Dynamic Models of Portfolio Credit Risk: A Simplified Approach

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Portfolio Credit Derivatives

- Key product is a CDO
- Protection seller agrees to insure all losses on the portfolio that are between X% and Y% of the portfolio principal for life of contract (e.g. 5 yrs)
- Initial tranche principal is (Y–X)% of the portfolio principal
- Protection buyer pays a spread on the remaining tranche principal periodically (e.g. at the each quarter)
- Tranches of standard portfolios (iTraxx, CDX IG, etc) trade very actively

CDO models

- Standard market model is one-factor Gaussian copula model of time to default
- Alternatives that have been proposed: t-, double-t, Clayton, Archimedian, Marshall Olkin, implied copula
- All are static models. They provide a probability distribution for the loss over the life of the model, but do not describe how the loss evolves

Dynamic Models for Portfolio Losses: Prior Research

- **Structural:** Albanese et al; Baxter (2006); Hull et al (2005)
- **Reduced Form**: Duffie and Gârleanu (2001), Chapovsky et al (2006), Graziano and Rogers (2005), Hurd and Kuznetsov (2005), and Joshi and Stacey (2006)
- **Top Down:** Sidenius et al (2004), Bennani (2005), Schonbucher (2005),Errais et al (2006), Longstaff and Rajan (2006)

Our Objective

- Build a simple dynamic model of the evolution of losses that is easy to implement and easy to calibrate to market data
- The model is developed as a reduced form model, but can also be presented as a top down model

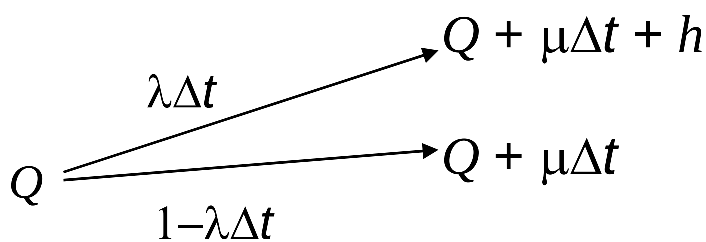
CDO Valuation

- Key to valuing a CDO lies in the calculation of expected principal on payment dates
  - Expected payment on a payment date equals spread times expected principal on that date
  - Expected payoff between payment dates equals reduction in expected principal between the dates
  - Expected accrual payments can be calculated from expected payoffs
- Expected principal can be calculated from the cumulative default probabilities and recovery rates of companies in the portfolio

The Model (Homogeneous Case)

$$dQ = \mu dt + dq$$

where Q is the an obligor's cumulative default probability and dq represents a jump that has intensity λ and jump size h



μ and λ are functions only of time and h is a function of the number of jumps so far. $\mu > 0$, $h > 0$, and Q is set equal to the minimum of 1 and the value given by the process

Implementation of Model

- Instruments such as CDOs, forward CDOs, and options on CDOs can be valued analytically
- Model can be represented as a binomial tree to value other more complicated structures such as leveraged super seniors with loss triggers

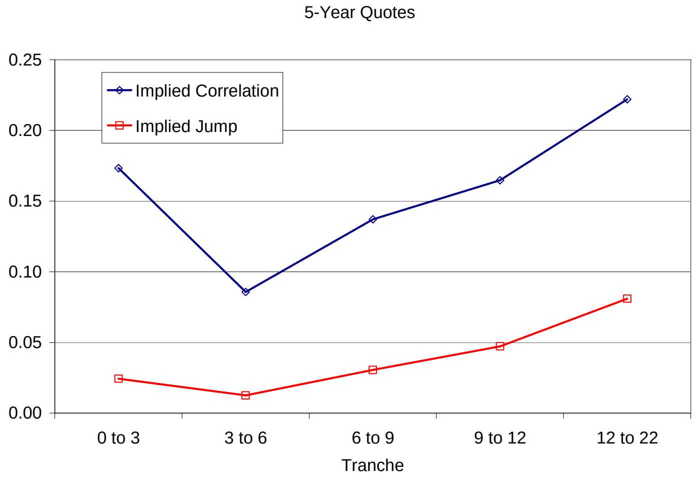
Illustrative Data

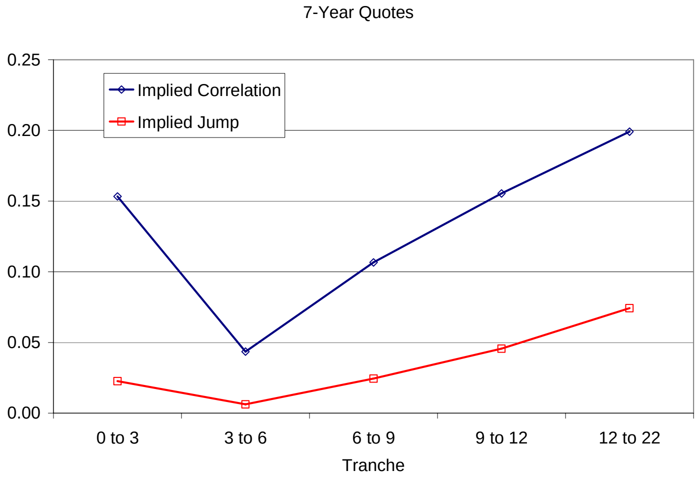
Table 1 iTraxx CDO tranche quotes December 4, 2006.					
a_L	a_H	3 yr	5 yr	7 yr	10 yr
0	0.03	n/a	12.38	27.00	40.75
0.03	0.06	n/a	56.00	135.00	337.00
0.06	0.09	n/a	14.00	37.00	99.00
0.09	0.12	n/a	5.00	18.00	42.00
0.12	0.22	n/a	2.00	5.50	14.00
Index		12.69	24.75	32.83	43.59

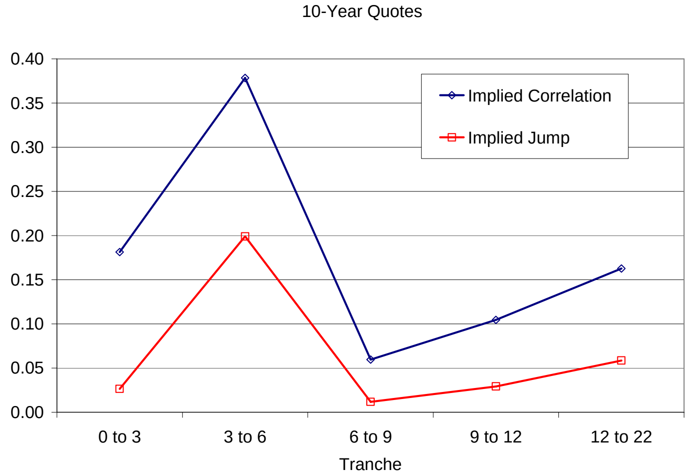
Simplest Version of Model

- Jump size is constant and $\mu(t)$, is zero
- Jump intensity, $\lambda(t)$ is chosen to match the term structure of CDS spreads
- There is then a one-to-one correspondence between tranche quotes and jump size
- Implied jump sizes are similar to implied correlations

Comparison of Implied Jump Sizes with Implied Tranche Correlations



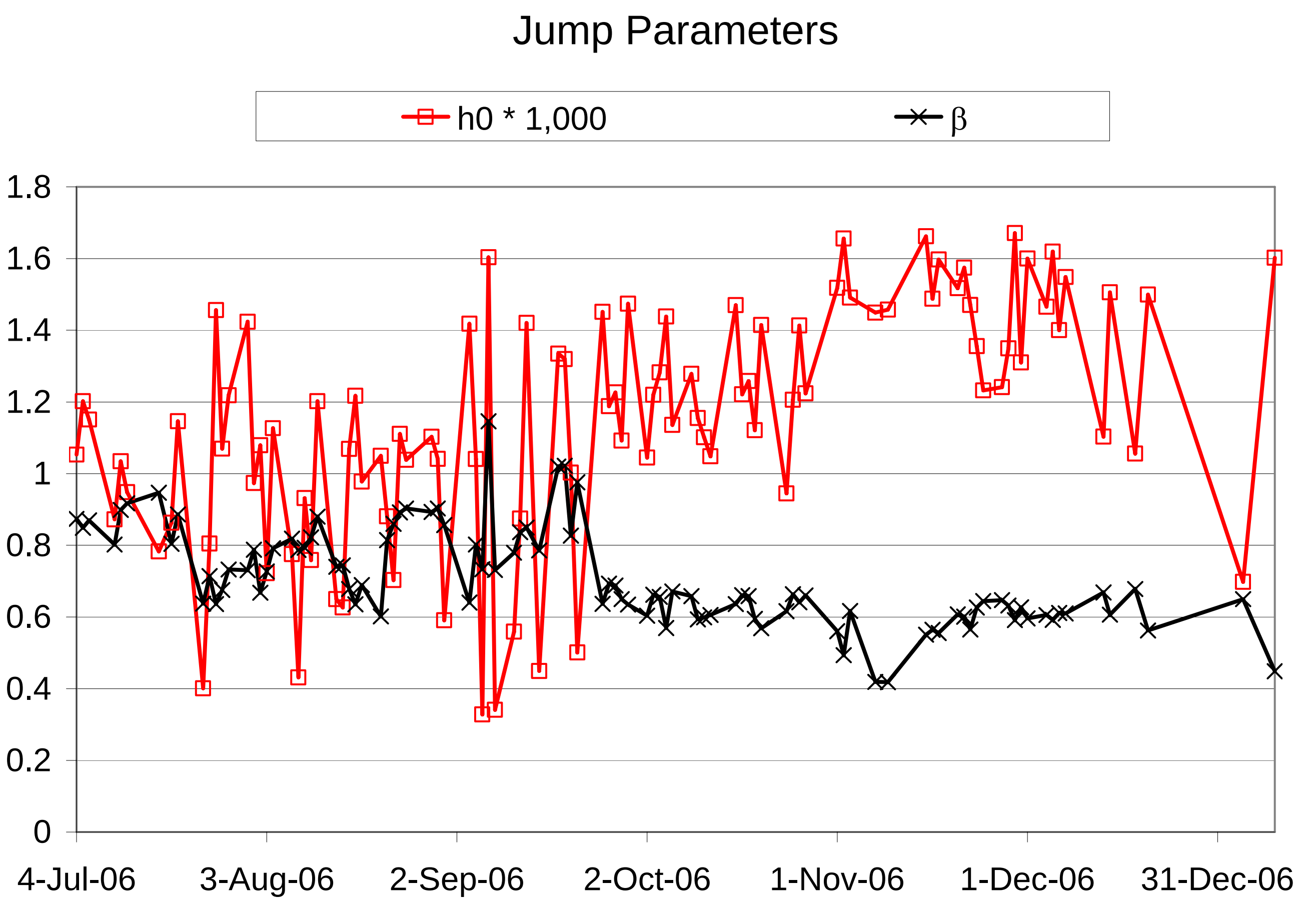




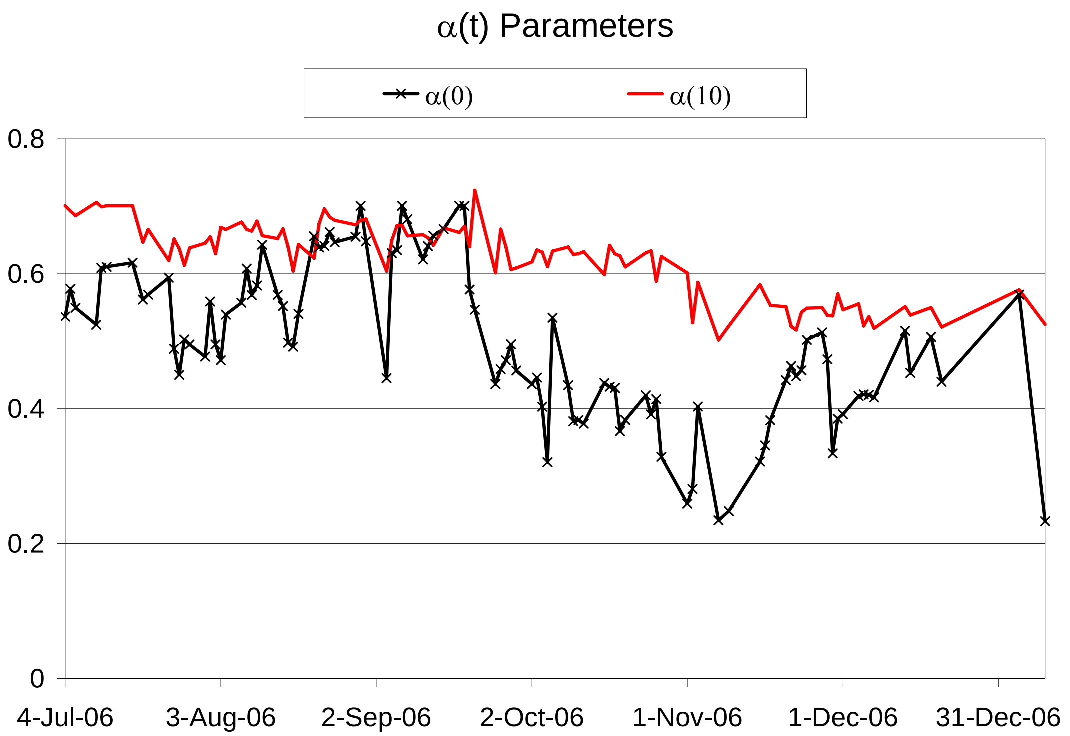
More Complex versions of the model. $\alpha(t)=\mu(t)/\mu_{max}(t)$. In all cases $\lambda(t)$ is chosen to fit CDS term structure

- Constant $\alpha(t)$, constant jumps
- Constant $\alpha(t)$, size of Jth jump, $h_J = h_0 e^{\beta J}$. This provides a good fits to all tranches for a particular maturity.
- $\alpha(t)$ linear function of time, size of Jth jump, $h_J = h_0 e^{\beta J}$. This provides a good fit to all tranches for all maturities.

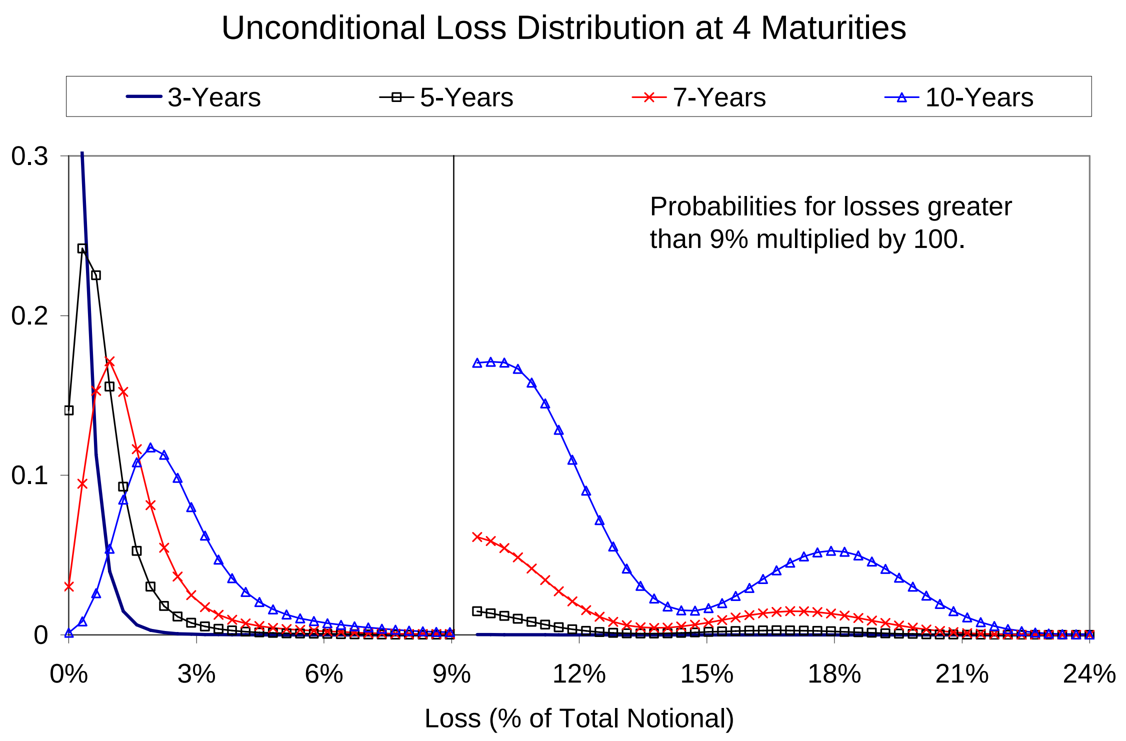
Variation of best fit h_0 and β across time



Variation of best fit $\alpha(0)$ and $\alpha(10)$ across time



Evolution of Loss Distribution on Dec 4, 2006 for 4 parameter model.



Conclusions

- It is possible to develop a simple dynamic model for losses on a portfolio by modeling the cumulative default probability for a representative company
- The only way of fitting the market appears to be by assuming that jumps in the cumulative default probability get progressively bigger.