, 2007 - November 30, 2009

The above results provide some interesting observations. First, there is very strong correlation between these two indexes, as expected. Second, for 70% of the study period, the VXV Index was higher than the VIX Index, indicating that the market expected the medium term stock market volatility to be higher than the short term volatility. This effect is very pronounced for the Post-Crisis period where the VXV Index was higher than the VIX Index for almost 98% of the time and the average VIX:VXV ratio was the smallest. The behavior of the VIX:VXV ratio during the Crisis period was different from other periods – during the Crisis period, on average, the VIX Index was higher than the VXV Index, indicating more near-term uncertainty. Moreover, the VIX:VXV ratio during the Crisis period was twice as volatile as this ratio in other periods, as seen by the standard deviation of this ratio. Appendix B shows the results of T-tests which indicate that the average VIX:VXV ratio during the crisis is different from 1 and different from the ratios for the other periods at 95% significance levels.

IV.3. Volatility Skew

Figure 7 and Table 7 provide a summary of the regression of 30 days Implied Volatility Skew of the S&P 500 Options on the 30 days Implied Volatility of the At-The-Money S&P 500 Options. The results indicate that there is a strong correlation between the Volatility Skew and the ATM Implied Volatility during the Crisis period, whereas during other periods, the correlation is very weak. For the Post-Crisis period, the small t-statistic for the regression

indicates that the linear relationship between Volatility Skew and ATM Volatility cannot be established at high significance levels. We posit that the reason for the weak correlation during the Post-Crisis period could be due to the low variance of both the Volatility Skew and the ATM Volatility during this period. The standard deviation as a percentage of the average was the lowest for both ATM Volatility (19.4%) and the Volatility Skew (15.2%) during the Post-Crisis period. As a result, the observed data may not have had sufficient variability to establish a linear relationship with a high level of significance. This suggested to us that the volatility skew might be level dependent but insensitive for small changes in ATM Volatility. So, we divided the data into groups based on bands of ATM Volatility and performed a regression between the Average ATM Volatility and Average Volatility skew. Table 8 shows the ATM Volatility and Volatility Skew data by bands and Table 9 shows the summary of the regression analysis using the bands.

Figure 7: S&P 500 Implied Volatility Skew Vs ATM Implied Volatility

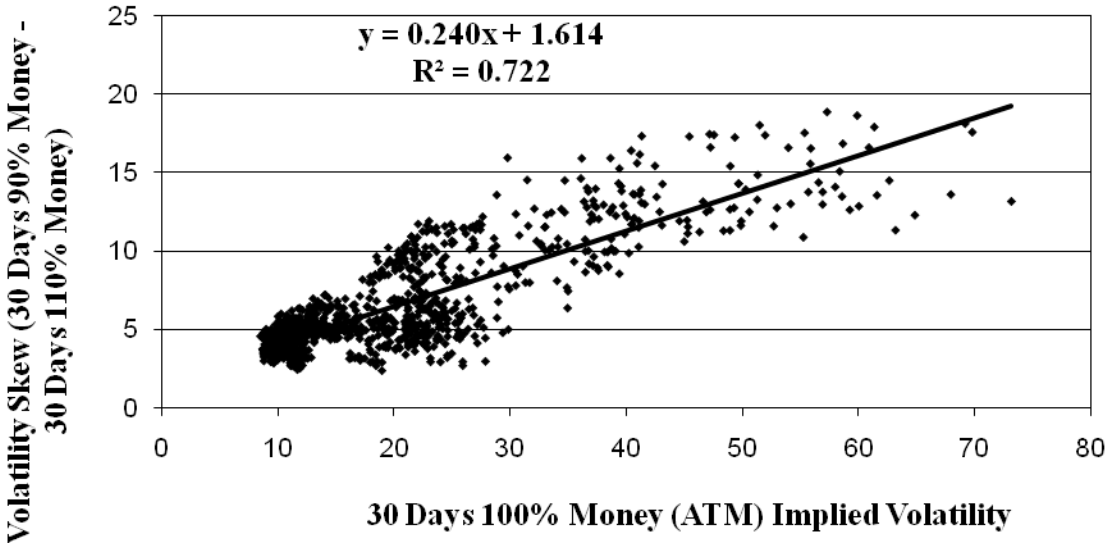

Figure 7: Regression of Implied Volatility Skew of 30 days S&P 500 Options on the At-The-Money Implied Volatility of 30 days S&P 500 Options. The regression results are significant and

indicate that a strong correlation volatility skew and the level of volatility. The sample period for this study is January 2005 – November 2009. Data Source: Bloomberg

Table 7: Summary of Regression of S&P 500 Implied Volatility Skew on ATM Implied Volatility

Period	Correlation	Average ATM IV[1]	Std Dev ATM IV[1]	Average Vol Skew[2]	Std Dev Vol Skew[2]	Coefficient of ATM Volatility	t-Statistic
Pre-Crisis	0.406	13.4	4.7	4.6	1.05	0.089	12.58
Crisis	0.853	33.1	14.1	9.4	4.43	0.268	26.46
Post-Crisis	0.019	25.3	4.9	9.3	1.42	0.005	0.232
All Data[3]	0.849	19.1	11.4	6.2	3.23	0.240	56.05

1. 'ATM IV' refers to the At-The-Money Implied Volatility (100% money) of 30 days S&P 500 Options

2. 'Vol Skew' refers to the implied volatility skew of 30 days S&P 500 Options (90% money Implied Volatility - 110% money Implied Volatility)

3. 'All Data' refers to the time period January 2005 - November 2009

Table 8: Summary of data grouped by ATM Implied Volatility Bands

Group	30 Days 100% Money Implied Volatility	30 Days Implied Volatility Skew
5-10	9.50	3.93
10-15	11.60	4.52
15-20	17.64	5.43
20-25	22.29	6.70
25-30	26.82	7.70
30-35	32.80	10.36
35-40	37.82	11.70
40-45	41.49	13.01
45-50	47.72	13.56
50-55	52.02	14.35

55-60	57.35	14.83
60-65	62.37	14.32
65-70	68.93	16.40
70-75	73.11	13.12

Table 9: Summary of Regression of S&P 500 Volatility Skew on 30 days ATM Volatility

	Coefficient	t-Statistic	Regression R-square	Regression F
Intercept	3.157	3.35	0.87	80.3
30 Days 100% Money	0.188	8.96		

1. The sample period for this study is January 2005 - October 2009
2. The regression equation is: Volatility Skew = 3.157 + 0.188 * ATM Volatility

The large t-statistic for the regression indicates with a high level of significance that there is linear relationship between the Volatility Skew and the ATM Volatility. Moreover, an R-square of 0.87 indicates that the ATM Volatility explains 87% of the variability in the Volatility Skew. These results and the regression results for the Post-Crisis period shown in Table 7 (where the correlation was weak due to low variance of the independent and dependent variables) support our hypothesis that the Volatility Skew is dependent on the level of ATM Volatility but is insensitive to small changes in ATM Volatility.

IV.4. Leverage Effect: Relationship between S&P 500 returns, VIX and VIX Futures

Table 10 below shows the results of our analysis. Appendix C shows the complete results of the regression analysis for each period that we analyzed.

Table 10: Regression Results – Relationship between daily SPX returns (dependent variable) and daily changes in VIX (independent variable)

Period	VIX Increases 100 bps[3]	VIX Decreases 100 bps[3]	R-Square	Regression F	Intercept
Pre-Crisis	-0.745%	0.539%	0.71	961	0.115%
Crisis	-1.468%	0.557%	0.76	423	0.265%
Post-Crisis	-0.645%	0.700%	0.56	111	0.073%
All Data[1]	-0.861%	0.593%	0.72	1588	0.118%
Whaley[2]	-0.707%	0.469%	0.56		

1. 'All Data' corresponds to the time period January 2005 - November 2009
2. In 2000, Robert Whaley estimated the relationship between weekly changes in VIX values and impact on S&P 500 based on data from 1986 - 1999
3. The data in these columns represents the changes in S&P 500 associated with a 100 bps increase or decrease in VIX

During the Pre-Crisis and All-Data scenarios, the relationship between the SPX Index returns and changes in VIX is comparable to the results reported by Whaley. During the Crisis period, however, the relationship between S&P 500 returns and VIX change is different from that in other periods. During the crisis, a -1.468% return of S&P 500 index value is associated with a 100 bps increase in VIX, whereas during the other periods, S&P 500 index returns of -0.65% to -0.86% were associated with a 100 bps increase in VIX. Although we regressed S&P 500 returns on VIX Change, we do not imply that the change in VIX values causes the S&P 500 to decrease or increase. The mechanics of the interaction could be described as follows: if an exogenous negative shock impacts the system, it would cause a drop in the value of the S&P 500

index. This could cause an instantaneous increase in the volatility, which increases the value of the VIX.

The results show that the VIX index was less sensitive to drops in the value of S&P 500 during the crisis period compared to other periods. It is possible that during the crisis, the implied volatility on the S&P options was very high and thus bigger changes in S&P 500 were required during this period, compared to other periods, to cause the same change in implied volatility. The implied volatility data for the At-The-Money (ATM) SPX options that we obtained from Bloomberg confirms this hypothesis – the average ATM implied volatility during the crisis period was 33.1 compared to 19.1 for the entire study period. Additionally, during the Post-Crisis period, the impact on S&P 500 was higher when VIX dropped than when VIX increased. Again, without implying causality, what this means is that VIX dropped less for a certain increase in the S&P 500 value during this period compared to other periods. This could be because investors were very risk-averse after experiencing the turbulent markets during the crisis period and thus were slow to change their expectations about future volatility despite the improvements in S&P500.

Table 11: Correlation of SPX Returns with VIX and 30-day maturity VIX Futures

Period	SPX Returns Correlation with VIX	SPX Returns Correlation with 30-day maturity VIX Futures
Pre-Crisis	-0.84	-0.80
Crisis	-0.87	-0.85
Post-Crisis	-0.75	-0.82
All-Data[1]	-0.85	-0.84

1. All-Data corresponds to the period January 2006 - November 2009

Table 11 shows that the 30 day maturity VIX Futures contract has a strong negative correlation with SPX returns. Moreover, the results show that the correlation of the VIX Futures contract with SPX returns is quite comparable to the correlation between SPX returns and VIX changes, indicating that VIX Futures provide a good hedge against market volatility.

V. SUMMARY

The stock market volatility, as measured by the volatility of S&P 500 Index, increased from 13.4% during the Pre-Crisis period to 43.6% during the Crisis (325% of Pre-Crisis level). Even after the S&P 500 Index rebounded from its March 2009 lows, the market volatility reverted only to 20.9%, which is 156% of the Pre-Crisis level. Similar behavior was also observed in the other measures of Volatility that we analyzed, i.e., VIX, VIX Futures and VXV. All three measures of volatility increased significantly during the Crisis period compared to the Pre-Crisis values. Moreover, as the market rebounded during the Post-Crisis period, all three measures decreased from their Crisis period highs, but did not revert back to the pre-Crisis level, indicating that market participants continued to expect higher market volatility despite the rally in the S&P 500 Index. The behavior of observed Volatility of Volatility (VIX, VIX Futures and VXV) and expected volatility of volatility (Implied Volatility of VIX Call Options) was a little different from that of Market Volatility. The Volatility of Volatility during the Crisis period increased from the Pre-Crisis levels, similar to the behavior of market volatility. However, during the Post-Crisis period, the volatility of volatility reverted to levels lower than those observed during the Pre-Crisis levels for most measures that we analyzed, unlike the market volatility values which remained above their Pre-Crisis levels.

We found the leverage effect during the study period. The relationship between market returns and volatility during the Pre-Crisis period was similar to that found by Whaley (2000). However, during the Crisis and Post-Crisis periods, this relationship seemed different. During the Crisis period, VIX seemed to be less sensitive to decreases in SPX Index, whereas during the Post-Crisis period, VIX seemed to be less sensitive to increases in SPX Index.

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Appendix A – Bloomberg Implied Volatility Calculations

I. Introduction

Bloomberg equity implied volatility datasets consist of implied volatilities for fixed maturities and moneyness levels based on out of the money option prices (4 pm closing mid prices). Their methodology is split into 2 parts: calculation of the implied forward price and calculation of implied volatility surface consistent with the implied forward price.

II. Implied Forward Price

First, Bloomberg calculates the European Call and Put option prices from mid prices of American options, mid-underlying price (S), rates from Bloomberg S23 curve and dividends based on Bloomberg forecast model. Next, using put call parity, the implied forward price is calculated from the European call and put prices closest to the at-the-money and the interpolated risk-free rate as follows:

$$F_{impl} = Strike + e^{rt}\,(c^E - p^E)$$

Where c^E and p^E are the European Call and Put option prices.

To calculate the implied forwards for fixed maturity points (30, 60 days etc), the forward prices are transformed into returns using the following formula:

$$r_{impl}\,(T) = \left(\frac{1}{T}\right) \ln\left(\frac{F_{impl}\,(T)}{S}\right)$$

Finally,

$$F_{impl}\,(T) = Spot * \exp(r_{impl} * T)$$

III. Volatility Surface

The implied volatility σ_{impl} for each maturity and strike level is computed by equating the Black-Scholes formula to the European option price calculated using the methodology of section II and the implied forward also calculated in section II.

$$c^E = e^{-rt} F_{impl} N\left(\frac{\ln\left(\frac{F_{impl}}{K}\right) + 0.5 * \sigma_{impl}^2 * T}{\sigma_{impl} \sqrt{T}} \right) - Ke^{-rt} N\left(\frac{\ln\left(\frac{F_{impl}}{K}\right) - 0.5 * \sigma_{impl}^2 * T}{\sigma_{impl} \sqrt{T}} \right)$$

To calculate the implied volatility at a fixed level of moneyness, Bloomberg uses non-parametric interpolation in variance space across strikes and to interpolate in time, they use a Hermite cubic spline interpolation in total implied variance space.

IV. Definition

$$\% \text{ Moneyness} = \frac{Strike}{Spot}$$

Appendix B – VIX:VXV T-test results

Two-Sample T-Test and CI: Crisis Period, Post-Crisis Period

Two-sample T for Crisis Period vs Post-Crisis Period

	N	Mean	StDev	SE Mean
Crisis	263	1.021	0.111	0.0069
Post-Crisis	191	0.9278	0.0434	0.0031

Difference = mu (Crisis) - mu (Post-Crisis)
Estimate for difference: 0.09280
95% CI for difference: (0.07796, 0.10765)
T-Test of difference = 0 (vs not =): T-Value = 12.29
P-Value = 0.000 DF = 361

Two-Sample T-Test and CI: Crisis Period, Pre-Crisis Period

Two-sample T for Crisis Period vs Pre-Crisis Period

	N	Mean	StDev	SE Mean
Crisis	263	1.021	0.111	0.0069
Pre-Crisis	70	0.9929	0.0526	0.0063

Difference = mu (Crisis) - mu (Pre-Crisis)
Estimate for difference: 0.02777
95% CI for difference: (0.00944, 0.04610)
T-Test of difference = 0 (vs not =): T-Value = 2.98
P-Value = 0.003 DF = 241

One-Sample T: Crisis Period

Test of mu = 1 vs not = 1

Variable	N	Mean	StDev	SE Mean	95% CI	T	P
Crisis	263	1.02064	0.11133	0.00686	(1.00713, 1.03416)	3.01	0.003

Appendix C – Leverage Effect: Regression Analysis Results

Regression Results - Pre Crisis

Multiple R	0.8428
R Square	0.7104
Adj R Square	0.7096
Standard Error	0.0046
Observations	787

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.0401	0.0200	961.4	1.11E-211
Residual	784	0.0163	0.0000		
Total	786	0.0564			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001	0.000	4.538	0.000	0.001	0.002	0.001	0.002
ΔVIX	-0.005	0.000	-30.125	0.000	-0.006	-0.005	-0.006	-0.005
ΔVIX^+	-0.002	0.000	-4.802	0.000	-0.003	-0.001	-0.003	-0.001

Regression Results - Crisis

Multiple R	0.875
R Square	0.765
Adj R Square	0.763
Standard Error	0.013
Observations	263

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.153	0.076	423.1	0.000
Residual	260	0.047	0.000		
Total	262	0.200			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.003	0.001	2.073	0.039	0.000	0.005	0.000	0.005
ΔVIX	-0.006	0.000	-19.77	0.000	-0.006	-0.005	-0.006	-0.005
ΔVIX^+	-0.009	0.002	-4.278	0.000	-0.013	-0.005	-0.013	-0.005

Note: $\Delta VIX^+ = \Delta VIX$ if $\Delta VIX > 0$; otherwise $\Delta VIX^+ = 0$

Regression Results - Post Crisis

Multiple R	0.747
R Square	0.559
Adj R Square	0.554
Standard Error	0.009
Observations	178

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.017	0.009	110.8	0.000
Residual	175	0.014	0.000		
Total	177	0.031			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001	0.001	0.659	0.511	-0.001	0.003	-0.001	0.003
ΔVIX	-0.007	0.001	-9.96	0.000	-0.008	-0.006	-0.008	-0.006
ΔVIX^+	0.001	0.002	0.266	0.790	-0.004	0.005	-0.004	0.005

Regression Results - All Data

Multiple R	0.8495
R Square	0.7217
Adj R Square	0.7212
Standard Error	0.0081
Observations	1228

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.2082	0.1041	1588	0
Residual	1225	0.0803	0.0001		
Total	1227	0.2884			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0012	0.0003	3.45	0.0006	0.0005	0.0019	0.0005	0.002
ΔVIX	-0.0059	0.0001	-43.79	0.0000	-0.0062	-0.006	-0.006	-0.006
ΔVIX^+	-0.0027	0.0006	-4.79	0.0000	-0.0038	-0.002	-0.004	-0.002

Note: $\Delta VIX^{+} = \Delta VIX$ if $\Delta VIX > 0$; otherwise $\Delta VIX^{+} = 0$