



Managing the Newest Derivatives Risks

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Derivatives 2007: New Ideas, New Instruments, New markets

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Some Practical Aspects of Option Modelling:

- I. The Case of FX, Fixed Income and Equity Derivatives**
- II. The Case of Credit Derivatives**

I. The Case of FX, Fixed Income and Equity Derivatives

- ***There are two approaches to dealing with pricing models for derivatives:***
 - ***“Fundamental approach” : assumes ex-ante some specification for the dynamics of the underlying instruments (diffusion, jump-diffusion, local-volatility diffusion model,...) that best recovers the market prices of the plain-vanilla options actively traded in the market.***
 - ***“Instrumental, or the trader’s approach” : market quotes and model prices are compared using implied volatility. Traders are not interested in the true process for the underlying but are concerned by the “smile, the spot and forward term structures of volatility and how they evolve through time.***

- ***Now, could the process for the underlying be chosen with total disregard for the true process as long as it reproduces the correct behavior of the implied volatilities?***
 - **The answer is NO for the reason that the trader will have to, at the very least, delta hedge the positions.**
 - **Clearly, the effectiveness of the hedging program depends on the current specification of the dynamics of the underlying.**

- ***In practice we judge the quality of a model from two different angles:***
 1. ***Does the model produce prices within the market consensus?***
 2. ***How effective is hedging?***

A model is considered attractive not only if it prices correctly, but also if the parameters of the model remain stable when the model is recalibrated every day, and the hedge ratios in terms of the hedging instruments also remain stable.

- ***Which model for which product?***

Issue: incorporate all the available market information on the liquid hedging instruments when calibrating a model.

FX derivatives:

- **Highly liquid market for plain-vanilla and barrier options.**
- **As a consequence prices cannot be replicated by simple local volatility models (not enough degrees of freedom):**
LSV (local stochastic volatility) models plus jump (short-term smile)

Highly liquid market for plain-vanilla options (200 – 300 calibration points) and, more recently, liquid market for variance swaps (15 – 20 calibration points).

Local or stochastic volatility models?

Local volatility models aim at a full replication of the market smile seen from today, using a local variance dependent on the spot level.

No genuine financial interpretation.

The most famous examples are Dupire local volatility model and Derman- Kani (discrete time binomial tree version).

Equity Derivatives

- The rationale for stochastic volatility models is to introduce a process on the local variance in order to control the smile dynamics (its evolution over time)
A common example is the Heston model

▪ Local or Stochastic Volatility Models: Pros and Cons

▪ Local volatility

- Good market replication
- Consistent modelling at the book level

but

- Poor smile dynamics
- Delta and gamma get mixed up

▪ Stochastic volatility

- Finer modelling through decorrelation
- Allows some control over the smile dynamics
- Separate between the risk factors

but

- Many, many choices...
- Calibration may be difficult
- Dynamic vega hedge required to achieve replication

Equity Derivatives

- **Options on single stocks: jump to default models that incorporate the information on the CDS market (asymptotic smile for low strikes)**

- **Market Standard for Stochastic Volatility Models**
 - No model is really the market standard – some are more popular than others.
 - Several features to take into account:
 - Calibration
 - Numerical tractability
 - Induced smile dynamics
 - Hedge ratios

- **Affine models** (Heston, Heston-Bates – square-root process for the local variance together with Poisson-type jumps)
 - Tractable numerical solutions for plain-vanilla options using Fourier-Laplace transforms.
 - Parsimonious models but calibration does not produce stable parameters, e.g. correlation between the spot and volatility very unstable.
 - However, these models are useful to produce smooth volatility surfaces.

- **Local Stochastic Volatility (LSV):**

- The best of both worlds: a self-calibrated model with flexible smile dynamics

$$\frac{dS_t}{S_t} = a(S_t, t)b(Y_t, t)dW_t^1 + \mu_t dt$$

$$dY_t = \alpha(Y_t, t)dW_t^2 + \xi_t dt$$

- LSV for products that depends on the forward smile:
 cliquet options, options on volatility and variance, options with payoff conditional on realized volatility,...
- LSV + Jump when steep short-term smile

Implementation issues:

Pricing based on Monte-Carlo:

- **Server farm with 3,000 processors used to conduct parallel computing.**
- **Variance reduction techniques:**
 - **Antithetic method;**
 - **Control variate technique;**
 - **Importance sampling: difficult to implement in practice as distribution shift is payoff specific.**

Next challenges:

- **Correlation smile:**

Basket of indexes: Euro Stoxx, S&P, Nikkei

Arbitrage: index vs. individual stock components

- **Dynamic management of the hedge:**

How to rebalance the hedge portfolio provided we cannot trade in continuous time but only once every Δt (one day, 15 mns,...)?

Fixed Income Derivatives

Products:

Reverse Floater Target Redemption Notes (TARN)

Callable Snowballs

CMS spread options

Models:

“Hull & White” is the model that traders like very much. HW can fit the:

- zero-coupon yield curve,
 - term structure of implied volatilities for captions or swaptions.
- and has become the standard approach to price American options.

Shortcomings:

Does not capture the smile (at-the-money calibration: for a given maturity all the caplets have the same volatility).

Practical solutions:

- H&W with shifted strikes or stochastic volatility (1 or 2 factors depending on the products: easy to price but difficult to calibrate).
- “Smiled BGM”: easy to calibrate but difficult to price (Monte Carlo for American options difficult to implement).

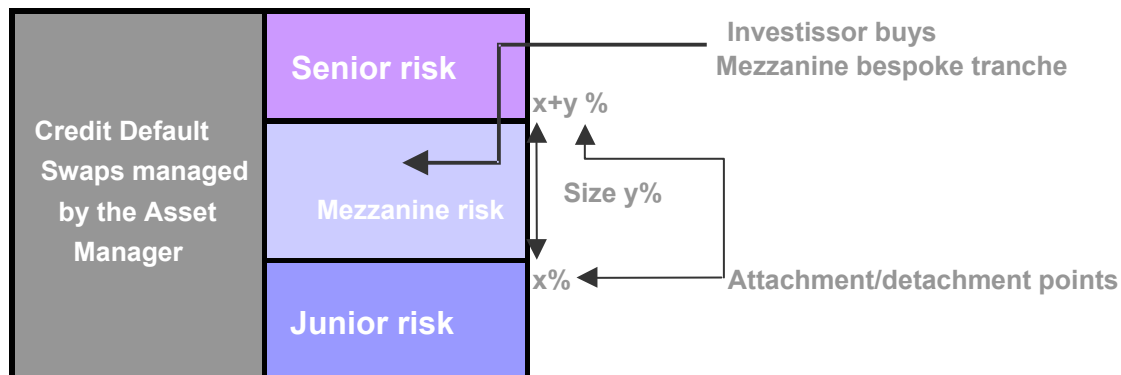
It is a local volatility extension of BGM model that allow almost arbitrary terminal distributions for Libor rates, while keeping pricing by simulation feasible. Also, shifted log-normal BGM with stochastic volatility.

- HK (Hunt Kennedy): a Markovian arbitrage-free, one factor model that allows exact numerical calibration of market caplet smiles. (Analogy with Dupire’s model for equity derivatives.) Traders don’t like HK as it generates unstable hedge ratios.
- SABR: static model but flexible to control the smile. SABR is used (Bi-SABR) to price CMS spread options.

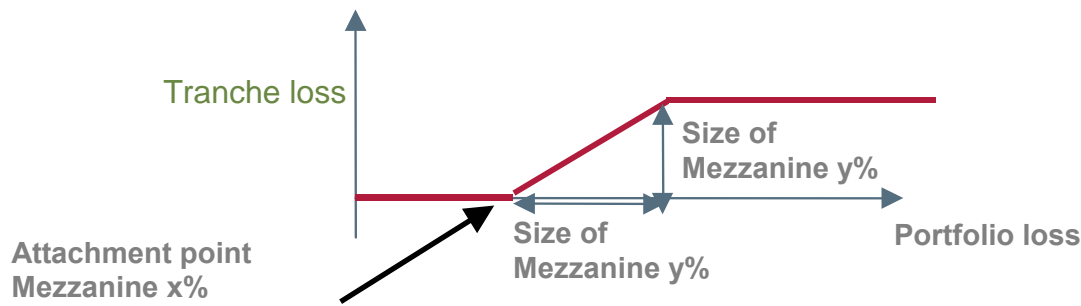
II. The Case of Credit Derivatives

Bespoke Single Tranche CDO

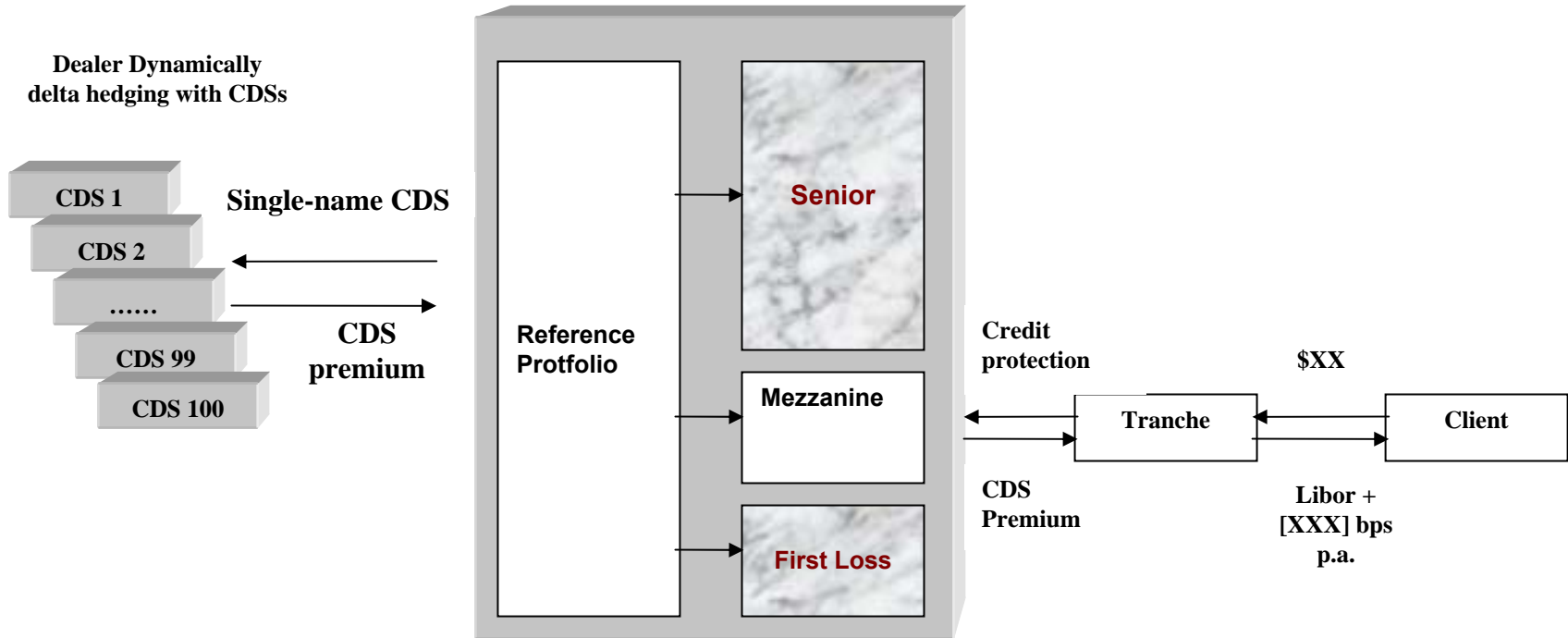
- Example of a **mezzanine** risk bought by an investor (the junior and senior risks being borne by the bank)



- The mezzanine tranche can be viewed as a call spread position on defaults in the underlying portfolio.



Bespoke Single Tranche CDO



Bespoke Single Tranche CDO

- End investors have now the opportunity to purchase customized credit portfolio exposures at pre-specified risk-reward trade-offs. To provide these bespoke portfolios, dealers delta hedge their exposures using single-name CDSs, credit indexes and index tranches.
- Standardization of indices is currently a significant driver of growth in the credit derivatives market.
- Multiplication of indices: ABX, CMBX,...

The DJ.CDX.NA.IG is the US benchmark for tradable 5, 7 and 10 year index products

- Static portfolio of 125 diverse names (CDSs) which are equally weighted at 0.8%.
- Tranching for CDX: 0-3% (equity tranche), 3-7%, 7-10%, 10-15%, 15-30%, 30-100%.
- Tranching for iTraxx: 0-3%, 3-6%, 6-9%, 9-12%, 12-22%, 22-100%.
- Tranching of the European and US indices is adjusted so that tranches of the same seniority receive the same rating.
- Active market for 5 and 10 year tranches.

CDX.IG

CDX.IG.7 7 market data from 1/22/07				
Maturity 12/20/13				
Protection Start	Protection End	Premium	upFt fee	
0,00%	3,00%	5,000%	41,00%	
3,00%	7,00%	1,895%		
7,00%	10,00%	0,365%		
10,00%	15,00%	0,155%		
15,00%	30,00%	0,060%		
30,00%	100,00%	0,031%		
0,00%	100,00%	0,450%		

Itraxx 6 - 7 market data from 1/22/07				
Maturity 12/20/13				
Protection Start	Protection End	Premium	upFt fee	
0,00%	3,00%	5,000%	25,75%	
3,00%	6,00%	1,120%		
6,00%	9,00%	0,338%		
9,00%	12,00%	0,163%		
12,00%	22,00%	0,054%		
22,00%	100,00%	0,021%		
0,00%	100,00%	0,320%		

- The spread of each tranche is determined so that the risk-neutral expectation of the fixed leg is equal to the risk-neutral expectation of the loss:
 - Need to specify pricing model / **risk-neutral probability**;
 - Need to specify the **joint default probabilities** of the underlying pool of debt instruments;
 - Need a model for **default correlations** – the only “observable” default correlations are for standard baskets (iTraxx, CDX,...)

Pricing of Credit Derivatives

- Goal of credit derivative pricing models: assign prices to various credit-risky payoffs in a manner which is:
 - Arbitrage free
 - Consistent with market prices of benchmark instruments used for hedging (this is a calibration issue and many models don't satisfy this constraint)

Pricing of Credit Derivatives

In the case of credit risky payoffs involving an entity i the computation of

$$E^{\mathbb{Q}}[B(t, \tau_i) f(L_{\tau_i})]$$

involves knowledge of law under \mathbb{Q} of two quantities:

- Time of default τ_i
- Loss given default L_{τ_i}

so two basic quantities we need to model are

- Risk-neutral conditional default probabilities:

$$q^i(t, T) = \mathbb{Q}(\tau_i \leq T) \quad (5)$$

- Recovery rate R defined by $L_{\tau} = (1 - R)N$ or, in the random case, risk-neutral distribution of recovery rate.

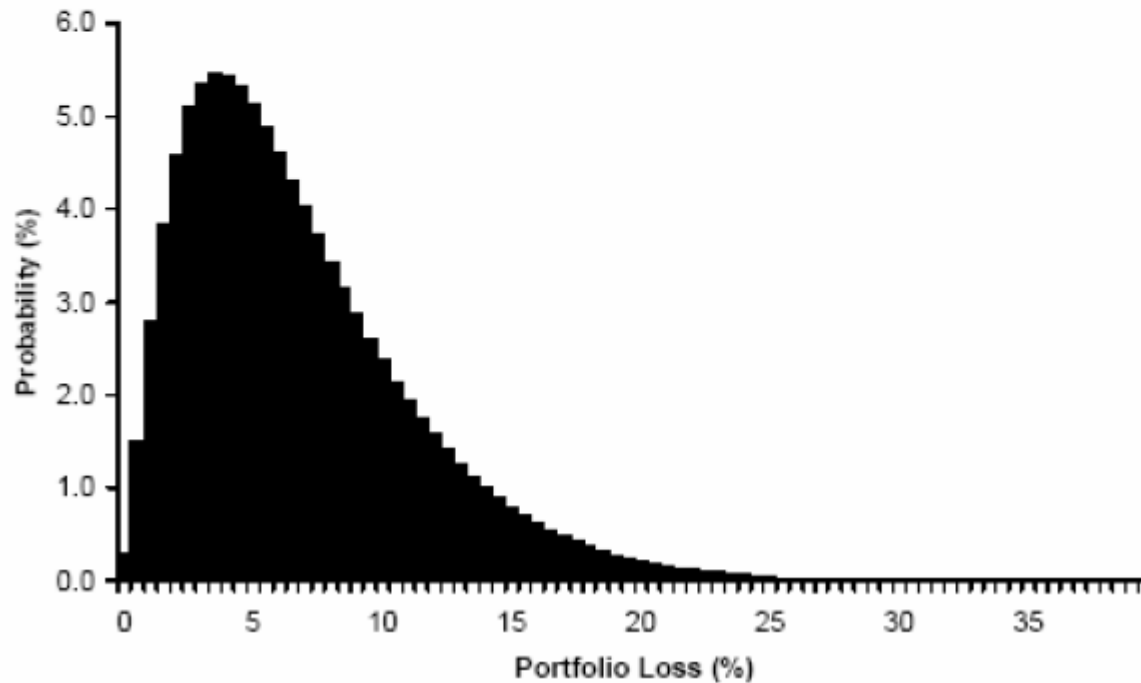
We are looking for the joint risk-neutral distribution of time to default and loss given default.

The Gaussian Copula Model

- Has become the market standard for quoting CDO tranche spreads
- Equivalent to the Black Scholes model for equity options
- Simple to implement, single parameter

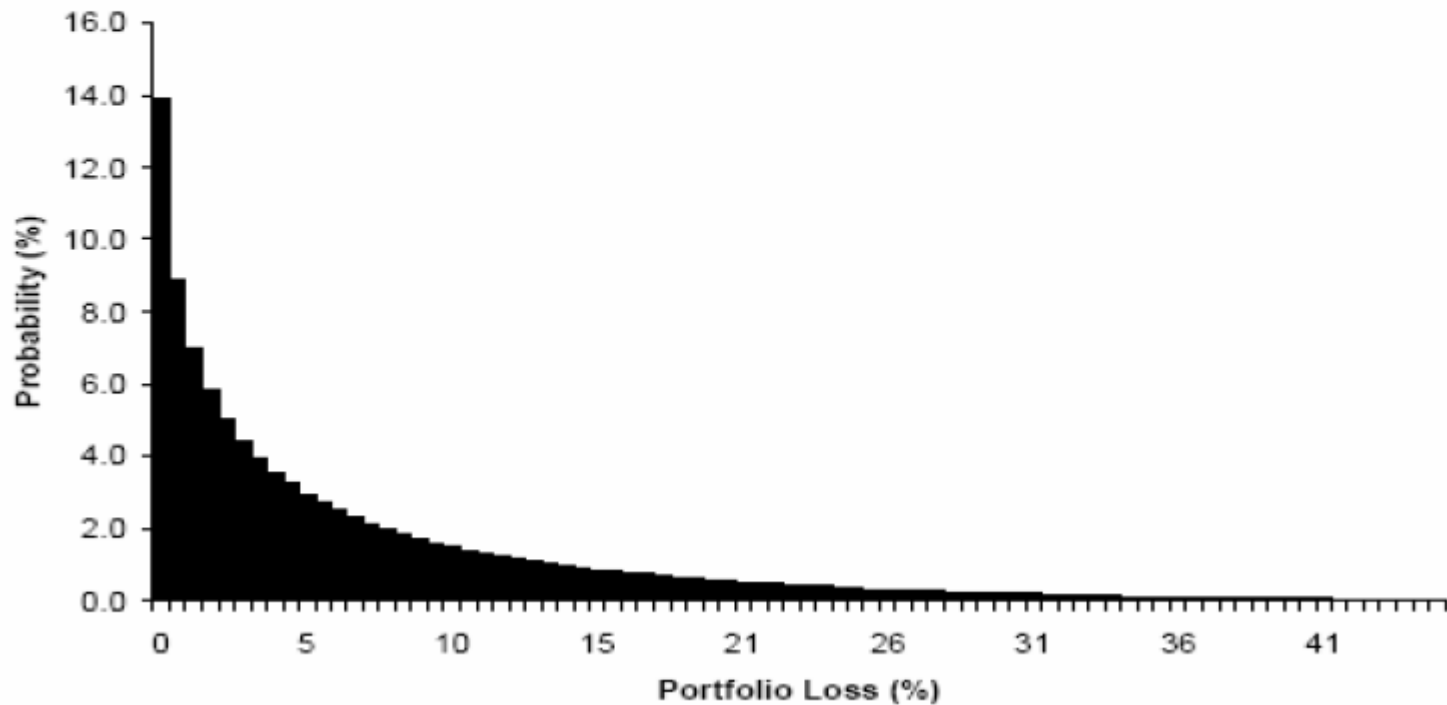
Effect of default correlation on loss distribution

Figure 32. **The Loss Distribution for a Portfolio of 100 Assets with a Default Correlation of 15%. Each Asset Has a 6-Year Default Probability of 14% and a Recovery Rate of 50%.**



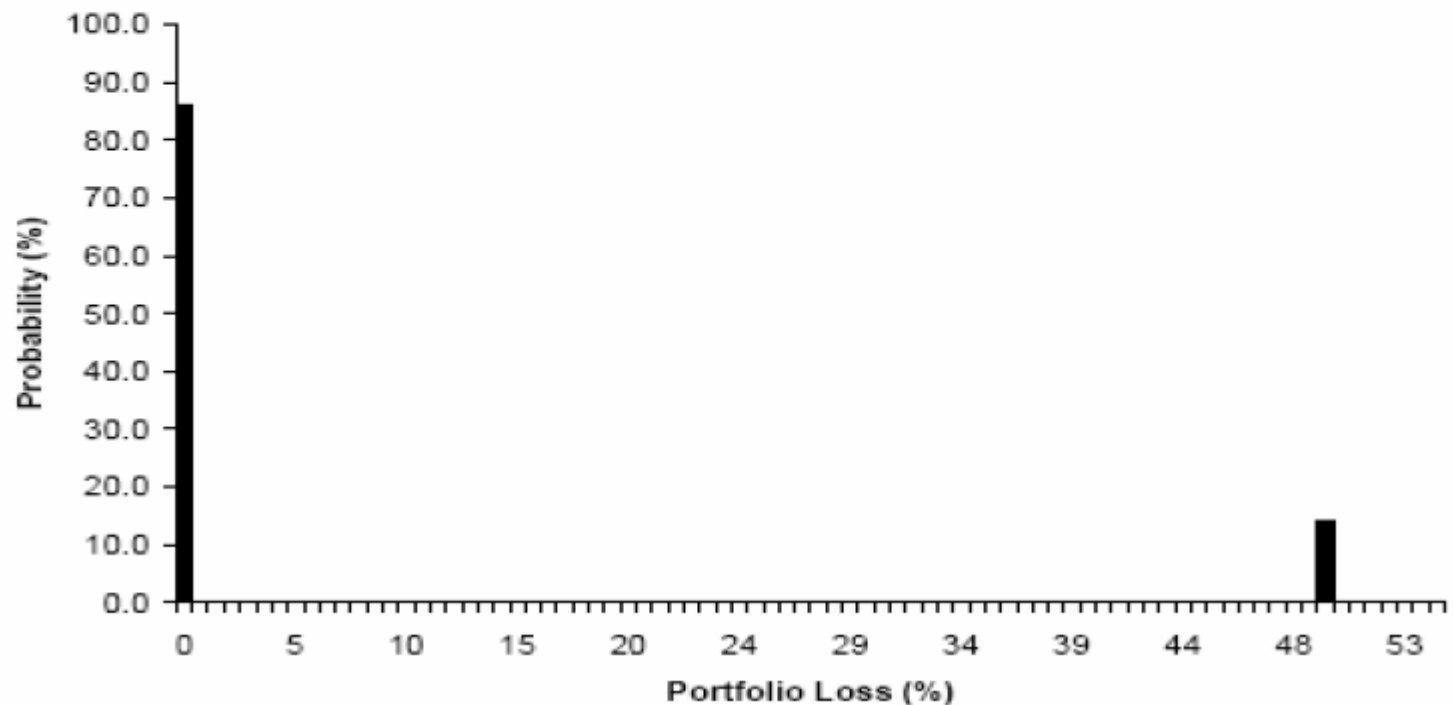
Effect of default correlation on loss distribution

Figure 33. **The Loss Distribution for a Portfolio of 100 Assets Each with a 40% Default Correlation**



Effect of default correlation on loss distribution

Figure 34. **The Loss Distribution for a Portfolio of 100 Assets with a 100% Default Correlation**

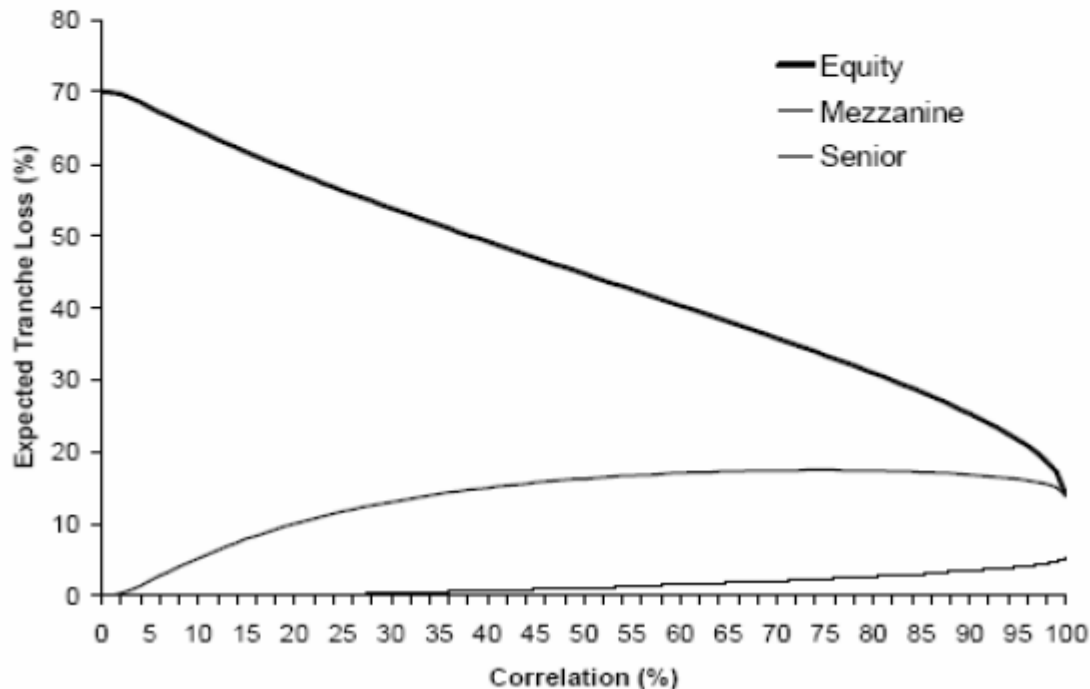


Effect of Correlation on Tranches

- At low correlation, there is very little likelihood that the mezzanine or senior tranche will be affected by defaults, so their expected loss is small.
- This is why senior tranches can receive high ratings even if the underlying portfolio is not investment grade.
- The higher the default correlation, the more likely it is that higher tranches will be affected by default.

Effect of correlation on tranche losses

Figure 35. The Percentage Expected Loss of Each of the Tranches of the Portfolio as a Function of the Default Correlation Between the Assets in the Collateral Pool



Disadvantages of default time copula models

- Copula models are unable to reproduce implied correlations for quoted CDO tranches in a simple manner.
- Static: no dynamics for spreads.
- Deltas with respect to CDS computed under these models are inconsistent since they do not contain spread risks.
- Unclear how new information should be incorporated into the model: conditional default probabilities?

Dynamic models

Many dynamic models have been proposed in the literature but very few have actually reached implementation stage:

- Multi-name default barrier models
- Multi-name random intensity models
- Aggregate loss models

Advantages of aggregate loss models

Calibration to initial “base correlation skew” is automatic.

Standardized tranches are calibrated so model prices are consistent with tranche-based hedging.

Provides a joint model for spread and default risk.

Avoids the specification of cumbersome copulas.

Allows the pricing of more complex derivatives such as options on CDO tranches, forward start tranches, ..

Drawbacks of aggregate loss models

At present: just a framework, specification needs to be done.

Aggregate loss models are very new: little experience with specification/ implementation.

No deltas/ sensitivities to underlying portfolio.

Applicable to indices only, not to bespoke portfolios.

Portfolio-specific approach: non-transposable from one portfolio to another.

High dimensional ($N=125$) stochastic model: heavy Monte Carlo needed.

Open questions

- Good descriptions of default dependence across time.
- Good models for dynamics of term structures.
- Efficient pricing algorithms in dynamic models for loss term structure models.
- Hedging of spread and default risk with credit default swaps
- Hedging of "default correlation" risk.
- Efficient calibration algorithms for CDS in random intensity models.
- Efficient calibration algorithms for single tranche CDOs in all models.

An exciting area for research!