Unequal Growth*

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Abstract

Over the past 50 years US households have experienced changes in earnings dynamics that resulted in a large increase in inequality. This paper assesses the direct impact of these changes on aggregate growth and welfare. We first use a panel of U.S. household data, for the period 1967-2018, and apply a simple statistical decomposition of aggregate earnings growth. The decomposition expresses aggregate growth as the sum of two terms. The first is the covariance between the level and growth of household earnings, which only depends on micro earnings dynamics. The second is average growth across households, which depends both on micro and macro factors, such as a common TFP growth. In order to identify the impact of the changes in the micro dynamics on aggregate outcomes we map a simple model of micro-founded growth onto the terms of the decomposition. We find that changes in household earnings dynamics that are consistent with data involve *unequal growth* across the earnings distribution. That is, a transition period during which there is a change in the *shape* of the distribution which is not mean preserving, so that aggregate growth is affected. These changes have a small positive effect on aggregate growth, and, with incomplete markets, an ex-ante large negative welfare effect.

JEL Classification Numbers: D31, O4

Key Words: Income distribution, Inequality, Growth slowdown

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1 Introduction

Over the past 50 years U.S. households have experienced changes in earnings/income dynamics that have generated a large increase in earnings/income inequality (see, among others, Katz and Murphy 1992 and Heathcote et al. 2010). The objective of this paper is to measure the direct impact of these changes on aggregate growth and welfare.

Our starting point is the observation that aggregate earnings growth can be thought as coming from two sources: the first is growth that is common (or evenly distributed) across the earnings distribution, such as aggregate productivity growth. This source has, by definition, no impact on the *shape* of earnings distribution, and on earnings inequality. The second source is growth that is systematically different across the earnings distribution. This source leads to a change in the shape of the income distribution and it can affect, at the same time, income inequality and aggregate growth. This is the source that we refer to as "unequal growth". In order to identify unequal growth we present a statistical decomposition showing that aggregate earnings growth can be written as the sum of two terms: the first is the cross sectional (across households) covariance between earnings growth and earning levels, the second is the (un-weighted) average of household/individual earnings growth.

The key insight is that the cross sectional covariance term is connected to aggregate growth, but only depends on micro earnings dynamics, so that we can identify changes in these dynamics from changes in this covariance term (and underlying correlations and standard deviations). Once changes in micro dynamics are identified we can assess their impact on aggregate growth. Moreover, by looking at the evolution of the second term of the decomposition, we can also identify changes in the growth that is common across the distribution.

Specifically we first document the evolution of the two terms of the decomposition for the United States using micro data from the Panel Study of Income Dynamics (PSID) over the period 1968-2018. The data shows the well known fact that aggregate growth is slowing down (see, among others, Gordon 2012 or Summers 2015) and that inequality is increasing. More importantly for our purposes the data shows that the correlation between earnings growth and levels is negative but increasing over time: that is over time high earnings households tend to grow faster, relatively to low earnings households.

Second we bring these data to a simple model of micro-founded growth à la Aiyagari-Bewley-Huggett, modified to include a labor participation margin. In the model we introduce changes in parameters governing income dynamics, and discipline these changes using standard studies on income micro dynamics and the observed aggregate moments, as they appear in the statistical decomposition described above. The idea is closely linked to the analysis by Gabaix et al. (2016), who frame the evolution of income inequality as a transition, from one invariant distribution to a new one, triggered by a change in the fundamentals of the household's income process. Our key contribution relative to the previous literature is the focus on the impact of these changes on aggregate growth.

The model shows that the changes in micro income dynamics that are consistent with the decomposition involve sizeable a decline in the common component plus a changing *unequal growth* across the income distribution, that is a distribution of growth opportunities across the earnings distribution that over time has favoured (relative to earlier periods) high earnings households. We then show that this changing unequal growth has resulted in a moderate increase in aggregate output (about 20% over our sample size). The intuition for this result is that since high earnings households comprise a large fraction of aggregate earnings, having them grow faster results in higher aggregate growth. So in a sense our first conclusion is that the increase in inequality in the US over the past 50 years has resulted in additional aggregate growth that has partly offset the slowdown in the common growth component. We then use our model to evaluate the ex ante welfare consequences of such changes, and our second conclusion is that, in an economy with incomplete markets, the ex-ante welfare effect of the increase in *unequal growth* is negative and sizeable. The reason is that the lower (relative to earlier periods) growth of low earnings households leads to prolonged income stagnation for these households, that lead to non participation and low consumption and welfare. These losses are only partially offset by the gains at the top.

2 Literature Review

Several insightful contributions have studied the the dynamics of income and wealth inequality in the US and other countries over the past decade. The contributions analyze the phenomenon from different angles: some focus on the measurement and documentation of the key facts, as in Atkinson, Piketty, and Saez (2011); Guvenen, Ozkan, and Song (2014); Guvenen, Karahan, Ozkan, and Song (2021), other propose formal models of the mechanisms behind the data such as Luttmer (2011); Gabaix (2011); Gabaix et al. (2016); Benhabib and Bisin (2016).

The papers by Jovanovic (2014); Jones and Kim (2018); Moll, Rachel, and Restrepo (2021) are closely related to the issues we explore. These papers present theoretical models to explain the *joint* evolution of income inequality and aggregate growth. They propose explicit mechanisms through which fundamental changes in the technology, or the market structure, simultaneously triggers a change of the cross sectional income inequality (for the top incomes in Kim and Jones) and a change of the aggregate growth. Jovanovic (2014) presents a model where an improvement in the technology for labor market matches between workers with complementary skills, leads to a reshuffling of the matches in the labor market which implies wage gains for the workers at the high end of the skill distribution and losses for the workers in the lower end of the distribution.[1] Assuming a lognormal distribution of skills, the model delivers an analytic characterization of the transition dynamics following an improvement of the matching technology, which illustrates the consequences for income inequality and for aggregate growth.[2] In a nutshell, better signals about the workers' skills lead to faster growth, to more income inequality and a smaller turnover in the distribution of rims' productivity.[3]

The paper by Jones and Kim (2018) presents a model of the right tail of the income distribution. Assuming an exponential income growth that is occasionally destroyed by the arrival of a new competitor the model generates an income distribution that is Pareto.[4] Changes in top income inequality reflect changes in the power law parameter that can be triggered by shocks to information technology, taxes, and policies related to innovation blocking. The paper shares with our investigation the focus on the linkages between the potential trade-offs between growth and income inequality, focusing on labor and entrepreneurial income (consistent with evidence in Piketty and Saez). Jones and Kim (2018) insightful model is designed to inspect the dynamics of the right tail of the income distribution, while our quantitative analysis focuses on the whole range of incomes, something that is also shared by Jovanovic (2014), with the aim to capture the interactions between inequality and growth over the whole range of the income distribution.

The paper by Moll, Rachel, and Restrepo (2021) presents a model that is closely related to the phenomena we study. These authors develop a tractable theory that links technological innovations, in particular automation of the tasks performed by labor. They characterize how technological changes affect the returns to capital and labor, as well as inequality. Automation has two effects: it increases inequality by affecting the returns to wealth, and it

[1] The quantitative model by Grigsby (2021), featuring heterogenous skills workers, provides an insightful complement to the theory illustrating how non-uniform labor demand shocks may lead to labor relocation and negatively affect the aggregate wage. See Haskel et al. (2012) for a critical review of the hypothesis that increased globalization triggered significant effects on the labor income inequality.

[2] A related model by Benabou and Tirole (2016) studies an imperfectly competitive labor market with asymmetric information about heterogenous workers type. It is shown how the increased competition for the best talents leads to (a possibly inefficient) increase of income inequality. This paper however does not discuss the consequences for aggregate growth.

[3] See Garicano and Rossi-Hansberg (2006) for a related analysis of an economy where agents organize production by matching with others in knowledge hierarchies. The authors discuss how changes in the cost of communication affect various dimensions of wage inequality.

[4] As usual, this is readily seen from the Kolmogorov forward equation for the distribution of incomes $f(y)$: assuming a growth rate γ and a killing rate δ the invariant distribution satisfies $0 = \gamma f' + \delta f$ which gives a Pareto distribution with parameter $\alpha \equiv -\frac{\delta}{\gamma}$.

leads to stagnant wages at the bottom of the distribution. An inelastic supply of capital implies that these effects are persistent, so that the wages remain of displaced worker remain low even in the long run.

3 A micro decomposition of aggregate growth

In this section we present a simple statistical decomposition that connects aggregate income growth to micro-level (household or individual) income growth, cross sectional income inequality, and the cross sectional correlation between income growth and income level. These types of decompositions have been widely used in industrial organization to connect sectoral productivity growth to productivity growth in individual firms (see, among others, Olley and Pakes 1996). We find it useful to apply this decomposition to household level data (as opposed to firms), because it connects aggregate growth with household income inequality, which has a more direct and relevant welfare content than firms income inequality.

Let y_{it} be level of income of household/individual i at time t. Let Γ_{t+T} be the economy's aggregate growth over an horizon T, which is

$$\Gamma_{t+T} = \frac{E(y_{it+T})}{E(y_{it})} = E\left(\frac{y_{it+T}}{y_{it}}\frac{y_{it}}{E(y_{it})}\right)$$

where $E(.)$ is the cross sectional average. Now define

$$g_{i,t+T} \equiv \frac{y_{it+T}}{y_{it}} \quad , \quad s_{i,t} \equiv \frac{y_{it}}{E(y_{it})}$$

so that $\Gamma_{t+T} = E(g_{i,t+T} \cdot s_{i,t})$ where $g_{i,t+T}$ is income growth of unit i and $s_{i,t}$ the ratio between income of unit i and average income. Then, using the definition of covariance and the fact that $E(s_{i,t}) = 1$ we get

$$\Gamma_{t+T} = cov(g_{i,t+T}, s_{i,t}) + E(g_{i,t+T}) \tag{1}$$

or equivalently

$$\Gamma_{t+T} = corr(g_{i,t+T}, s_{i,t})\sigma(s_{i,t})\sigma(g_{i,t+T}) + E(g_{i,t+T}) \tag{2}$$

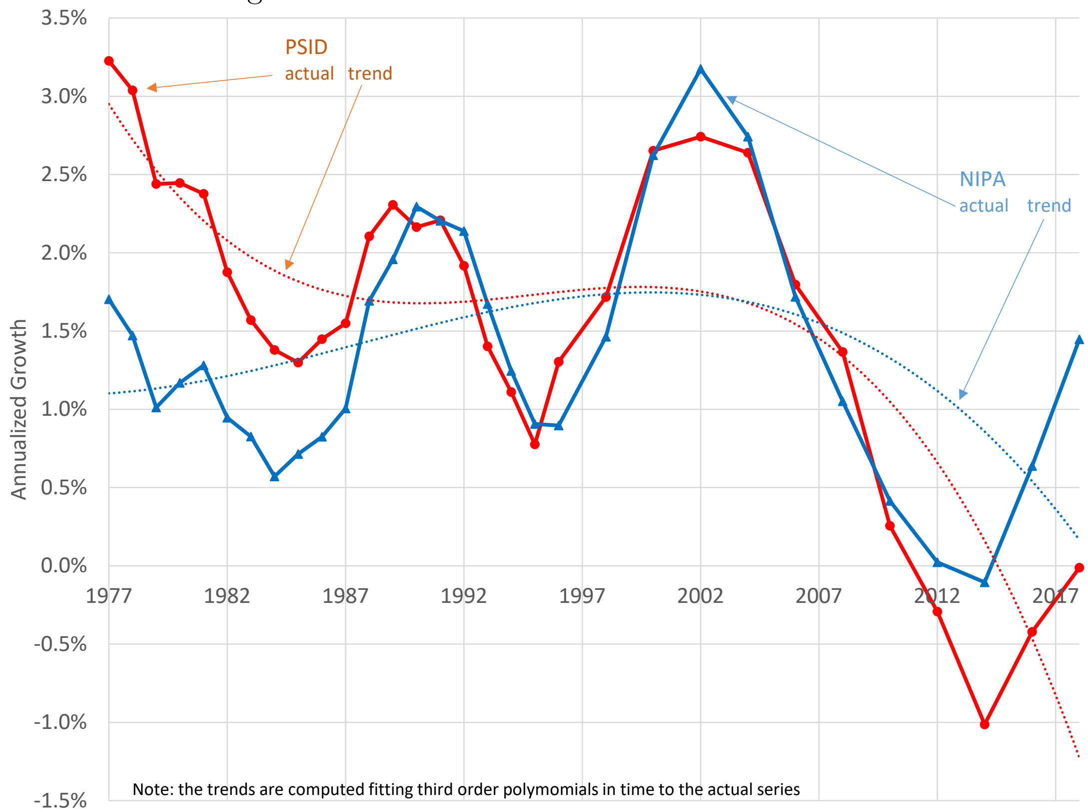
Equation (1) suggests that what matters for aggregate growth is not only the (unweighted) average individual growth $E(g_{i,t+T})$ but the distribution of growth opportunities, as summarized by $cov(g_{i,t+T}, s_{i,t})$. The intuition for why this is the case is straightforward: the higher the covariance, the faster higher income individuals grow; since they are high income they contribute more to aggregate growth and aggregate growth is higher. Equation (2) also suggests that $cov(g_{i,t+T}, s_{i,t})$ is linked to three cross sectional moments that have an intuitive economic interpretation. The first, $corr(g_{i,t+T}, s_{i,t})$, is the correlation between level and growth at the individual level. This measure captures the degree of mean reversion (or economic rank mobility) in individual income dynamics. The second, $\sigma(s_{i,t})$ is the standard deviation of $s_{i,t}$, which is essentially a measure of cross sectional income inequality. The third, $\sigma(g_{i,t+T})$, is the standard deviation of the growth rate of individual income, which is a measure of cross sectional income volatility. The equation suggests that changes in any of these three quantities will be associated, *ceteris paribus*, with changes in aggregate growth. It is important to note that this decomposition is a statistical identity, so, by itself, it cannot be used to make causal inferences on growth and inequality. Nevertheless it provides a useful starting point for assessing the impact of changing individual income dynamics on growth. To see why this is the case, note that all the moments in the first term of equation (2) are independent from the presence of a common growth factor, call it $\bar{g}$, that affects equally the growth of all households. All the terms in the product only depend on heterogenous individual income dynamics. The second term in equation (2) is instead potentially affected both by the common factor $\bar{g}$ and by individual income dynamics. So the evolution of the statistics in equation (2) will help us, with the aid of a simple statistical model, to identify the impact on growth of the changes in income dynamics, that drive in income inequality, from the changes in growth that are common across all households. For this reason the next section uses a panel of micro data to document how the terms in the decomposition has changed over time.

4 A decomposition of U.S. growth: 1967-2018

Both equation (1) and equation (2) involve cross-sectional moments as well as moments related to individual income growth, so in order to bring them to the data we need panel data on household/individual earnings. Since our main focus is aggregate growth in the United States we also want a panel which captures well aggregate US growth. For these reasons we work with the Panel Study of Income Dynamics (PSID), which is a panel of U.S. households, selected to be representative of the whole population, collected from 1967 to 1996 at the annual frequency and from 1996 to 2018 at bi-annual frequency. Figure 1 reports aggregate growth in per capita labor income (earnings) both in the PSID and the National Income and Product Accounting (NIPA).[5] The solid lines report the actual annualized growth, (computed

[5]The income measure in PSID is total (for each household in the sample) wage and salary income plus farm income plus 50% of business income for each household in the sample, divided by the total number

Figure 1: Growth in labor income: NIPA and PSID



over 5 years intervals), while the dotted lines are polynomial trends.[6]. The figure shows that aggregate growth in PSID does not match growth in NIPA perfectly but that the two series show a strong co-movement, suggesting that the PSID sample is a good laboratory to study the connections between household income dynamics and aggregate growth.

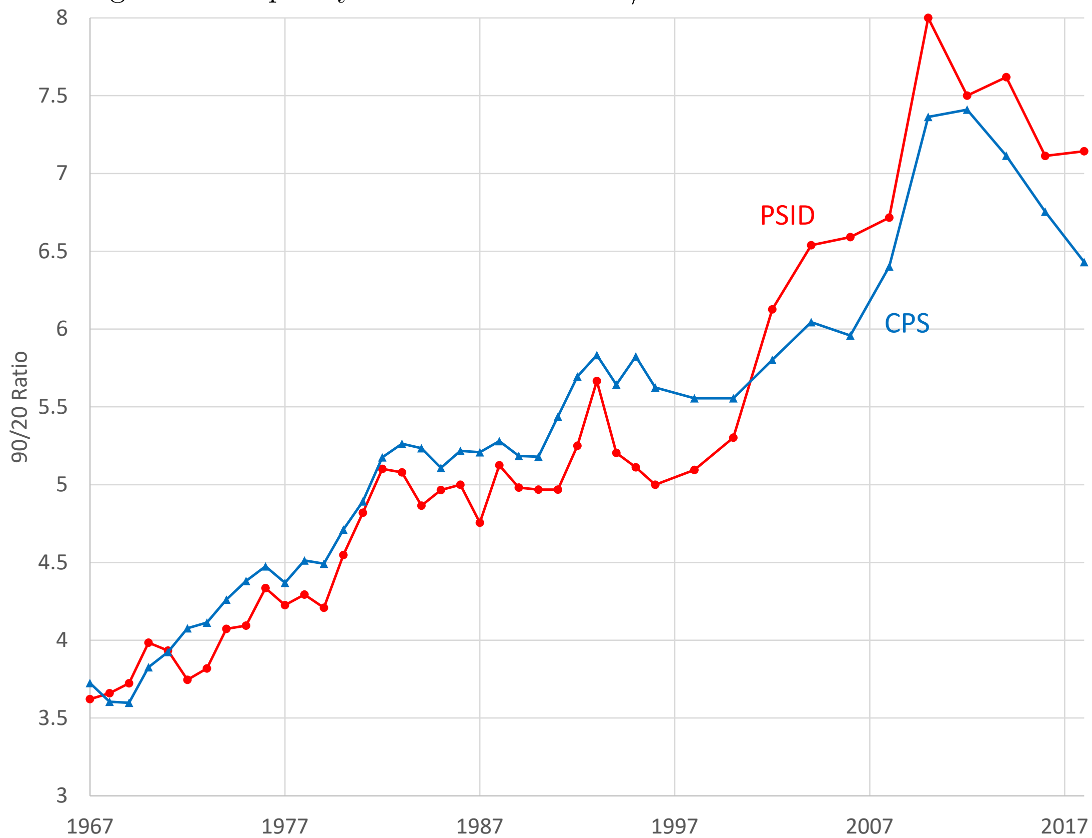
Figure 2 also shows that the PSID captures well the patterns of US household income inequality, as documented in from a much larger cross sectional survey, i.e. the March Current Population Survey (CPS). The figure plots a commonly used measure of inequality, that is the ratio of 90th to the 20th percentile of the household earnings distribution [7] The figure

of persons in the sample. The income measure in NIPA is compensation of employees, wages and salaries disbursement plus 50% of proprietors income, divided by the U.S. population. All measures are deflated using the PCE deflator. See the data appendix for more details on data construction and for similar figures for different (narrower and broader) income measures. The reason why we focus on labor income is that other categories of income are notoriously not well measured in the PSID and in other micro surveys.

[6]Due to bi-annual sample of PSID we only use the first, third and fifth year of each interval. So, for example, the observation for 2018 measures the growth between average income in 2018,2016,2014, and average income in 2012,2010,2008

[7]The earnings measure in both PSID and CPS is total wage and salary income plus 50% of household business and farm income. Inequality measures are computed for households with heads between age 25 and 60. The average sample size in the PSID is around 4000 household per year, the size in CPS is 10 times

Figure 2: Inequality in labor income: 90/20 ratio in PSID and CPS



shows that both surveys capture the well known secular increase in income inequality in the United States.

Since Figure 1 and Figure 2 show that the data in PSID capture well the evolution of aggregate growth and inequality, we now proceed to compute the data equivalent in PSID of s_i and g_i, which are the basic elements of the decomposition in equations equation (1) and equation (2). In order to reduce measurement error, we aggregate individual PSID data along two dimensions.[8] First instead of using current labor earnings, y_{it}, we use an average of real (PCE deflated) labor earnings over a 5 year window, so $\bar{y}_{it} = y_{it} + y_{it-3} + y_{it-5}$ is our measure of earnings.[9] Second we aggregate households in 10 deciles of $\bar{y}_{it}$. Formally let I_t by the group of households who are in the i_{th} decile of the $\bar{y}_{it}$ distribution in period t. We define

$$g_{i,t} = \frac{\sum_{j \in I_t} \bar{y}_{j,t+6}}{\sum_{j \in I_t} \bar{y}_{j,t}} \frac{\bar{P}_t}{\bar{P}_{t+6}} \text{and } s_{i,t} = \frac{\sum_{j \in I_t} \bar{y}_{j,t}}{\sum_{I_t} \sum_{j \in I_t} \bar{y}_{j,t}} \tag{3}$$

larger.

[8] Guvenen et al. (2014) who also analyze the relation between level and growth in individual earnings data use a similar aggregation.

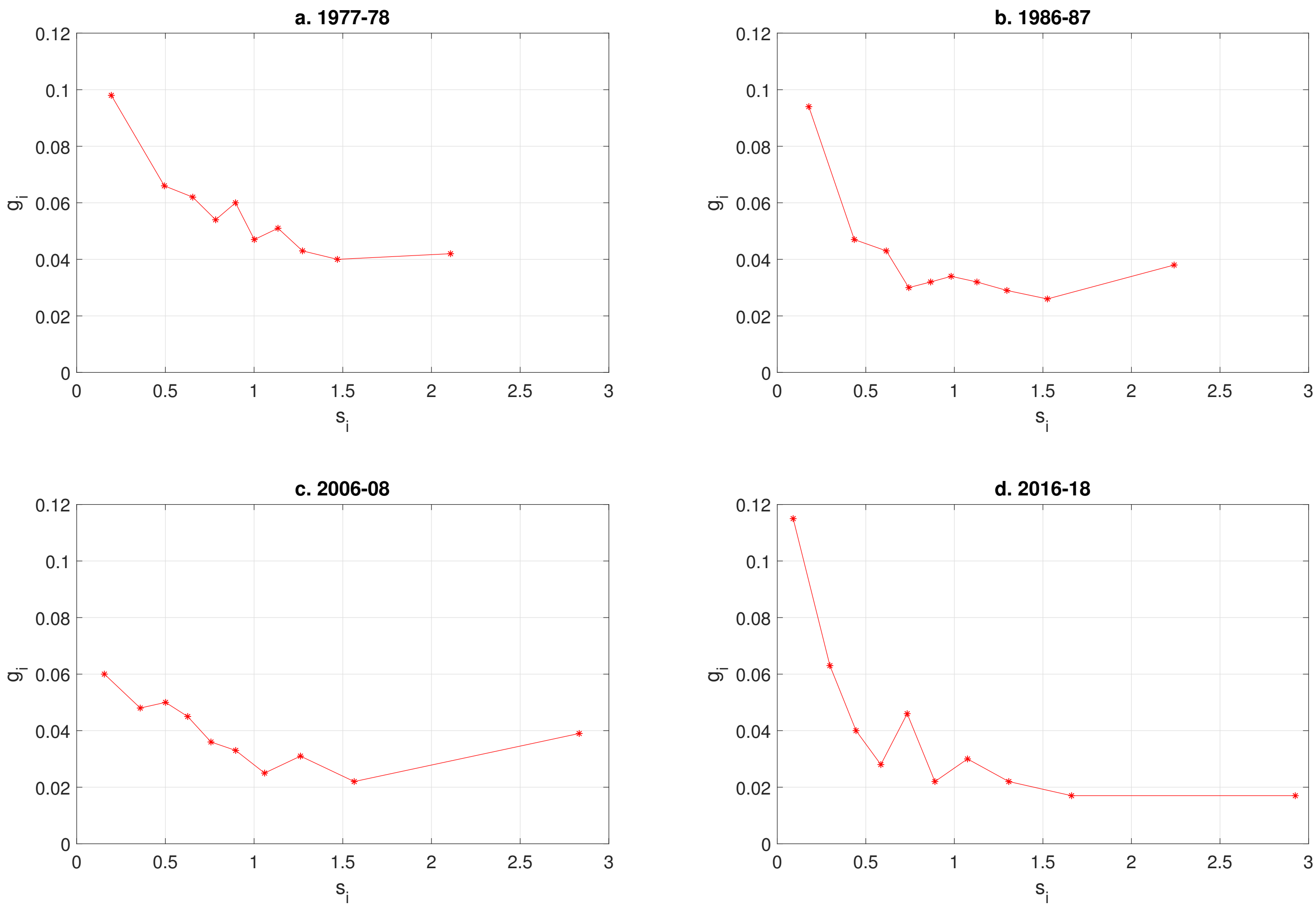
[9] The reason why we don't use all years in the window is that PSID data is bi-annual after 1996.

where $\bar{P}_t$ is average population in the PSID sample in periods t,$t-3$ and $t-5$. Our sample includes all households with head between age 25 and 60, which are in the sample for at least 11 years (from $t+6$ to $t-5$). Note finally that the growth rate of earnings in a given decile is computed using the same group of households in t and $t+6$. In figure 3 we show the s_{it} and g_{it} for 4 points in our sample: the beginning (final years of the growth window are 1977-78), two mid points (final years are 1986-87 and 2006-08) and the end (final years are 2016-18). Starting with the curve in panel (a) we want to highlight three features. The first is that earnings growth is unequal across the earnings distribution, with households at the bottom of the distribution experiencing faster growth. The second is that the curve is L-shaped, i.e. quite steep at the bottom end of the distribution (for $s_i < 1$) and fairly flat at the top of the earnings distribution (for the $s_i > 1$). The final feature is that the support of the curve is fairly concentrated, with income of the top-decile being only twice income of the middle decile. Moving now, to the middle periods (panels b and c) the first notable changes we highlight is that the curve is becoming more U-shaped with growth of the top decile being faster than the growth of the middle deciles. This faster growth at the top results in a widening of the support of the earning distribution (i.e. increasing inequality). Finally notice that the curve shifts down over time, suggesting, for most deciles a reduction in growth. Panel 4 finally shows that in the last years of the sample (growth ending in 2016-18) the curve turns back to be L-shaped, with a more noticeable spike of growth at the bottom.

After showing the evolution of unequal growth in the United States, we provide some evidence on the connection between unequal growth, inequality and aggregate growth. The top panel of 4 shows two components of the decomposition in equation 2. The solid line depicts the standard deviation of s_i, a measure of income inequality, while the dashed line depicts $corr(s, g)$, i.e. the correlation between income level and income growth. The panel shows that there is co-movement between earnings inequality ($\sigma(s)$) and the correlation between income levels and growth. In particular, there are two periods in our sample (highlighted by the shaded areas, which correspond to panels b and c in Figure 3) when the correlation between level and growth peaks, that are associated with large increases in income inequality. As discussed above an increase in the correlation between income and growth, implies that high earnings households tend to grow faster and hence increase income inequality. The most interesting insight for our purpose, however, comes from comparing the top with the bottom panel, which reports the term Γ_t in equation (1), measured aggregating all households in our PSID sample.

Comparing the two panels highlights that during both episodes when the correlation between income and growth peaks, we also see high aggregate growth (or a reduction in the growth decline). We find this to be suggestive evidence that changes in unequal growth can

Figure 3: Evolution of Unequal Growth in the United States: 1977-2018



drive, at the same time, increase in income inequality and changes in aggregate growth.

So far we have documented a series of facts relating growth and inequality in the United States over the past 50 years. Aggregate growth has declined and inequality has increased. The decline in growth has not been uniform across the income distribution, and in the middle years of our sample we have documented faster earnings growth of households at the very top of the distribution. Towards the end of our sample, on the other hand, we observe faster earnings growth of households at the very bottom of the distribution. In the following section we present a simple model of household earnings formation, and then we use it together with the facts to identify changes in the process of earnings formation. The model will allow us to derive our two main results, which are to measure the impact of changes in earning dynamics on aggregate growth and on welfare of the US population.

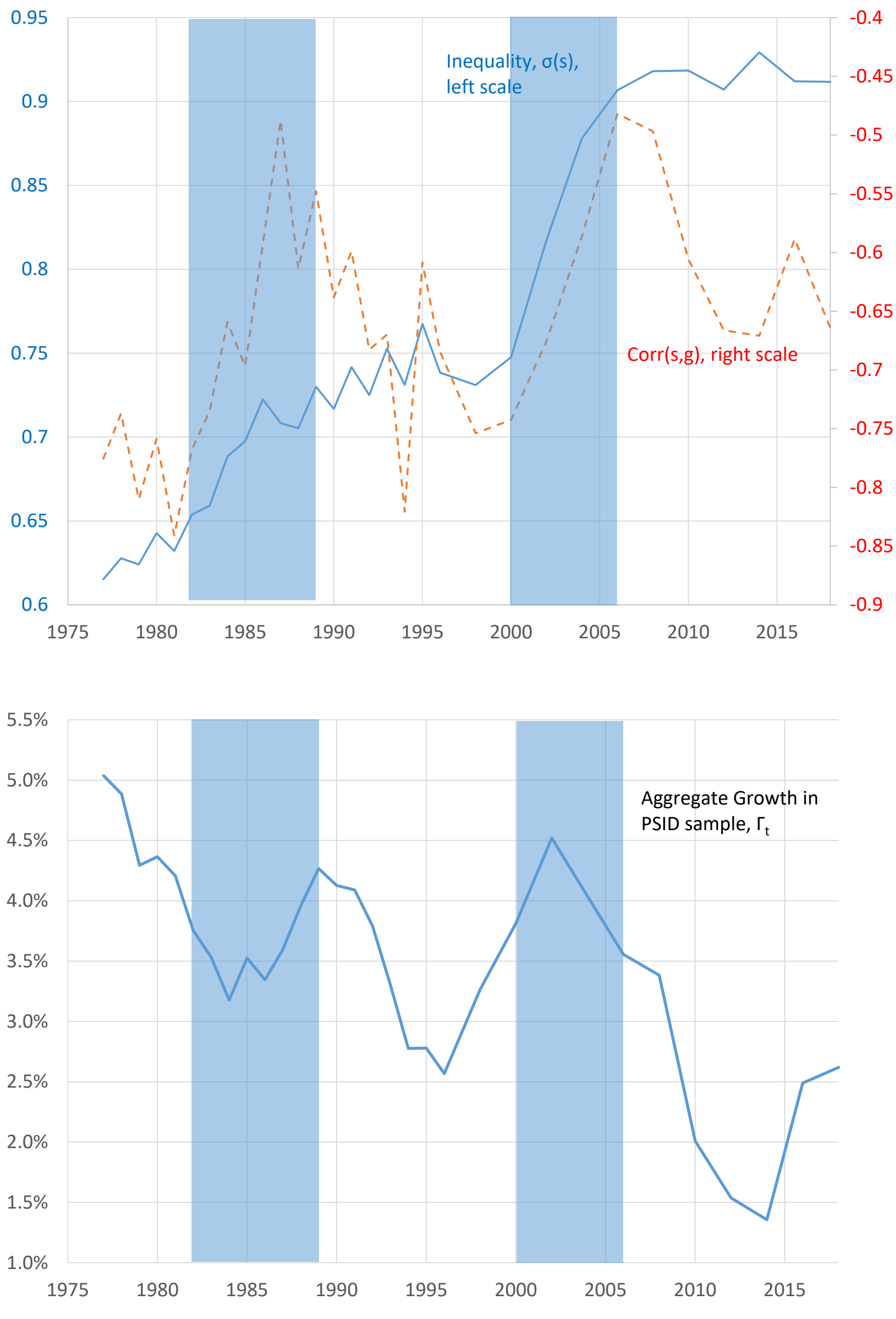


Figure 4: Level-Growth Correlation, Inequality and Aggregate Growth

5 A Bewley-Aiyagari-Huggett model

We consider a standard Bewley-Aiyagari-Hugget small open economy, with few simple modifications to the household income process, introduced to capture the features and the changes in the income distribution documented above.[10] We then explore the effect of these changes on aggregate growth and on welfare. The economy is inhabited by a continuum of infinitely lived households with standard preferences over consumption flows, denoted by

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}).$$

where $\beta > 0$ is the discount factor and $u(.)$ is a standard utility function, which is assumed to be $CRRA$, i.e. $u(c) = \frac{c^{1-\theta}}{1-\theta}$ with $\theta > 0$.

5.1 Earning Potential

Each household in each period receives an idiosyncratic realization of its earning potential Y_{it}. We model earning potential as

$$\log Y_{it} \equiv y_{it} = \alpha_i + f_{it} + e_{it}. \tag{4}$$

The first two components are meant to capture permanent differences in earnings potential across households, so we define $p_{it} \equiv \alpha_i + f_{it}$ and $\tilde{s}_{it} \equiv \frac{e^{p_{it}}}{E_i(e^{p_{it}})}$ to be the relative position in terms of permanent earnings potential of household i.

The first component, α_i, is a standard fixed effect, meant to capture initial permanent differences in earnings potential across households. We assume

$$\alpha_i \sim N(0, \sigma_\alpha)$$

The second component of the earnings potential process, f_{it}, which we name the growth factor, is going to be the driver of the increase in income inequality and it evolves according to

$$f_{it} = f_{it-1} + \bar{g}_t \;\; + \delta_t \frac{(\tilde{s}_{it} - 1)}{(1 + \tilde{s}_{it})} \tag{5}$$

The important element in equation (5) is that earnings growth of household i can depend on $\tilde{s}_{it}$. First consider the case in which $\delta_t = 0$. In this case each household experiences a

[10] The assumption of small open economy is made for computational convenience. For completeness we also solve a closed economy version of the model, where the interest rate is endogenous, and where we explicitly model production.

common earnings growth rate $\bar{g}_t$. In our experiments this is going to be the relevant case in the initial and final steady state. During finite time transitions, however, we will allow the parameter δ_t to be different from 0, and in particular to be positive, so that households with permanent earnings above the mean ($\tilde{s}_{it} > 1$), can have faster growth than households with permanent earnings below the mean. As we will show below, when $\delta_t > 0$, inequality is increasing, so this will be our modelling device to obtain the observed trends the distribution of earnings. One possible more structural interpretation of the two components α_i and f_{it} is that α_i captures the value of the initial skill endowment of household i and f_i captures the changing value of this skill (see, for example, Lochner and Shin 2014).

The final component e_{it} is a standard autoregressive process, which we model as

$$\begin{aligned} e_{it} &= \rho e_{it-1} + \varepsilon_{it} \quad , \quad \varepsilon_{it} \sim N(\mu_\varepsilon(\tilde{s}_{it}), \sigma^2_\varepsilon(\tilde{s}_{it})) \\ \sigma^2_\varepsilon(\tilde{s}_{it}) &= \frac{\sigma^2_\varepsilon}{\tilde{s}^\chi_{it}} \end{aligned}$$

Note that the parameter χ links the volatility of shocks of the income process, $\sigma^2_\varepsilon(\tilde{s}_{it})$, to $\tilde{s}_{it}$, the position of household i in the permanent earnings distribution. When $\chi = 0$, shocks have the same variance across households, when $\chi > 0$ poorer households have higher volatility of earnings shocks. [11] This is motivated by a large body of research which has documented that households at the bottom of the income distribution face higher volatility in their earnings shocks (see, among others, Meghir and Pistaferri (2004)).

5.2 Work choices and earnings

In each period each household with earning potential Y_{it} has the option to work on the market, and earn its potential minus taxes, or work at home and earn a transfer income $\exp(\phi_t)$, which grows at the common growth rate of the economy

$$\phi_t = \phi_{t-1} + \bar{g}_t$$

When households work on the market they pay a flat tax that the government uses to finance the transfer income. The process for earnings (before transfer and taxes) of household

[11] Since $\exp(e_{it})$ is distributed log normally changing the volatility of e_{it} also mechanically change its mean. To eliminate this effect we also allow also the mean of the shocks $\mu_\varepsilon(\tilde{s}_{it})$ to depend on $\tilde{s}_{it}$ and we set it so that $E(\exp(\varepsilon_{it}))$ does not vary across the income distribution. This is done to separate heterogeneity in variance (captured in the autoregressive component of income) from heterogeneity in means, which in our specification is captured by the fixed effects and by the growth factor.

i, which we denote by $h(Y_{it})$ is thus given by

$$h(Y_{it}) = \begin{cases} Y_{it} & \text{if } Y_{it}(1-\tau) \geqslant \exp(\phi_t) \\ 0 & \text{if } Y_{it}(1-\tau) < \exp(\phi_t) \end{cases}$$

This feature of the model will generate household earnings that feature positive as well as 0 values.

5.3 The household problem

The household consumption saving problem is standard. In the baseline case we assume incomplete markets so that each household can borrow and save using an uncontingent bond, which pays an exogenously given interest rate r. Bond holdings have to be above a borrowing constraint $\bar{b} \leq 0$. The problem can then be written as

$$\begin{aligned} & \max_{c_{t+j}, b_{t+j}} E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}) \qquad (6) \\ & s.t. \\ c_{t+j} \; = \; & b_{t+j-1}(1+r) + \max(h(Y_{it+j}), \exp(\phi_{t+j})) - b_{t+j},\; b_{t+j} \geq \bar{b} \text{ for every } j \\ & b_{t+j} \geq \bar{b} \qquad b_{t-1} \text{ given} \end{aligned}$$

5.4 Equal growth stationary equilibria

We first analyze stationary equilibria in which there is no unequal growth ($\delta = 0$) and in which all parameters, including the long run growth rate of the economy α are constant. An *equal growth* equilibrium is a distribution of households over earning potential and asset of $\mu(Y, b)$, plus household decision rule $b'(b, Y)$ satisfying the following conditions

1. The decision rules solve the household decision problem 6
2. Given the decision rules of the households the distribution is time invariant
3. The government budget constraint is satisfied

$$\int \tau h(Y) d\mu = \int \phi I(h(Y) = 0) d\mu$$

   where $I(.)$ is the indicator function.

Note that in an *equal growth* equilibrium, all individual and aggregate variables grow at the constant rate of α, hence when we solve for it, we solve for equilibrium in an economy where all variables are detrended by the growth factor f_t and where the discount factor β and the interest rate on bonds $1 + r$ are suitably rescaled. [12]

5.5 Unequal growth equilibria

We label *unequal growth* equilibria, the equilibria that arise during a transition from one stationary distribution to another. We assume the economy start in an stationary equilibrium and at time t_0, then experiences a change in parameters for $N < \infty$ periods. In particular we will consider the case in which $\delta_t > 0$ and in which α_t is not constant for $t \in [t_0, t_0 + N]$. After period $t_0 + N + 1$, we assume that the economy settles to a constant growth rate $\bar{\alpha}$ and that $\delta_t = 0$. An unequal growth equilibrium is a sequence of distributions $\mu_t(Y, b)$, and a sequence of decision rules $b'_t(b, Y)$, for $t \in [t_0, \infty]$, satisfying the following conditions:

1. Given perfect foresight on the path of parameters changes, the decision rules solve the household decision problem 6
2. The sequence of distributions are consistent with the decision rules
3. The government budget constraint is satisfied in every period

$$\int \tau_t h(Y) d\mu_t = \int \phi_t I(h(Y) = 0) d\mu_t$$

Note that the assumption of perfect foresight might sound a bit extreme, as it implies that high income households in 1979 (the date at which we will start our transition), learn that they have faster growth for the next N years (which in the baseline calibration we set to 30). For this reason we will also present results for *unequal growth* equilibria where agents do not expect the change in parameters and are "surprised" every period.

5.6 Calibration

Table 1 summarizes our parameter values for the equal growth equilibrium, which we calibrate to match features of the earnings distribution in PSID in the late 1960s and mid 1970s, before the increase in inequality started. In particular the first 5 parameters of the table are chosen so that the relation between earnings level and growth (g_i and s_i) in the model matches the one in the PSID data in the first two years of our sample (1977-1978). Figure 5 illustrates the matching between data and model. Note that a crucial element of the income process that

[12] In particular the interest rate in the detrended economy is equal to $\frac{1+r}{1+\alpha}$ and the discount factor, in the case where utility is CRRA with risk aversion parameter equal to θ, is equal to $\beta * (1 + \alpha)^{(1-\theta)}$

Table 1: Parameters in the initial stationary equilibrium

Income Process Parameters		
Name	Symbol	Value
Variance of fixed effects	σ_α	0.45
Persistence of shocks	ρ	0.95
Baseline sd of shocks	σ_ε	0.07
Standard deviation gradient	χ	1
Common growth	$\bar{g}$	4.7%
Transfer income (% of average Y)	ϕ	0.3
Tax rate	τ	1.5%
Unequal growth	δ	0
Preference Parameters		
Discount Factor	β	0.97
Risk Aversion	θ	2
Other Parameters		
Borrowing Constraint	$\bar{b}$	0
Risk free rate	r	2.5%

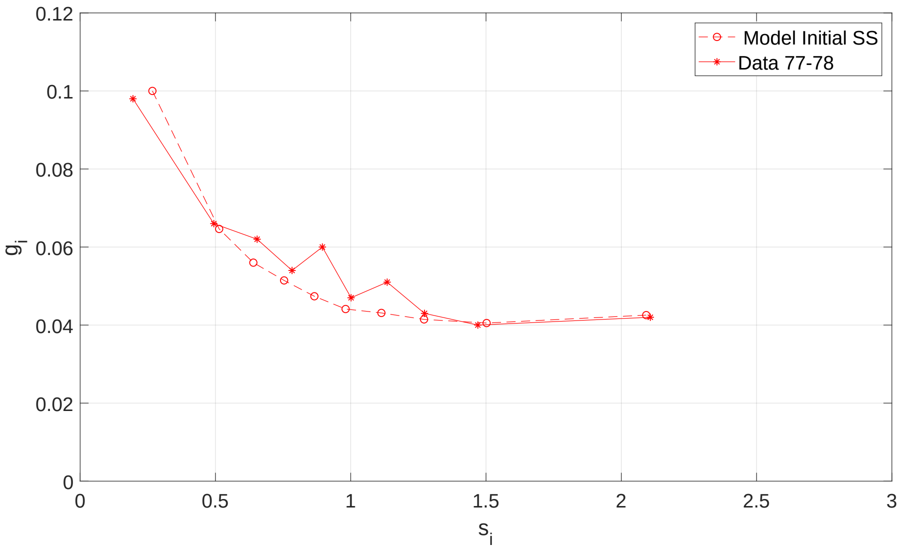
allows the model to match the data is the variance of shocks σ_ε^2 that declines with the level of earnings (*i.e.* $\chi > 0$). To understand why this is the case note that the curve in the data in figure 5 is fairly flat for the top of the distribution (for $s > 1$) and steep and downward sloping for the bottom (for $s < 1$). The autoregressive component (e_{it}) of the earnings process generates a downward sloping curve, while the fixed effect (α_{it}) generates a flat curve, along the whole distribution, so a simple mixing of the two processes cannot match the data well. However when the variance of the shocks of the autoregressive component declines with income, the fixed effects are mainly responsible for dispersion at the top, while shocks are mainly responsible for dispersion at the bottom, so the data and the model match.

Finally we set the interest rate on bonds to 2.5% and the discount factor to 0.97, so to generate, in the initial steady state, a wealth to income ratio of around 3.0.

5.7 Results

Once we have calibrated the model to the initial steady state, we consider a transition period. In particular we assume that starting in 1979 the parameter δ increases from 0, its steady state value, to 0.036, during a period of 30 years. This implies, for example, that during that period a household with earnings that are twice the mean ($s_i = 2$) grows at roughly 1% more per year than a household with earnings at the mean ($s_i = 1$). After 40 years the parameter δ reverts to 0. This parameter change is chosen so that the model exactly replicates

Figure 5: Initial steady state: data and model



the increase in earnings inequality (the increase in standard deviation of the s_i) documented in figure **??**. the other important change is a reduction in aggregate growth from 4.7% to 1.5% chosen so that the model matches the aggregate reduction growth. Our estimate of the reduction in the common factor of earnings growth is large, suggesting that changes such as technological slowdown (see, for example Gordon 2012), or the decline in labor share (see, for example, Elsby et al. 2013) have had an important effect on the evolution of the growth in labor earnings in the United States. Our results below suggest that this effect has been partly muted by the unequal growth in earning dynamics.

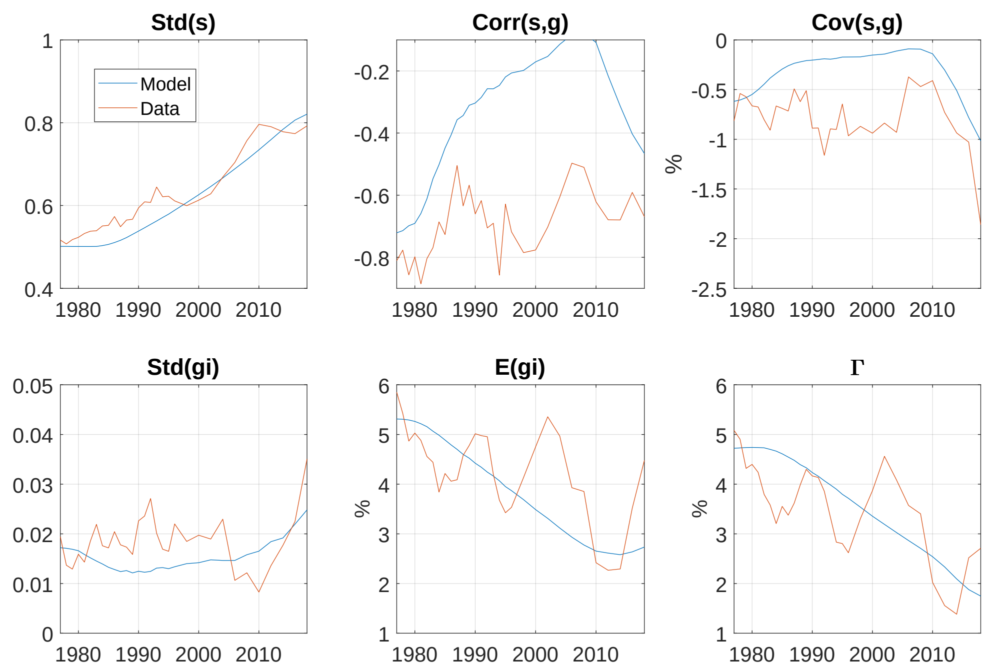
If that was the only change in the transition the model would imply a share of non participant households that would rise "too much" relative to the data. Low s_i households experience negative growth in their potential income which induce them not participate, so the share of non working households would rise too much relative to the data. For this reason in our baseline calibration we also change the time path for the transfer income ϕ_t so that along the transition the model has a consnat fraction of non working households set to 4%.[13]

[13] In order to match a constant fraction of non working households the model calls for a *decline* in the transfer income. The reason is that unequal growth would imply too much non participation, and we need a reduction in transfer income to induce households keep participation constant. We view this decline in transfer income as a reduced form way to capture an increasing incentive for labor force participation, which is particular relevant for women.

5.7.1 Growth impact

Figures 6 and 7 shows the time paths implied by the model and constrasti with PSID data. The figures suggests that the increase in unequal growth captures well the type of income dynamics in the data. Note that unequal growth is able to generate an increasing path for correlation between level and growth, together with a declining pattern for covariance between the two variables. Initially, as unequal growth takes place, it increases both the covariance and the correlation between income levels and growth.

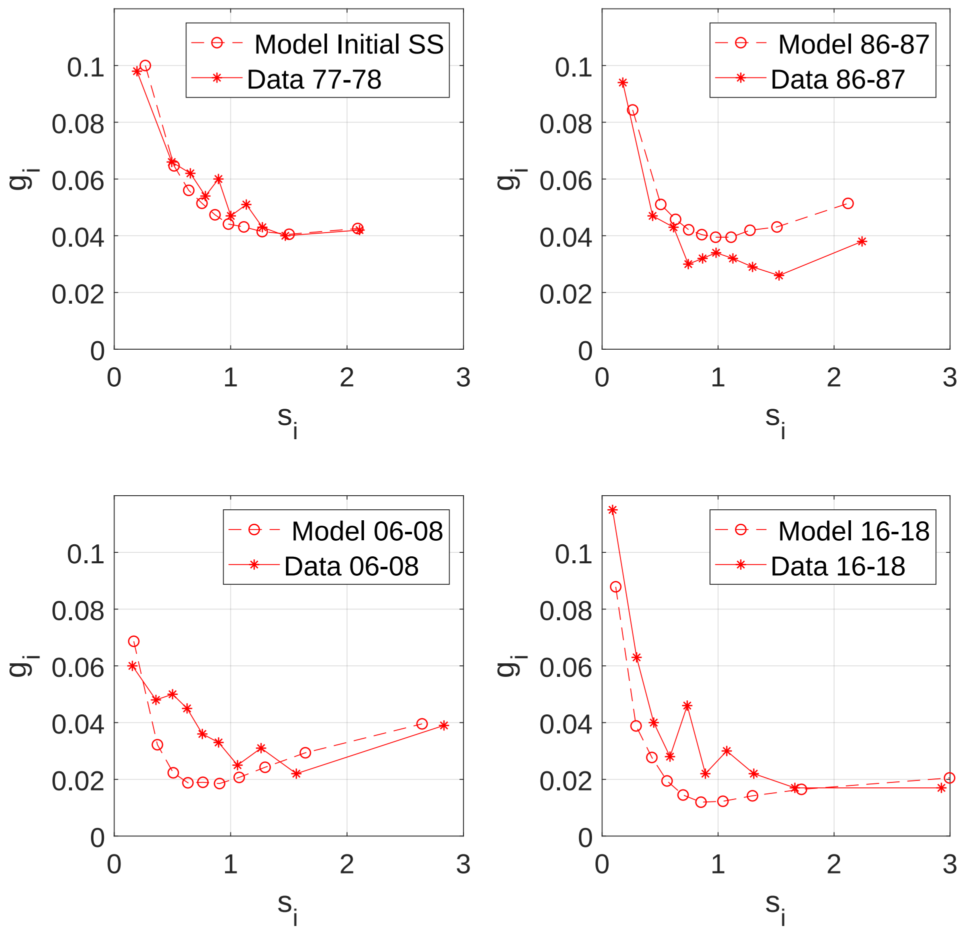
Figure 6: Time paths: data and model



As time goes by, more unequal growth results in poor agents experiencing larger shocks, because the variance of earning shocks increases when income falls, and because they move more between working and not working. These larger shocks at the bottom result in higher $\sigma(s_i)$ and higher $\sigma(g_i)$ which result in falling covariance.

Our final result involves assessing the aggregate impact of unequal growth. To do so, we simply compute aggregate growth during the transition with and without unequal growth and in Figure 8 we plot the differences between the two. The figure shows that unequal growth can account for an increase in growth, along the transition, of an average 0.25% per year.

Figure 7: Unequal Growth over time: data and model



5.7.2 Welfare

We conclude this section with an analysis of the welfare impact of "unequal growth", that is of the changes that are triggered by the new income dynamics. As is intuitive, the impact crucially depends on two factors: the curvature in utility, which in this class of models captures the social cost of consumption inequality, and the degree of market incompleteness. In Table 2 we measure the welfare cost (in lifetime consumption equivalent units) of moving from a steady state with equal growth, to an unequal growth equilibrium. In other words, the number in the table measure the percentage of lifetime consumption a household, under the veil of ignorance, is willing to give up to avoid the period of unequal growth. We consider two values of the risk aversion (2 and 4) and three market structures, complete markets (CM), bond economy (BE, the economy described above) and autarky (A), the economy in which household simply consume their (after transfer) earnings. In the bond and the complete markets economy the welfare numbers are computed assuming that households are surprised

Figure 8: Aggregate Impact of unequal growth

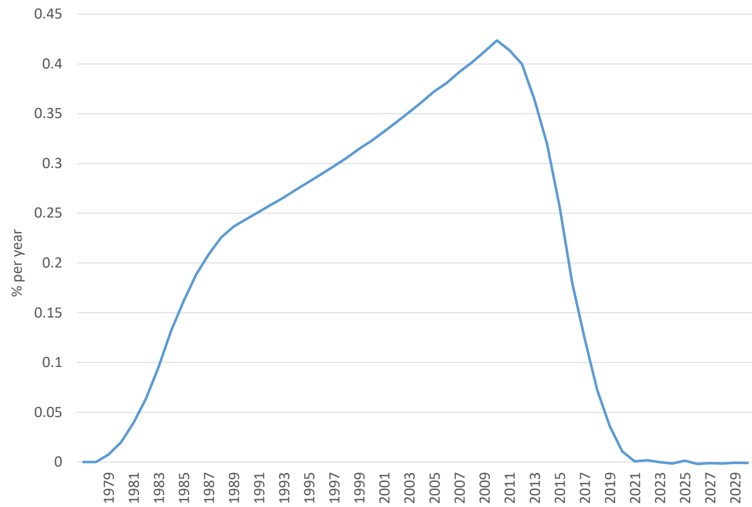


Table 2: Welfare costs after shocking the income process

	Market Structure		
Risk aversion (θ)	CM	BE	A
$\theta = 2$	-3.3%	+4.1%	+22.3%
$\theta = 4$	-1.6%	+18.5%	+63.6%

by changes in the growth factor every period (but expect them to be permanent).[14]

The table shows first that in complete markets unequal growth produce welfare gains. The logic is obviously that the benefits of the higher aggregate growth are shared among all households. Welafre gaing are declining with more curvature, and extra resources are value less with more curvature. When markets are incomplete, however, high earnings household benefit and low earnings lose, and this results in ex-ante welfare losses, that with curvature equal to 4 can be very substantial. It is useful to think of the losses in incomplete markets as arising from two features. The first is that poor agents experience negative growth and thus are stuck with permanently lower component of their income. The other is that with lower income, they also experience more volatile shocks. In financial autarky both these features affect welfare negatively, hence the large welfare losses. In the bond economy agents can (partly) insure against the more volatile shocks, but still suffer the adverse consequences of the permanently lower component of income and that explain why the welfare losses in the bond economy are also quite high. Another way to understand the large welfare losses in the

[14] In the autarky economy the welfare impact is independent on whether or not the changes in the income process are anticipated.

bond economy is that the process of unequal growth causes increase dispersion in "permanent income" (see Bowlus and Robin (2004), Abbott and Gallipoli (2019) and Straub (2019)) which translates in dispersion in consumption and in welfare losses. Note also that with when curvature is high ($\theta = 4$) the gap in the welfare impact of unequal growth between complete markets and incomplete markets gets very large. This is not surprising, but it highlights that a period of unequal growth increases the social value of better risk sharing or social insurance mechanisms.

6 Conclusions

We have shown that a statistical process for household earnings that involve more "unequal growth" , i.e. high earnings households growing (over time) faster and low earnings growing (over time) slower can account well for the evolution of the US earnings distribution over the past 50 years. We have also shown that more unequal growth has a mild (between 0.5 and 1% per year) positive effect on aggregate growth, and a potentially very large (as high as 50% of lifetime consumption) negative welfare effect, when markets are incomplete. The natural next question is what is the driver of this increase in unequal growth? For some times there has been a lot of very exciting work that thinks about sources of unequal growth (see, for two recent examples of such work, Fogli and Guerrieri 2019, Moll et al. 2019), and we believe that integrating our framework to these works can help us understand better the aggregate consequences of changes in the formation of individual earnings. We also find that, with the increase in unequal growth the social value of better (private or public) insurance mechanisms increase tremendously, and thus another relevant research direction is how to improve such mechanisms.

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